ROBUST MPC OF CONSTRAINED DISCRETE-TIME NONLINEAR SYSTEMS BASED ON ZONOTOPES

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Abstract

A robust MPC for constrained nonlinear system with uncertainties is presented. Uncertain evolution sets are used to predict the evolution of the system under any admisible uncertainty. The robust stability is guaranteed by a terminal and contractive constraint that drives the system to a robust positively invariant set. A new method to calculate the uncertain evolution sets is used. The method uses zonotopes to represent the uncertain sets.

1 Introduction

Model Predictive Control is a control strategy that has been widely adopted in industry and academia. The reason for this success is the ability to deal with constraints and multivariable systems [2].

When uncertainties are present, they must be taken into account in the computation of the control law in order to get robust stability. Some authors have tackled this problem as in [11] where a dual-mode receding horizon controller is proposed and robustness under decaying additive uncertainties is achieved by a proper choice of the terminal region. In [9] a robust MPC strategy based on H_∞ cost function is presented and in [13] a closed-loop min-max technique is shown.

In [10] a robust dual MPC based on uncertain evolution sets is presented. The control of a discrete nonlinear plant with additive uncertainty is considered. Uncertain evolution sets are used to represent all possible states of the system when uncertainty is considered. While traditional MPC use a nominal prediction to calculate the optimal input, in this case the prediction is based on the uncertain evolution sets. Obtaining the uncertain evolution sets to form the prediction is difficult when the system is nonlinear. In order to reduce the complexity of the computation, these sets are substituted by conservative approaches. Some of them are based on the linear differential inclusion of the non linear system [1][3][4]. In this paper, natural interval extension is used to bound the uncertain evolution of the system. When natural interval extension is used, an overestimation of the exact uncertain evolution set that is accumulative is produced. The consequence can be an unfeasible optimization problem. In this work, the uncertain evolution sets are approximated by zonotopes [7] that provide better approximation. In [10] the prediction horizon decreases each sample time. In this work, a new formulation is presented that provides a constant prediction horizon and hence, an improvement of the perfomance. The proposed controller is applied to a highly nonlinear system: an simulation model of a CSTR for a exothermic reaction. The results are compared to the controller proposed in [10].

2 Problem statement

2.1 System description

Consider an uncertain nonlinear discrete-time system of the form:

$$x_{k+1} = f(x_k, u_k, w_k) \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state of the system and $u_k \in \mathbb{R}^m$ is the control vector at sample time k. The vector $w_k \in \mathbb{R}^n$ is the uncertainty. It is assumed that the uncertainty is bounded in a compact set $w_k \in W$ that contains the origin. The system is subject to constraints on the state an on the control action: $x_k \in X$ and $u_k \in U$ where X is a closed set and U a compact set, both containing the origin.

The model given by $\hat{x}_{k+1} = f(x_k, u_k, 0)$ is the nominal model of the system. If x_k is the state of the system at sample time k, and given a sequence of control inputs denoted $u(k)_0^{N-1} = \{u(k|k), u(k+1|k), \dots, u(k+N-1|k)\}$, the sequence of future states of the nominal system is denoted $x(k)_0^N = \{x(k|k), \hat{x}(k+1|k), \dots, \hat{x}(k+N|k)\}$, where $x(k|k) = x_k$.

2.2 Uncertain evolution sets

Definition 1 *(Range). The range of a continuous function* $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ *over a set* $X \subset \mathbb{R}^n$ *is defined as* $f(X) = \{f(x) \mid x \in X\}$.

Definition 2 *(Exact uncertain evolution set). Consider a sys-*

tem given by (1), consider also that the state at sample time k is x_k and that a sequence of control inputs $u(k)_0^{j-1}$ is given, then the uncertain evolution set at sample time $k+j$ is $X_j = f(X_{j-1}, u(k+j-1|k), W)$ where $X_0 = \{x_k\}$.

Note that the set X_j, also denoted by $X(k+j|k)$, is the set of all states that can be reached by the evolution of the uncertain system at sample time $k+j$ applying the sequence control inputs $u(k)_0^{j-1}$.The exact computation of these sets is a difficult task due to the nonlinear nature of the model. In order to reduce the complexity of the computations of these sets, these can be substituted by conservative approaches. These approximate sets, denominated uncertain evolution sets, may be computed more easily.

Definition 3 *(Uncertain evolution set). Consider a system given by (1) and method $\Psi(\cdot,\cdot,\cdot)$ to bound the function $f(\cdot,\cdot,\cdot)$. Consider also that the state at sample k is x_k and that a sequence of control inputs $u(k_0^{j-1})$ is given, then the uncertain evolution set at sample time $k+j$ is $\tilde{X}_j = \Psi(\tilde{X}_{j-1}, u(k+j-1|k), W)$ where $\tilde{X}_0 = x_k$ and $\tilde{X}_i \subseteq X_i$, $i = 0,\ldots,j$.*

Direct natural interval extension has been used to calculate the uncertain evolution sets in [10]. Although it is an efficient solution, direct natural interval extension can produce a large overestimation of the exact uncertain evolution set. To obtain better approaches in this paper it is proposed to use zonotopes [7].

3 Kuhn's method for computing uncertain evolution sets

3.1 Interval arithmetic

An interval number $X = [a,b]$ is the set $\{\, x \,:\, a \le x \le b \,\}$ of real numbers between and including the endpoints a and b. Interval arithmetic is an arithmetic defined on sets of intervals, rather than sets of real numbers. The interval arithmetic is based on operations applied to sets of intervals.

Let $\mathbb{I}$ be the set of real compact intervals $[a,b]$ with $a,b \in \mathbb{R}$. Operations in $\mathbb{I}$ satisfy the expression:

$$A \; op \; B = \{\, a \; op \; b \,:\, a \in A, b \in B \,\} \tag{2}$$

In this way, the four basic interval operations [12] are:

$$[a,b] + [c,d] = [a+c, b+d] \tag{3}$$

$$[a,b] - [c,d] = [a-d, b-c]$$

$$[a,b] * [c,d] = [\min(ac,ad,bc,bd), \max(ac,ad,bc,bd)]$$

$$[a,b]/[c,d] = [a,b] * [1/d, 1/c], \text{ if } 0 \notin [c,d]$$

An extension of the interval arithmetic to include 0 in division can be found in [5]. The interval extension of standard functions $\{sin, cos, tan, arctan, exp, ln, abs, sqr, sqrt\}$ is possible too.

Definition 4 (Box) *A box is an interval vector. An interval hull of a set $X \subseteq \mathbb{R}^n$, denoted by $\Box X$, is a box that satisfies $X \subseteq \Box X$. Given a box $\Box X = ([a_1,b_1],\ldots,[a_n,b_n])^\top$, $mid(\Box X)$ denotes its center and $diam(\Box X) = (b_1 - a_1, \ldots, b_n - a_n)^\top$.*

Definition 5 (Interval Matrix) . *An interval matrix is a matrix whose components are intervals.*

Definition 6 (Natural interval extension) *If $f : \mathbb{R}^n \to \mathbb{R}$ is a function computable as an expression, algorithm or computer program involving the four elementary arithmetic operations interspersed with evaluations of standard functions then, a natural interval extension of f, denoted $\Box f$, is obtained replacing each occurrence of each variable by the corresponding interval variable, by executing all operations according to formulas (3) and by computing ranges of the standard functions.[6]*

Theorem 1 *A natural interval extension $\Box f$ of a continuous function $f : \mathbb{R}^n \to \mathbb{R}$ over a box $X \subseteq \mathbb{R}^n$ satisfies that $f(X) \subseteq \Box f(X)$. This is the fundamental theorem of the interval arithmetic [12].*

Theorem 2 (Mean Value Theorem) *Let $f : \mathbb{R}^n \to \mathbb{R}$ be differentiable at every point in an open set containing the line segment L joining two vectors $x, y \in \mathbb{R}^n$. There is a vector $x_0 \in L$ such that: $f(x) - f(y) = \nabla f(x_0)(x-y)$.*

Definition 7 (Minkowski sum) *The Minkowski sum of two sets X and Y is defined by $X \oplus Y = \{\, x+y \,:\, x \in X, y \in Y \,\}$.*

Definition 8 (Mean value extension) *Suppose a function $f : \mathbb{R}^n \to \mathbb{R}$ with continuous derivatives in $X \in \mathbb{I}^n$. Suppose also a real vector $c \in X$. Then, the mean value extension for f over X is defined by $f(X) \subseteq f(c) \oplus \Box\nabla_x f(X)(X - c)$, where $\Box\nabla_x f(X)$ is an interval enclosure for the range of $\nabla_x f(X)$ over X.*

3.2 Künh´s method

The Künh´s method is a procedure that allows us to bound the orbits of discrete dynamical systems [7]. The evolution of the system is approximated by a high order zonotope. A zonotope is the Minkowski sum of a set of parallelepipeds. In [7] sub-exponential overestimation is proven. These concepts are defined below.

Definition 9 (Zonotope of order m) *A zonotope Z of order m is the Minkowski sum of m parallelepipeds: $Z = P_1 \oplus P_2 \oplus \ldots \oplus P_m$*

A parallelepiped is a linear image $P = MI$ where M is a square matrix and I is the unitary box. The order m is a measure for the geometrical complexity of the zonotopes.

The uncertain evolution sets at time k are represented by $\tilde{X}_j = z_j + Z_j$ where $z_j \in \mathbb{R}^n$ is a real vector and Z_j a centered zonotope. The exact calculation of the next uncertain evolution set

$X_{j+1} = f(X_j, u, W)$ is a difficult task. It is possible to approximate X_{j+1} by a bound $\tilde{X}_{j+1}$ where the non lineal function $f(\cdot)$ is substituted by a lineal transformation.

Theorem 3 *Let X_j be a set, W a box, T a matrix, $\tilde{X}_j = z_j + Z_j$ a zonotope such that $X_j \subseteq \tilde{X}_j$ and a system given by (1). Then $X_{j+1} = f(X_j, u_j, W) \subseteq \tilde{X}_{j+1}$ where $\tilde{X}_{j+1}$ is a zonotope that is given by $\square f(z_j, u_j, W) \oplus TZ_j \oplus (\square \nabla f(\square \tilde{X}_j, u_j, W) - T)\square Z_j$*

Proof:

Let

$$X_{j+1} = f(X_j, u_j, W) \subseteq f(\tilde{X}_j, u_j, W) \subseteq$$

$$f(z_j, u_j, W) \oplus \nabla f(\tilde{X}_j, u_j, W)(\tilde{X}_j - z_j) =$$

$$f(z_j, u_j, W) \oplus (\nabla f(\tilde{X}_j, u_j, W) - T + T)Z_j \subseteq$$

$$f(z_j, u_j, W) \oplus TZ_j \oplus (\nabla f(\tilde{X}_j, u_j, W) - T)Z_j \subseteq$$

$$\square f(z_j, u_j, W) \oplus TZ_j \oplus (\square \nabla f(\square \tilde{X}_j, u_j, W) - T)\square Z_j$$

Note that $\tilde{X}_{j+1}$ is a zonotope because the Minkowski sum of two zonotopes is a zonotope. So it can be rewritten as $\tilde{X}_{j+1} = z_{j+1} + Z_{j+1}$ where $z_{j+1} = mid(\tilde{X}_{j+1})$ and $Z_{j+1} = \tilde{X}_{j+1} - z_{j+1}$. A possible selection for T is $mid(\square \nabla f(\square \tilde{X}_j, u_j, W) - T))$. ■

Note that this inclusion is not very conservative because it makes a sort of linearization of the range of the function. With this proposal, at each sample time, the order of the zonotope is increased. The computational cost increases quadratically, so it is interesting to dispose of an algorithm to bound a high order zonotope by a lower order one. This algorithm can be found in [7].

4 Robust MPC

4.1 Robust MPC dual

If the system is uncertain, then the stability, and probably the feasibility, may be lost. In order to achieve robustness, the controller must stabilize the system and satisfy robustly the constraints for all possible realizations of the uncertainty along the prediction horizon.

This section presents a dual predictive controller [11]. These controllers split the space into two parts. One of them is a control invariant set around the origin that constitutes the terminal region Ω. In the terminal region a local control law is used $u = h(x)$. In the rest of the state space, a predictive controller with terminal region Ω is used.

The predictive controller has finite horizon N. The first of the sequence of controls signal calculated solving the optimization problem is applied each sample instant. In the prediction the uncertainties are included, obtaining the uncertain evolution sets presented. So, if in the initial state, the optimization problem is feasible then there is a sequence of control signals that drives the system to the terminal region in M sample instants. This means that the future feasibility of the optimization problem is guaranteed.

The optimization problem at instant k is represented by $P(x_k, k)$ and it is defined by:

$$P(x_k, k) = \min_{u(k)_0^{N-1}} J(k, x(k)_0^N, u(k)_0^{N-1})$$

$$= \sum_{i=0}^{i=N-1} L(\hat{x}(k+i|k), u(k+i|k)) + V(\hat{x}(k+N)|k)$$

subject to:

$$u(k+j|k) \in U, \; j = 0, \ldots, N$$

$$\tilde{X}_j(x_k, u_F(k)) \subseteq X, \; j = 1, \ldots, M$$

$$\tilde{X}_M(x_k, u_F(k)) \subseteq \Omega$$

$$J_{EXT}(k, \tilde{X}(k)_0^M, u(k)_0^{M-1}) -$$

$$J_{EXT}(k-1, \tilde{X}(k-1)_0^M, u^*(k-1)_0^{M-1}) < -\alpha \; if k > 0$$

where $L(\cdot,\cdot)$ is the positive definite state cost, $V(\cdot)$ is a positive definite terminal cost, $\alpha \in \mathbb{R}, \alpha > 0$ is a parameter that it will be defined below and $J_{EXT}(\cdot,\cdot,\cdot)$ is a cost function to assure convergence to the terminal region and represents the cost of the uncertain prediction horizon that is out of the terminal region. It is defined by:

$$J_{EXT}(k, \tilde{X}(k)_0^M, u(k)_0^{M-1}) = J_{EXT}(k, u(k)_0^{M-1}) =$$

$$L_{EXT}(x_k) + sup(\sum_{i=0}^{i=M} \square L_{EXT}(\square \tilde{X}_i))$$

with $L_{EXT}(\cdot)$ a positive definite state cost function. $J_{EXT}(\cdot)$ is evaluated with interval arithmetic to obtain an upper bound. The parameter α is a scalar such that $\alpha < L_{EXT}(x) \; \forall x \in X, x \notin \Omega$. The optimization algorithm, to obtain a smaller cost as possible, adjusts the parameter M.

A new constraint is added to the optimization problem when $k > 0$. This constraint assures convergence to the terminal region. As it will be shown in the next section, this contractive constraint assures convergence to the terminal region. The system reaches the terminal region in a finite number of sample instants. The constraints are applied to the uncertain evolution sets so, given an initial state and a sequence of control signals, it is assured that the state satisfies the constraints for any uncertainty considered. The dual controller applies the control signal $u_k = K_{MPC}^d(x_k)$ at time k. The next algorithm calculates the control signal:

ControllerAlgorithm(k)

Alg

if $x_k \in \Omega$ $u = h(x_k)$

if $x_k \notin \Omega$

if $(k = 0)$

$u^*(k)_0^{N-1} = P(x_0, 0)$

else

$u^F(k)_0^{N-1} = u^*(k)_1^{N-1}, h(\hat{x}(N|k-1))$

$u^0(k)_0^{N-1} = P(x_k, k)$

if $J(k, x(k)_0^N, u^F(k)_0^{N-1}) < J(k, x(k)_0^N, u^0(k)_0^{N-1})$

$u^*(k)_0^{N-1} = u^F(k)_0^{N-1}$

else

$u^*(k)_0^{N-1} = u^0(k)_0^{N-1}$

endif

endif

$u = u^*(0|k)$

End

4.2 Stability analysis

Since the uncertainties are merely bounded and they may not be decaying, the origin is not a steady state of the uncertain system. Hence, the aim of a stabilizing controller is to steer to a neighborhood of the origin and keep the state evolution in it. This set is a robust positively invariant set for the closed loop system, and its size depends on the bound on the uncertainties.

The controller proposed in this paper steers the uncertain system to the terminal region, which is a robust invariant set.

Assumption 1 *There is a region $\Omega \subseteq X$ such that it is a robust positively invariant set for the uncertain system and the associated local control law $u = h(x) \in U$ for all $x \in \Omega$.*

Theorem 4 *Consider a system given by (1). Consider a robust invariant set for the system Ω with an associated local controller $u = h(x) \in U$ such that the assumption is satisfied. Consider the proposed procedure to compute the uncertain evolution sets, then the system controlled by $u_k = K^d_{MPC}(x_k)$ is ultimately bounded for all x_0 such that the optimization problem $P_0(x_0, 0)$ is feasible.*

Proof:

To prove theorem 4 two lemmas are enunciated below.

Lemma 1 *Let $\alpha \in \mathbb{R}, \alpha > 0$ be such that $\alpha < L_{EXT}(x)$ $\forall x \in X, x \notin \Omega$. If $\forall k$ $J_{EXT}(k, \tilde{X}(k)_0^M, u(k)_0^{M-1}) - J_{EXT}(k-1, \tilde{X}(k-1)_0^M, u^*(k-1)_0^{M-1}) < -\alpha$ for every $x_{k-1} \notin \Omega$, then there exists N_α such that $x_{N_a} \in \Omega$.*

This can be proved by means of the following relation:

$$0 < J_{EXT}(k, u^*(k)_0^{M-1}) < J_{EXT}(k-1, u^*(k-1)_0^{M-1}) - \alpha <$$
$$\ldots < J_{EXT}(0, u^*(0)_0^{M-1}) - k\alpha$$

There is a $i < \frac{J_{EXT}(0)}{\alpha}$ that makes the cost function $J_{EXT}(i)$ negative, which is a contradiction, therefore, $x_i \in \Omega$. The value α can be considered a performance parameter that must be chosen conveniently in order to assure feasibility.

Lemma 2 *If $P(x_0, 0)$ is feasible then, the constraint $J_{EXT}(k, \tilde{X}(k)_0^M, u^*(k)_0^{M-1}) - J_{EXT}(k-1, \tilde{X}(k-1)_0^M, u^*(k-1)_0^{M-1}) < -\alpha$ with $\alpha \in \mathbb{R}, \alpha > 0$ and $\alpha < L_{EXT}(x)$ $\forall x \in X, x \notin \Omega$, is fulfilled for the closed loop evolution of the system out of the terminal region.*

In effect,when the algorithm selects $u^0(0|k)$, the constraint is satisfied by the optimization problem definition. If the algorithm selects $u^F(0|k)$, each uncertain evolution region can be built by the expression $\tilde{X}(i|k) = \tilde{X}(i|k) \bigcap \tilde{X}(i+1|k-1)$ and then $\tilde{X}(i|k) \subseteq \tilde{X}(i+1|k-1)$. The consequence of this is that the constraint $J_{EXT}(k, u^*(k)_0^{M-1}) - J_{EXT}(k-1, u^*(k-1)_0^{M-1})$is satisfied. So if $\alpha < L_{EXT}(x)$ $\forall x \in X, x \notin \Omega$ then the constraint is fulfilled if $P_0(x_0, 0)$ is feasible.

■

Note that the stability is guaranteed by the feasibility of the computed control action at each sample time. The optimality is not required and a suboptimal solution of the optimization problem suffices to guarantee stability. Furthermore, the election of the cost function affects only the performance but not the stability of the closed-loop system.

5 Example

The proposed MPC controller is applied to a highly nonlinear system: a continuos stirred tank reactor (CSTR) simulation model. The continuous time model of a CSTR for an exothermic, irreversible reaction $A \longrightarrow B$ with constant liquid volume is given by [8] :

$$\frac{d\,C_A}{dt} = \frac{q}{V} \cdot (C_{Af} - C_A) - k_0 \cdot \exp\left(-\frac{E}{R \cdot T}\right) \cdot C_A$$

$$\frac{d\,T}{dt} = \frac{q}{V} \cdot (T_f - T) - \frac{\Delta H \cdot k_0}{\rho \cdot C_p} \cdot \exp\left(-\frac{E}{R \cdot T}\right) \cdot C_A + \frac{U \cdot A}{V \cdot \rho \cdot C_p} \cdot (T_c - T)$$

where C_A is the concentration of A in the reactor, T is the reactor temperature and T_c is the temperature of the coolant stream. The parameters of the model are: $\rho = 1000$ g/l, $C_p = 0.239$ J/g K, $\Delta H = -5 \times 10^4$ J/mol, $E/R = 8750$ K, $k_0 = 7.2 \times 10^{10}$ min^{-1}, $U \cdot A = 5 \times 10^4$J/min K. The nominal operating conditions are given by: $q = 100$ l/min, $T_f = 350$ K, $V = 100$ l, $C_{Af} = 1.0$ mol/l. The steady state is $C_A^o = 0.5$ mol/l, $T^o = 350$ K, $T_c^o = 300$ K. The temperature of the coolant is constrained to $280K \leq T_c \leq 370$. The state of the system is defined as $x = [C_A - C_A^o, T - T^o]^T$, and the input as $u = T_c - T_c^o$.

The model is discretized with a sampling period $T_s = 0.03$ min. The additive uncertainty on the discrete-time model of the system are bounded by

In Fig 1 the solution of $P(x_0, 0)$ is shown. It can be seen the cloud of points that represent the exact uncertain evolution set and two sequences of uncertain evolution sets. The first sequence are boxes calculated by natural interval extension [10]. The overestimation makes $P(x_0, 0)$ infeasible $\tilde{X}_{11} \not\subset \Omega$. The second sequence are zonotopes calculated by Kühn's method. $P(x_0, 0)$ is now feasible because the approximation to the exact uncertain evolution set is better and $\tilde{X}_{11} \subseteq \Omega$. The result is tighter envelopes that enlarge the feasible region. The proposed method is a better solution to calculate uncertain evolution sets.

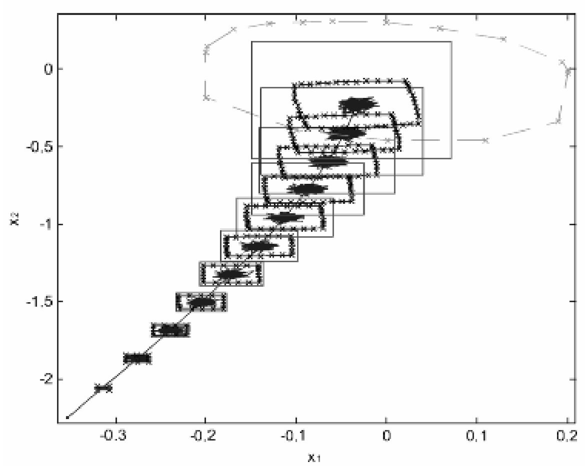

Figure 1: Uncertain evolution sets

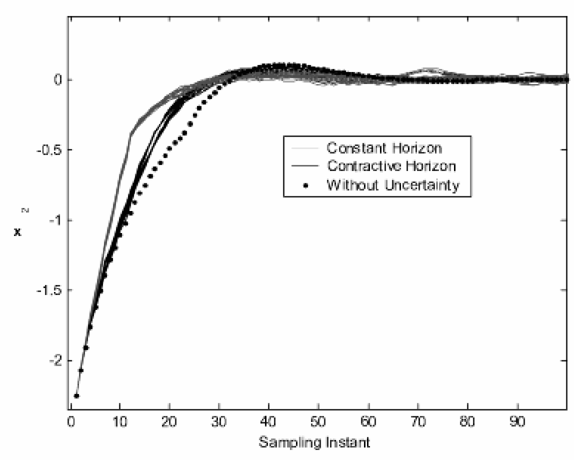

Figure 2: Closed loop trajectories

In Fig 3 closed loop trajectories of the system are shown. A nominal trajectory without uncertainties is shown. Two kind of trajectories with random bounded uncertainties are calculated too. The first kind uses the contractive horizon proposed in [10], the second uses the fixed horizon proposed here. The conclusion is that a fixed horizon provides a better performance. For all the uncertainties, the optimization problem is feasible and hence, the system is steered to Ω despite the uncertainties.

6 Conclusion

A robust dual-mode MPC controller for constrained discrete-time nonlinear system with uncertainties has been presented. Uncertain evolution sets has been added to the MPC optimization problem. Terminal and contractive constrains have been considered to obtain robust stability. A new technique for the computation of the uncertain evolution set has been applied. This technique is based on zonotopes. The proposed controller has been compared to the controlled presented in [10].

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