ADAPTIVE VISION-BASED PATH FOLLOWING CONTROL OF A WHEELED ROBOT

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Abstract

This paper addresses the problem of vision-based path following control for a wheeled robot of the unicycle type. A non-linear controller is derived that yields convergence of the vehicle's position to a straight line in the presence of model parameter uncertainty. Control design starts with a kinematic model of the system that describes the relationship between robot inputs and the pose of the robot with respect to the path directly in the camera plane, using selected path features. A kinematic control is then derived to drive path following errors to zero. The kinematic controller is further extended to cope with vehicle dynamics by resorting to backstepping and Lyapunov based techniques. Robustness to system parameter uncertainty is addressed by incorporating a parameter adaptation scheme. Simulations results illustrate the performance of the controller derived.

1 Introduction

This paper addresses the problem of vision-based path following control of a wheeled robot of the unicycle-type. The reader is referred to [3], [6], [7], [8], [11] and the references for previous work on *visual servoing*. See also [1], [2], [5], [10] [14] for important results on the general problem of *path following* for wheeled robots. In the paper, basic concepts and techniques from these two areas are used to derive an adaptive non-linear controller that deals explicitly with vehicle dynamics and yields convergence of the vehicle's position to a straight line in the presence of camera/robot model parameter uncertainty.

Control design starts with a kinematic model of the system that describes the relationship between robot inputs (linear and rotational speeds) and the pose of the robot with respect to the path directly in the camera plane, using selected path features. A kinematic control is then derived to drive path following errors to zero. The kinematic control is further extended to cope with vehicle dynamics by resorting to backstepping and Lyapunov based techniques ([9], [12], [13],[14]). Robustness to camera and plant parameter uncertainty is addressed by incorporating a parameter adaptation scheme. In this set-up,the camera that is mounted on the robot is simply viewed as a *non-linear sensor that provides information of the relative position and attitude of the mobile robot with respect to an observed feature*. The relationship between robot inputs and feature-related path following errors is naturally catpured in the so-called Interaction Matrix (see ([4], [6], [7] for examples of the computation of the Interaction Matrix in a general setting). In this paper, this matrix is found by direct derivation, thus allowing for an intuitive interpretation of its structure using remarkable geometrical relations.

The main contribution of the paper is threefold:

- controller design is done by using directly variables available in the camera plane (that is, no explicit transformation of variables from the camera to an inertial frame is performed).
- the controller derived deals explicitly with vehicle dynamics.
- an adpation scheme is included in the path following control law proposed, in order to cope with camera and robot parameter uncertainty.

The paper is organized as follows: Section 2 formulates the problem of path following of a wheeled robot using vision. Section 3 derives an adaptive path following controller. Section 4 includes the results of simulations. Finally, Section 5 contains the main conclusions and describes problems that warrant further consideration.

2 Problem formulation

This section introduces some basic notation, presents the kinematic equations of motion of a mobile robot equipped with a camera, and formalizes the problem of driving the robot along a desired path. The first part (2.1) of the section summarizes the notation that will be used throughout the paper. The second part (2.2) summarizes the kinematic equations of a wheeled robot of the unicycle type depicted in figure 1. The third part (2.3) deals with the vision sensor and establishes the interaction relation between the parameterization space and the Cartesian space. Finally, the last part (2.4) formulates the problem of finding a controller that yields convergence of the robot to the path.

2.1 Notation

Throughout this paper, the following notation will be used.

- The symbol $\{R_A\} := \{x_A, y_A, z_A\}$ denotes a reference frame with origin O_A. We let $\{R_0\}$, $\{R_B\}$, $\{R_C\}$ and $\{R_{Im}\}$ be inertial, body axis, camera, and image frames, respectively.
- $P|_A$ denotes the position of a point P in frame $\{R_A\}$.
- $O_B|_0 = [x_0\ y_0\ 0]^T$ is the position of the origin of $\{R_B\}$ with respect with $\{R_0\}$ (i.e. inertial position of the robot).
- α denotes the orientation of $\{R_B\}$ with respect to $\{R_0\}$ (i.e. yaw angle).
- v, w are the linear and angular velocities of the robot, respectively, with respect to $\{R_0\}$, resolved in $\{R_B\}$.
- $p_{im}^{pol} = [\rho\ \theta]^T$ are polar parameters of a straight line as seen in the image plane, resolved in $\{R_{Im}\}$.
- $p_{im}^{cart} = [a\ b]^T$ are cartesian parameters of the path, resolved in $\{R_0\}$.
- Given two frames $\{A\}$ and $\{B\}$, R_A^B denotes the rotation matrix from $\{A\}$ to $\{B\}$.

Note that since the study is made in the horizontal plane, then $z = 0$ for any point attached to the robot or the camera, and $z = -h$ for any point attached to the ground. Furthermore, $\dot{z} = 0$.

2.2 Robot kinematic model

The vehicle has two identical parallel, non-deformable rear wheels that are controlled by two independent motors, and a steering front wheel. It is assumed that the plane of each wheel is perpendicular to the ground and that the contact between the wheels and the ground is pure rolling and non-slipping, i.e., the velocity of the center of mass of the robot is orthogonal to the rear wheel axis. By assuming that the wheels do not slide, a non-holonomic constraint on the motion of the mobile robot of the form $\dot{x}\sin\alpha - \dot{y}\cos\alpha = 0$ is imposed. It is further assumed that the masses and inertias of the wheels are negligible and that the center of mass of the mobile robot is located in the middle of the axis connecting the rear wheels. Each rear wheels is powered by a motor which generates a control torque $\tau_i, i = 1, 2$.

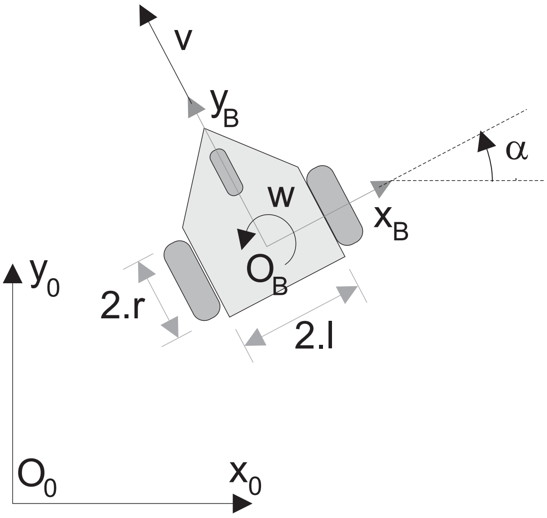

Figure 1: A wheeled robot of the unicycle type

With the above notation, the kinematics of the mobile robot are modeled by

$$\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\sin\alpha & 0 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} v \\ w \end{bmatrix} \tag{1}$$

2.3 Vision sensor: the interaction matrix

For the sake of simplicity we assume that the vehicle is equipped with a camera that is rigidly attached to it and points downwards. The image plane $\{x_{Im}, y_{Im}\}$ is parallel to the ground and the camera is mounted at the center of mass of the vehicle. Assume the simple pin-hole model for the camera, see figure (2), where $\{R_C\}$ and $\{R_{im}\}$ denote the camera and image frame, respectively.

2.3.1 Projective model

Define $\gamma = f/h$, where f is the camera focal length and h the distance of the camera above ground. Let $\Pi : \Re^3 \to \Re^2$ be a projection operator defined as $\Pi([x\ y\ z]^T) = [x\ y]^T$. Further let $P|_C = [x_C^P\ y_C^P\ \ -h]^T$ be the coordinates of a point on the ground, expressed in the camera frame. Its projection P_{Im} on the image plane is the end point of vector

$$P_{Im} = [x_{Im}^P\ y_{Im}^P]^T = \Pi\gamma R_0^C (P|_0 - O_C|_0)$$

where R_0^C is the rotation matrix from $\{R_0\}$ to $\{R_C\}$ and $O_C|_0$ is the position of the origin of $\{R_C\}$ with respect to $\{R_0\}$.

2.3.2 Interaction matrix

Consider a straight line D_T (path) on the ground. Then, there exists a parameterization vector $p_{Terrain}^{cart} = [a\ b]^T$ such that for every $M|0 = [x_0^M y_0^M 0]^T \in D_T$ the equality $D_T(M|_0, p_{Terrain}^{cart}) := ax_0^M + b - y_0^M = 0$ holds. In this case, the camera image of D_T is also a straight line D_{Im} with two

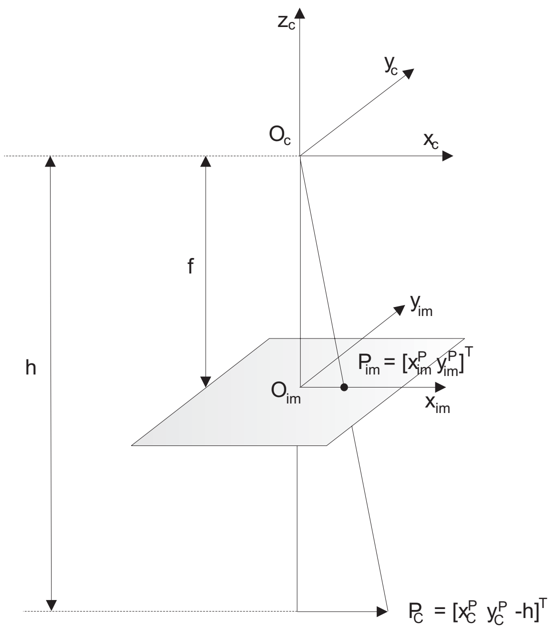

Figure 2: Coordinate frame for the vision system

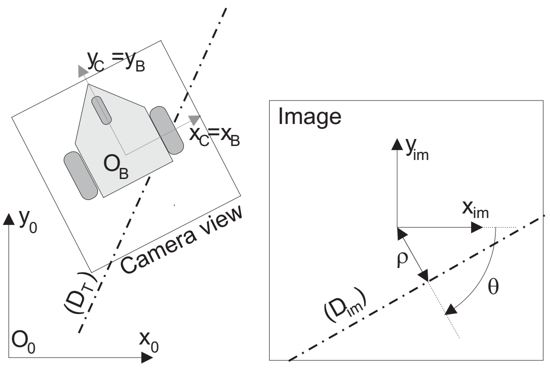

Figure 3: Image feature parameterization

possible parameterizations $p_{Im}^{pol} = [\rho\ \theta]^T$ and $p_{Im}^{cart} = [A\ B]^T$ such that for every point in the image plane such that $M_{Im} = [x_{Im}^M\ y_{Im}^M]^T \in D_{Im}$, the two following equalities hold:

$$M_{Im} \in D_{Im} \Leftrightarrow$$
$$\left(\begin{array}{lcl} D_{Im}(M_{Im}, p_{Im}^{cart}) & := & Ax_{Im}^M + B - y_{Im}^M = 0 \\ D_{Im}(M_{Im}, p_{Im}^{pol}) & := & x_{Im}^M \cos\theta + y_{Im}^M \sin\theta - \rho = 0 \end{array} \right)$$

As pointed out in [7] the parameterization p_{Im}^{cart} is inadequate. We therefore select the parameterization $p_{Im}^{pol} = [\rho\ \theta]^T$, where the ambiguity is partly overcome by fixing the sign of ρ. A geometrical analysis of the figure (3) yields

$$\left(\begin{array}{lcl} \rho & = & sign(a)\gamma\frac{ax_0+b-y_0}{\sqrt{1+a^2}} \\ \theta & = & -[\arctan\frac{1}{a} + \alpha] \end{array} \right. \quad (2)$$

It is now straightforward to compute the Jacobian Matrix L_0 that relates the inertial velocity of the robot with the rate of evolution of the image feature parameters to obtain

$$\begin{bmatrix} \dot\rho \\ \dot\theta \end{bmatrix} = \begin{bmatrix} \gamma\frac{asign(a)}{\sqrt{1+a^2}} & \gamma\frac{-sign(a)}{\sqrt{1+a^2}} & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \dot x_0 \\ \dot y_0 \\ \dot\alpha \end{bmatrix}$$

Using (1) and noting that $a\sin\alpha + \cos\alpha = -sign(a)\sin\theta\sqrt{1+a^2}$ allows for the computation of a second Jacobian Matrix L_C that relates the robot velocity in body frame with the rate of evolution of the image feature parameters, that is,

$$\begin{bmatrix} \dot\rho \\ \dot\theta \end{bmatrix} = L_C \begin{bmatrix} v \\ w \end{bmatrix} \quad (3)$$

with

$$L_C = \begin{bmatrix} \gamma\sin\theta & 0 \\ 0 & -1 \end{bmatrix} \quad (4)$$

Matrix L_C, henceforth referred to as the Interaction Matrix, will be used extensively and can be considered as the kinematic model of the complete sytstem (robot + camera). Note that if the camera is placed elsewhere, the interaction matrix becomes

$$L_C = \begin{bmatrix} \gamma\sin\theta & h(\theta) \\ 0 & -1 \end{bmatrix}$$

with $h(\theta) = \gamma d\sin(\beta + \theta - \phi)$, where d is the distance between the vehicle's center of mass and the center of the camera, β is the angle between x_B and x_{im}, and ϕ is the angle between x_B and $\overrightarrow{O_BO_C}$.

2.4 Path following: problem formulation

Equipped with this formalism, we now state the kinematic control problem $\mathbf{C}_1$ that is addressed in section 3.1.

$\mathbf{C}_1$: *Given the robot model (3) where $v : \mathbb{R}_+ \to \mathbb{R}$ is an arbitrary function such that $\lim_{t\to\infty} v(t) \neq 0$, compute a feedback control law for w such that ρ and θ converge to 0, as t goes to ∞.*

This problem will be extended in the sub-section 3.2 to deal with vehicle dynamics. The classic unicycle dynamic model is given by

$$\left\{ \begin{array}{l} \dot v = \frac{F}{m} \\ \dot w = \frac{\Gamma}{I} \end{array} \right. \quad (5)$$

where m is the mass of the vehicle, I its moment of inertia, and F and Γ are the forward force and moment torque applied by the wheel motors, respectively.

The dynamic control problem $\mathbf{C}_2$ is stated as follows.

$\mathbf{C}_2$: *Given the robot models (1) and (5) and a set of camera and robot related measurements, compute $U_{dyn} = [F(.)\ \Gamma(.)]^T$ so that ρ and θ converge to 0 as t goes to ∞.*

Notice that the dynamic model does not include the influence of unknown external disturbances. In fact this paper does not include the problem of external disturbance attenuation. However, it deals explicitly in section 3.3 with the problem of robustness against parameter uncertainty, where the uncertainty are confined to the parameters r (radius of the wheels), $2.l$ (distance between the wheels), m (vehicle mass), I (vehicle moment of inertia), and γ (camera aspect ratio). In order to include these parameters in the dynamic model, (5) is rewritten as

$$\left\{ \begin{array}{l} \dot{v} = \frac{u_{com}}{p_2} \\ \dot{w} = \frac{u_{dif}}{p_1} \end{array} \right. \text{ where} \left\{ \begin{array}{l} u_{dif} = \tau_1 - \tau_2 \\ u_{com} = \tau_1 + \tau_2 \end{array} \right. \tag{6}$$

with $p_2 = mr$, $p_1 = rI/l$, aand τ_1 and τ_2 are the left and right wheel torques, respectively. The adaptive dynamic control problem $\mathbf{C}_3$ can now be stated as follows.

$\mathbf{C}_3$: *Consider the kinematic model (3) together with the relations (6). Assume that parameters p_1, p_2 and γ are unknown. Compute an adaptive feedback control law $U_{adp} = [u_{com}\ u_{dif}]^T$, so that ρ and θ converge to 0, as t goes to ∞.*

A proposed solution of the $\mathbf{C}_i$, (i=1, 2, 3) problems is described in the following section.

3 CONTROLLER DESIGN

This section describes the solutions to problems $\mathbf{C}_1$, $\mathbf{C}_2$, and $\mathbf{C}_3$ stated above. The proofs of the theorems to follow are omitted in this paper. The interested reader is refered to [14] and [15] for related results.

3.1 Kinematic Controller

A solution to problem $\mathbf{C}_1$ is stated next.

Theorem 1: *Consider the kinematic model (3) and let $\delta(\rho)$ be a desired approach angle defined by*

$$\delta(\rho) = -sign(v)\theta_a \frac{e^{2k_\delta \rho} - 1}{e^{2k_\delta \rho} + 1} \tag{7}$$

where k_δ is a positive gain and $\theta_a \leq \pi/2$. Further assume that measurements of $[\rho\ \theta]^T$ are available from the camera. Then the control law described by

$$U_{kin}: \ w \ = \ K_\theta(\theta - \delta) - \dot{\delta} \tag{8}$$

solves the $\mathbf{C}_1$ problem.

The proof relies on the use of the Lyapunov function

$$V_1 = \frac{1}{2}(\theta - \delta)^2 \tag{9}$$

and the fact that $\dot{V}_1 = -K_\theta(\theta - \delta)^2 \leq 0$ (see [14] and [15]). Note : the approach angle is instrumental in shaping the approach to the path (see [10])

3.2 Dynamic Controller

The dynamic controller aims at driving the actual values of variables v and w to v_d and the "desired kinematic" profile specified by (8), respectively. Its design relies on standard backstepping techniques and leads to the controller structure described below.

Theorem 2: *Consider models (3) and (5). Let $\delta(\rho)$ be a desired approach angle defined in (7). Let v_d and $\dot{v}_d$ be the desired forward velocity and acceleration, respectively. Further assume that measurements of $[\rho\ \ \theta]^T$ are available from the camera and measurements of $[v\ \ w]^T$ are available from robot sensors. Then, the control law*

$$U_{dyn} = \left(\begin{array}{lcl} \Gamma & = & I \left[\begin{array}{l} -\ddot{\delta} + (K_\theta + K_c)(\dot{\theta} - \dot{\delta}) \\ +(1 + K_\theta K_c)(\theta - \delta) \end{array} \right] \\ F & = & m\dot{v}_d - K_v(v - v_d) \end{array} \right. \tag{10}$$

solves the $\mathbf{C}_2$ problem. Furthermore $\lim_{t\to\infty}(v - v_d) = 0$.

Notice that the control law naturally blends camera with robot motion measurements.

3.3 Adaptive Dynamic Controller

The derivation of the adaptive controller is done in a sequence of steps that start with the kinematic controller of (8). This controller is first modified to deal with camera parameter uncertainty, leading to a kinematic control law denoted U_{kin}^{adp}. This control law is then extended to deal with vehicle dynamics, using the strategy that allows transition from U_{kin} in theorem 1 to U_{dyn} in theorem 2. Finally the dynamic controller is modified to deal with vehicle parameter uncertainty. To perform the first step, define $\Delta\gamma = \gamma - \gamma^R$, where γ^R and γ are the actual and estimated value of the fundamental camera parameter, and rewrite the kinematic controller in (7) as

$$w = K_\theta(\theta - \delta) - \delta' \sin\theta v \gamma \ ; \ \delta' = \frac{\partial \delta}{\partial \rho}$$

Consider the Lyapunov function V_1 in (9). Then,

$$\dot{V}_1 = -K_\theta(\theta - \delta)^2 + \delta' \sin\theta v \Delta\gamma$$

that is, $\dot{V}_1$ is indefinite. To overcome this problem, construct the new candidate Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\frac{(\Delta\gamma)^2}{\lambda_\gamma} \ ; \ \lambda_\gamma > 0.$$

Straightforward computations show that the control law

$$U_{kin}^{adp} = \left\{ \begin{array}{lcl} w & = & K_\theta(\theta - \delta) - \delta' \sin\theta v \gamma \\ \dot{\gamma} & = & -\lambda_\gamma(\theta - \delta)\delta' \sin\theta v \end{array} \right. \tag{11}$$

yields $\dot{V}_2 \leq 0$. It can be shown that ρ and θ converge to 0 as t goes to ∞. The next step produces a dynamic controller that is robust with respect to error in parameter γ. To obtain it, start by defining

$$V_3 = \frac{1}{2}(w - w_r) + \frac{1}{2}(v - v_d)^2 + V_2 \tag{12}$$

with

$$\left\{ \begin{array}{lcl} w_r & = & K_\theta(\theta - \delta) - \delta' \sin\theta v \gamma_r \\ \dot{\gamma}_r & = & -\lambda_\gamma(\theta - \delta)\delta' \sin\theta v \end{array} \right.$$

where w is the actual value of rotational speed, w_r is the desired profile for w defined in (10), and v_d the desired velocity profile for v. The equation for updating γ_r is simply a re-writing of the second equation in (10) in the new dynamic setting. Then the choice

$$\begin{cases} u_{dif} &= p_1(\dot{w}_r - K_w(w - w_r)) \\ u_{com} &= p_2(\dot{v}_d - K_v(v - v_d)) \end{cases}$$

yields $V_3 \leq 0$. In the above development, it is assumed that the values of $p_i,\ i = 1, 2$ are known exactly. To tackle the problem of robustness against parameter uncertainty, assume $p_i,\ i = 1, 2$ are "estimated" values for the actual values p_i^R of the plant parameters and let $\Delta p_i = p_i - p_i^R$. Define

$$q_1 = p_1\ ;\ q_2 = p_1/\gamma\ ;\ q_3 = p_1/p_2\ ;\ q_4 = p_2$$

and let $q_i^R\ ;\ i = 1, 2, 3, 4$ be defined accordingly. Using (3) and (6), and expanding $\dot{w}_r$ yields

$$\begin{array}{lllll} u_{dif} &=& \sum_{i=1}^{3} q_i f_i &=& u_{dif}^{opt} + \Delta u_{dif} \\ u_{com} &=& q_4 f_4 &=& u_{com}^{opt} + \Delta u_{com} \end{array}$$

with $\Delta u_{dif} = \sum_{i=1}^{3} \Delta q_i f_i$ and $\Delta u_{com} = \Delta q_4 f_4$, where $\Delta q_i = q_i - q_i^R$ and

$$\begin{array}{lll} f_1 &=& -K_\theta w + \delta' w \cos\theta v \gamma_r \\ && -\delta' \sin\theta v \dot{\gamma}_r - K_w(w - w_r) \\ f_2 &=& -K_\theta \delta' \sin\theta v - \delta" [\sin\theta v]^2 \gamma_r \\ f_3 &=& -\delta' \sin\theta \gamma_r q_4(\dot{v}_d - K_v(v - v_d)) \\ f_4 &=& \dot{v}_d - K_v(v - v_d) \end{array}$$

Computing the derivative of V_3 yields

$$\begin{array}{lll} \dot{V}_3 &=& -K_w(w - w_r)^2 - K_v(v - v_d)^2 \\ &+& (w - w_r)\frac{1}{p_1^R} \sum_{i=1}^{3} \Delta q_i f_i \\ &+& (v - v_d)\frac{1}{p_2^R} \Delta q_4 f_4 \end{array}$$

which is indefinite. To overcome this problem, consider the new candidate Lyapunov function

$$V_4 = V_3 + \frac{1}{2p_1^R} \sum_{i=1}^{3} \frac{(\Delta q_i)^2}{\lambda_i} + \frac{(\Delta q_4)^2}{2p_2^R \lambda_4}$$

It can be shown that the control law

$$\begin{cases} u_{dif} &= \sum_{i=1}^{3} q_i f_i \\ u_{com} &= q_4 f_4 \end{cases}$$

and the adaptation scheme

$$\left(\begin{array}{lll} \dot{q}_i &=& -\lambda_i(w - w_r) f_i; i = 1, 2, 3 \\ \dot{q}_4 &=& -\lambda_4(v - v_d) f_4 \end{array} \right. \tag{13}$$

make $\dot{V}_4 \leq 0$. The proof that the complete adaptive path following system yields convergence of the robot to the path follows from standard method that resort to the Barbalat's lemma.

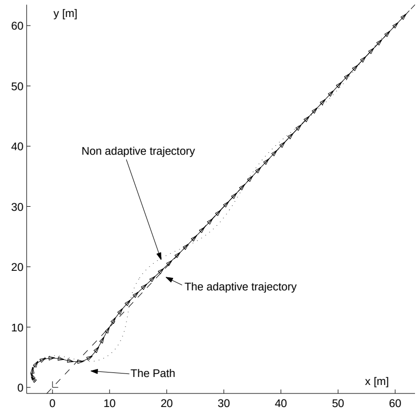

Figure 4: Desired and acutal paths

In summary, the adaptive control law is given by

$$U_{adp} = \left\{ \begin{array}{lll} u_{dif} &=& \sum_{i=1}^{3} q_i f_i \\ u_{com} &=& q_4 f_4 \\ \dot{q}_i &=& -\lambda_i(w - w_r) f_i\ ;\ i = 1, 2, 3 \\ \dot{q}_4 &=& -\lambda_4(v - v_d) f_4 \\ w_r &=& K_\theta(\theta - \delta) - \delta' \sin\theta v \gamma_r \\ \dot{\gamma}_r &=& -\lambda_\gamma(\theta - \delta)\delta' \sin\theta v \end{array} \right. \tag{14}$$

where $K_j\ ,\ j = \theta, w, v$ and $\lambda_i\ ,\ i = 1, 2, 3, 4, \gamma$ are positive gains.

4 SIMULATION RESULTS

The objective of the simulation is to drive the vehicle to the path, designed as a straight line. We consider that p_{im}^{pol} is extracted from the image and actualized each T_{im} (the camera acquisition and image treatment period). Measurements of $[v w]^T$ are available every T_s second. The control action is output every $T_{control}$ second. The robot parameters are: $m = 10Kg$, $I = 1N.m^{-1}$, $l = 0.5m$, $r = 0.25m$. The terrain feature parameters : $a = 1$ and $b = 0m$. The camera parameter $\gamma = z/h$ is set to 1. The different system periods adopted are $T_{im} = 0.1s$, $T_s = 0.02s$, and $T_{control} = 0.01s$. The control gains were set to $K_\theta = K_v = K_w = 1$, $\lambda_i = 10,\ ;\ i = 1, 2, 3, 4, \gamma$. The results are shown in figures (4), (5), (6), and (7), for the case where estimates of the real parameters were off by as much as 50%.

From figure (5), we note that the system converges to the desired path while the estimation of the parameters converges to a different value than the real one. This is a well known behavior of adaptive systems. Figure (4) compares the performance of the dynamic controller without the adaptation scheme (dotted line) and the adaptive one that clearly reduces the effects of parameter misestimation.

5 CONCLUSIONS

The paper proposed an adaptive vision based controller to steer a wheeled robot along a reference path. The resulting control law is robust against camera and plant parameter uncertainty.

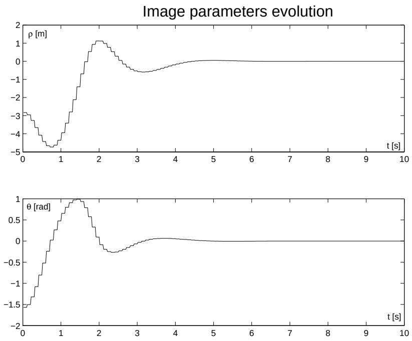

Figure 5: Evolution of parameters in the image plane

Future work will address the extension of the algorithm to deal with actuator saturation and camera misalignment. The extension of this circle of ideas to aerial robots deserves also further research.

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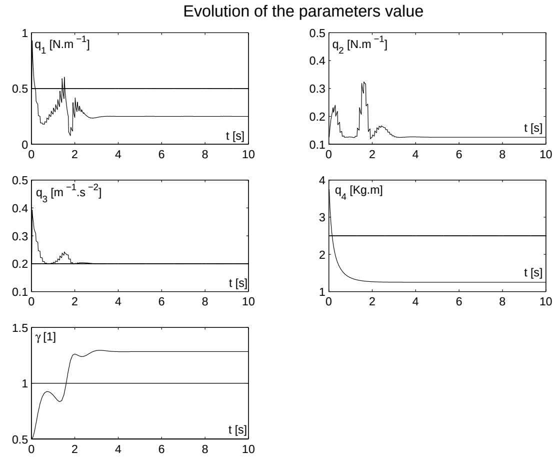

Figure 6: Evolution of robot and camera parameters

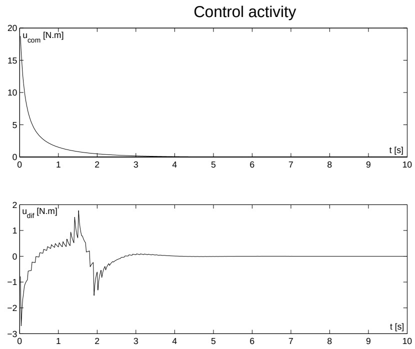

Figure 7: Evolution of control activity

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