NEARLY PASSIVE DYNAMIC WALKING OF A BIPED ROBOT

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Keywords : Biped robot, passive walking, Poincaré map, limit cycles, trajectory sensitivity

Abstract

The focus of this work is a study of the passive dynamic walking of a kneeless biped robot with torso on inclined slopes. We show that under a simple PD control applied between the torso and the stance leg the biped robot converges at a stable gait cycle.

1 Introduction

Biped robots form a subclass of legged or walking robots. The models for such robots are necessarily hybrid, consisting of ordinary differential equations to describe the swing phase of the walking motion, and a discrete map to model the impact when the leg touches the ground.

Biped robots exihibit periodic behavior. Discrete events, such as contact with the ground, can act to trap the evolving system state within a constrained region of the state space. Therefore even when the underlying continuous dynamics are unstable, discrete events may induce a stable limit set. Limit cycles (periodic behavior) are often created in this way.

Proof of existence and stability of periodic cycles are crucial problems for biped robots.

One can distinguish several approaches from the literature.Bipedal walking might be largely understood as a passive mechanical process, as shown for part of a stride by Mochon and MacMahon[2]. McGeer[1]demonstrated by both computer simulations and physical model construction, that some legged systems can walk in stable gait cycles on a range of shallow slopes with no actuation and no control (energy lost in friction and impact is recovered from gravity). Since then, many researchers have studied this topic.Goswami et al.[3][6], Coleman[5], Garcia et al.[4], Mark.W.Spong[7][8], F.Asano et al.[9], M.Haruna[10].

To date, for an under actuated biped robot with torso, none of the various approaches have found initial conditions under which passive dynamic walking in a downhill slope is generated.

In this paper we demonstrate that a kneeless biped robot with torso can walk in a stable gait cycle, downhill a slope, under a simple PD control applied between the torso and the stance leg[1].

Poincaré map is used to find periodic solution for the biped robot (limit cycles). To analyze the stability of such behaviours we use a recent generalization of trajectory sensitivity[14] [15].

The rest of this paper is organized as follows. Section 2 presents the dynamic model of a kneeless biped robot with torso. The stability of a passive gait and limit cycles is suggested in section 3. Simulation results have been evaluated in section 4. Conclusions and future work are presented in section 5.

2 The Model

The dynamic model of a simple planar biped robot is considered in this section .It's shown in figure (1). The robot has five degrees of freedom. It consists of a torso, two rigid legs, with no ankles and no knees, connected by a frictionless hinge at the hip. Masses M_t and M_h of the torso and the hip, respectively, are much larger than the leg mass m (M_t, $M_h >> m$) so that the motion of a swinging foot does not affect the motion of the hip and the torso. This linked mechanism moves on a rigid ramp of slope γ.

During locomotion, when the swing leg contacts the

[1]This work has been done in collaboration with L.R.V and the Coordinated Science Laboratory- University of Illinois (USA)

ground (ramp surface) at heelstrike, it has a plastic (no slip, no bounce) collision and its velocity jumps to zero.

The motion of the model is governed by the laws of classical rigid body mechanics. Following McGeer, we make the non physical assumption that the swing foot can briefly passes through the walk surface when the stance leg is near vertical. This concession is made to avoid the inevitable scuffing problems of stiff-legged walkers like the model analyzed in this paper. It's assumed that walking cycle takes place in the sagittal plane and the different phases of walking consist of successive phases of single support. With respect to this assumption the dynamic model of the biped robot consists of two parts : the differential equations describing the dynamic of the robot during the swing phase, and the algebraic equations for the impact (the contact with the ground).

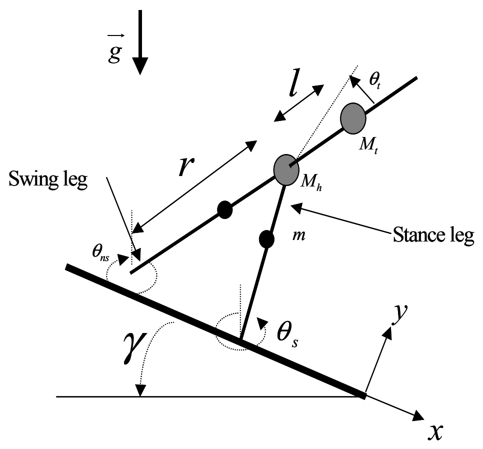

Figure 1: A passive dynamic walking model of the kneeless biped robot with torso

2.1 Swing phase model

During the swing phase the robot is described by differential equations written in the state space as follows:

$$\dot{x} = f(x) + g(x).u \tag{1}$$

where $x = (q, \dot{q})$.

(1) is derived from the dynamic equation between successive impacts given by :

$$M(q)\ddot{q} + H(q, \dot{q}) = B.u \tag{2}$$

where $q = (\theta_{ns}, \theta_s, \theta_t)$: θ_{ns} parametrizes the non support leg, θ_s the support leg, and θ_t the torso, $u = (u_1, u_2)$: u_1 and u_2 are the torques applied between the torso and the stance leg, and the torso and the swing leg, respectively.

$M(q) = [3 \times 3]$ is the inertia matrix and $H(q, \dot{q}) = [3 \times 1]$ is the coriolis and gravity term (i.e: $H(q) = C(q, \dot{q})\dot{q} + G(q))$ while B is a constant matrix such that $rank(B) = 2$. The matrices M, C, G, B are developped in [11] (see appendix)

2.2 Impact model

The impact between the swing leg and the ground (ramp surface) is modeled as a contact between two rigid bodies. The model used here is from[13], which is detailed by Grizzle & al.in [11][12].

The collision occurs when the geometric condition

$$\theta_{ns}(t) + \theta_s(t) + 2\gamma = 0 \tag{3}$$

is met. Yet, from biped's behaviour, there is a sudden exchange in the role of the swing and stance side members. The overall effect of the impact and switching can be written as :

$$h : S \rightarrow \chi \tag{4}$$

$$x^+ = h(x^-) = \begin{pmatrix} \theta_2^- \\ \theta_1^- \\ \theta_3^- \\ \theta_1^+(\theta_1^-) \\ \theta_2^+(\theta_2^-) \\ \theta_3^+(\theta_3^-) \end{pmatrix} \tag{5}$$

where

$$S = \{(q, \dot{q}) \in \chi / \ \ \theta_{ns}(t) + \theta_s(t) = -2\gamma\} \tag{6}$$

with $\theta_1^+, \theta_2^+, \theta_3^+$ are specified in [11] (see appendix).

The superscripts - and + denote quantities immediatly before and after impact, respectively.

2.3 The overall model

The overall biped model is written as follows :

$$\begin{cases} \dot{x} = f(x) + g(x).u & x^-(t) \notin S \\ x^+ = h(x^-) & x^-(t) \in S \end{cases} \tag{7}$$

With

$$f(x) + g(x)u = \frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M^{-1}(q)(-H(q, \dot{q}) + B.u) \end{bmatrix}$$

3 Passive dynamic walking on the downhill slope

3.1 Outline of procedure

At heelstrike, the impact is plastic, some energy is dissipated and support is transferred instantaneously. Because the model has a torso then an impulsive torque must be applied against the post-transfer stance leg to hold the torso in a desired region[1]. Then we decide to examine the possibility that a biped robot with torso can exhibit a passive dynamic walking in a stable gait cycle, downhill a slope, when torque is applied with a simple PD control scheme given by :

$$u_1 = K_p(\theta_t^d - \theta_t) - K_v\dot{\theta}_t \tag{8}$$

Both K_p and K_v are the control gain, θ_t^d is the desired angle of θ_t, and u_1 is the torque acting on the hip joint between the torso and the stance leg. No torque acts between the torso and the swing leg, that is, the free leg is a completely passive link.

The walker's motion can exhibit periodic behaviour. Limit cycles are often created in this way. At the start of each step we need to specify initial conditions $(q, \dot{q})$ such that after T seconds (T is the minimum period of the limit cycle) the system returns to the same initial conditions at the start.

A general procedure to study the biped robot model is based on interpreting a step as a Poincaré map. Limit cycles are fixed points of this function. And finally to evaluate the stability of a gait limit cycle we use trajectory sensitivity analysis.

3.2 Stability of limit cycles

In order to visualize the entire dynamics of the robot over a gait cycle it is useful to represent the dynamics by means of phase space trajectories.

In phase space, steady robot gaits are seen as stable limit cycles. We consider the notion of orbital stability to be the most appropriate in the context of biped robot dynamics. In what follows we introduce some basic definitions which cope with orbital stability notion[3][6][17]. Let us consider a second order system with state vector $x \in R^n$, initial conditions x_0 and $\{t_k\}$ a sequence of time instants with $\lim_{k\to\infty} t_k = \infty$

$$\dot{x} = f(x) \tag{9}$$

Definition 1: For the system (9) **a positive limit set** Ω of a bounded trajectory $x(t)$ ($\|x(t)\| < \mu$, $\forall t > 0$) is defined as :

$$\Omega = \{p \in R^n, \forall\varepsilon > 0, \exists\{t_k\} \ / \ \|p - x(t_k)\| < \varepsilon, \forall k \in N\} \tag{10}$$

Ω is a closed orbit (limit cycle or non-static equilibrium) for the system trajectory. Note that for second order systems the only possible types of limit sets are singular points and limit cycles. Let us consider the behavior of neighboring trajectories in order to analyze the orbital stability.

Definition 2 : **Orbital stability :** the system trajectory in the phase space R^n is a stable orbit Ω if $\forall\varepsilon > 0$ $\exists\delta > 0$ such that $\|x_0 - \Omega(x_0)\| < \delta \Rightarrow \inf_{p\in\Omega} \|x(t) - p\| < \varepsilon, \forall t > t_0$.

All trajectories starting near the orbit Ω stay in its vicinity. If all trajectories in the vicinity of Ω approach it as $t \to \infty$ then the limit set Ω is said to be attractive[19][18].

Definition 3: **Asymptotic orbital stability :**the system trajectory in the phase space R^n is asymptotically stable if it is stable and $\|x_0 - \Omega\| < \delta \Rightarrow \lim_{t\to\infty} \inf_{p\in\Omega} \|x(t) - p\| = 0$.

3.3 Poincaré map method

Since biped locomotion has periodic gaits, the idea of Poincaré map can be used. A Poincaré map samples the flow of a periodic system once every period [16]. The concept is illustrated in figure (2). The limit cycle Γ is stable if oscillations approach the limit cycle over time. The samples provided by the corresponding Poincaré map approach a fixed point x^*. A non stable limit cycle results in divergent oscillations, for such a case the samples of the Poincaré map diverge.

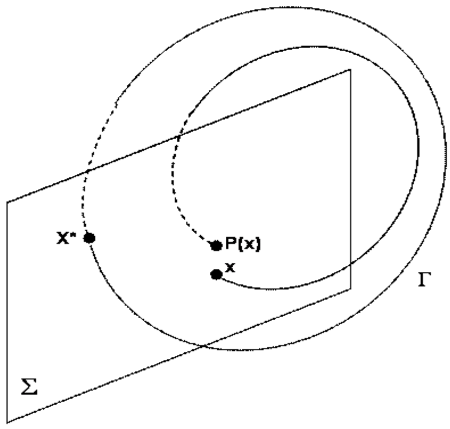

Figure 2: Poincaré Map

Let:

$$\begin{aligned} &P : \Sigma \to \Sigma \\ &P(x_k) = x_{k+1} = \phi_x(x_k, T) \end{aligned} \tag{11}$$

where the Poincaré hyperplane is defined by :

$$\begin{aligned} \Sigma \quad = \quad &\{(q, \dot{q}) \in \chi \ / \theta_{ns}(t) + \theta_s(t) = -2.\gamma, \\ &\theta_t = \theta_t^d, \dot{\theta}_t = 0, \dot{\theta}_{ns} + \dot{\theta}_s = 0\Big\} \end{aligned} \tag{12}$$

ϕ_x is the flow of (1), T is the time taken for the trajectory to return to Σ.

Stability of the Poincaré map (11) is determined by linearizing P around the fixed point x^*, leading a discrete evolution equation :

$$\Delta x_{k+1} = DP(x^*)\Delta x_k \tag{13}$$

The major issue is how to obtain $DP(x^*)$.- The jacobian matrix- .While the biped dynamic is rather complicated, a closed form solution for the linearized map is difficult to obtain. But one can be obtained by the use of trajectory sensitivity analysis.

3.4 Trajectory sensitivity

Trajectory sensitivity analysis is based upon linearizing the system around a nominal trajectory rather than around an equilibrium point[14][15]. It's therefore possible to determine directly the change in a trajectory due to (small) changes in initial conditions. Crucial to this method is the development of jump conditions describing the behavior of sensitivities at discrete events (impacts).

Away from events the dynamic of the biped robot is described as follows :

$$\dot{x} = f(x) + g(x).u = F(x) \tag{14}$$

The flow of x is defined as :

$$x(t) = \phi_x(x^*, t) \tag{15}$$

where x^* represents initial conditions :

$$\phi_x(x^*, t_0) = x^* \tag{16}$$

Sensitivity of the flow ϕ_x to initial conditions is obtained by linearizing (15) around the nominal trajectory and can be expressed as :

$$\frac{\partial \phi_x(x^*, t)}{\partial x^*} = x_{x^*}(t) = \Phi_x(x^*, t) \tag{17}$$

Away from events trajectory sensitivity x_{x^*} is given by :

$$\dot{x}_{x^*} = F_x(t) x_{x^*}(t) \tag{18}$$

where $F_x(t) = \frac{\partial F}{\partial x}$ is the jacobian matrix. Initial conditions for x_{x^*} are obtained from (16) as :

$$x_{x^*}(t) = I \tag{19}$$

where I is the identity matrix.

These equations describe the evolution of sensitivity between events (impacts). However at events, the sensitivity is generally discontinuous. Let $x(\tau)$ be the point where the trajectory encounters the triggering hypersurface Σ $(\theta_{ns} + \theta_s + 2\gamma = 0)$, i.e, the point where an event is initiated. This point is called the junction time. Just prior to the event we have :

$$x^- = x(\tau^-) = \phi_x(x^*, \tau^-) \tag{20}$$

Similarly x^+ is defined for time τ^+, just after the event. It is shown in [16]that the jump condition for the sensitivity is given by :

$$x_{x^*}(\tau^+) = h_x x_{x^*}(\tau^-) - (f^+ - f^-)\tau_{x^*} \tag{21}$$

where :
$h_x = \frac{\partial h}{\partial x}$ is the jacobian of the transition matrix.
$\tau_{x^*} = \frac{-s_x x_{x^*}(\tau^-)}{s_x f^-}$; $s_x = \theta_{ns} + \theta_s + 2\gamma$
$f^- = f(x(\tau^-))$
$f^+ = f(x(\tau^+))$.

The trajectory sensitivity $\Phi_x(x^*, t)$ is closely related to $DP(x^*)$ [16] as :

$$DP(x^*) = (I - \frac{f(x^*)\sigma^T}{\langle \sigma, f(x^*) \rangle})\Phi_x(x^*, T) \tag{22}$$

where σ is a vector normal to Σ, $\Phi_x(x^*, T)$ is the trajectory sensitivity after one period of the limit cycle, i.e, starting from x^* and returning to x^*. This matrix is called the *monodromy matrix.*

For an autonomous system [16], one eingenvalue of $\Phi_x(x^*, T)$ is always 1. The remaining eigenvalues of $\Phi_x(x^*, T)$ coincide with the eingenvalues of $DP(x^*)$, and are known as the characteristic multipliers λ_i of the periodic solution.Three cases can be notified:

1. $|\lambda_i| < 1, \forall i$. The map is stable, so the periodic solution is stable.
2. All λ_i lie outside the unit circle. The periodic solution is unstable.
3. Some λ_i lie outside the unit circle. The periodic solution is non-stable.

4 Numerical procedure

A numerical procedure is used to test the walking cycle via the poincaré map, it's resumed as follows :

1. With an initial guess we use the multidimentional Newton-Raphson method to determine the fixed point x^* of P^+ (immediatly prior the switching event).
2. Based on this choice of x^*, we evaluate the eigenvalues of the poincaré map after one period by the use of the trajectory sensitivity.

5 Simulation results

Consider the model (1), with the following values :
$m = 5kg$, $M_h = 10kg$, $M_t = 10kg$, $r = 1m$, $l = 0.5m$
$\gamma = 3°$, $K_p = 99$, $K_v = 79$, $\theta_t^d = \frac{\pi}{6}$.

We choose the hyperplane Σ as the event plane. Starting with a suitable initial guess we obtain the fixed point [2]:

$$\begin{aligned} x^* \; = \; & [-0.254973, 0.359693, -0.012109, \\ & 0.961480, 0.089247, 0.847713] \end{aligned} \tag{23}$$

[2]The fixed point x^* can be located by the use of multidimentional Newton-Raphson Method

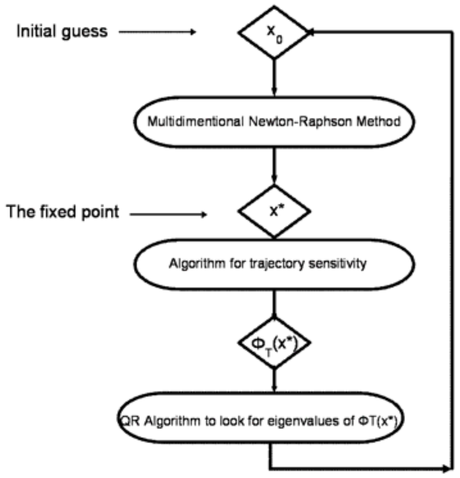

Figure 3: Algorithm of the numerical analysis

on the limit cycle before the impact event.

Based on this choice for x^*, the eigenvalues of the monodromy matrix $\Phi_x(x^*, T)$ after one period of the limit cycle are :

$$\begin{aligned} &0.987, -0.1245 \pm 0.6723, \\ &-0.3245 \pm 0.1241, 0.1578 \end{aligned} \tag{24}$$

The figure (4) shows the behavior of the stance leg in the phase space through a full cycle where T=3.2s.

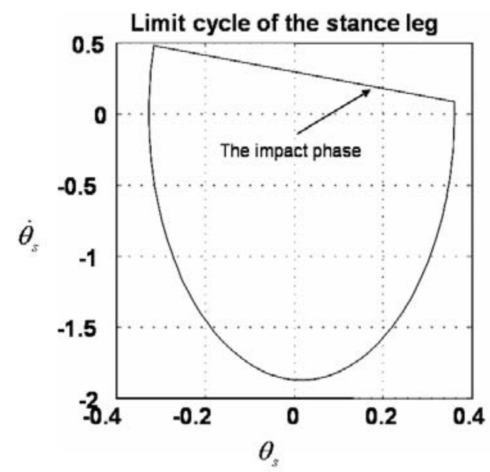

Figure 4: Nearly passive limit cycle of the stance leg

The figure (5) shows the behavior of the swing leg through a full cycle where T=3.2s.the control vector u is illustrated in figure(6), u_1 is the PD control applied between the torso and the stance leg, u_2 is the torque between the torso and the swing leg which is a passive link.

The figure (7) represents the stick diagram of the passive walking biped robot with torso.

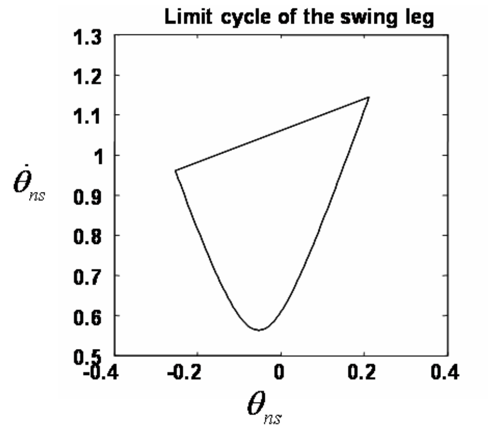

Figure 5: Nearly passive limit cycle of the swing leg

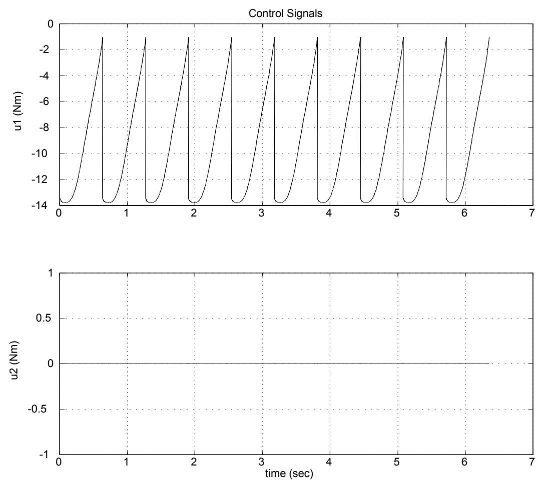

Figure 6: Control signals

6 Conclusion

A kneeless biped robot with torso is an underactuated system. It is shown in this paper that such system can exihibit a passive dynamic walking downhill a slope, under a simple PD control applied between the torso and the stance leg in order to stand the torso up.

An approach based on numerical optimization have been proposed and used to find appropriate and stable cycle for passive walking.

Poincaré map results are used to analyze the stability of limit cycle. The monodromy matrix is obtained by evaluating trajectory sensitivities over one period of the limit cycle.

Future research intends to examine a control law which realizes a robust continuous walking on the level ground to imitate a nearly passive dynamic walking on the downhill slope

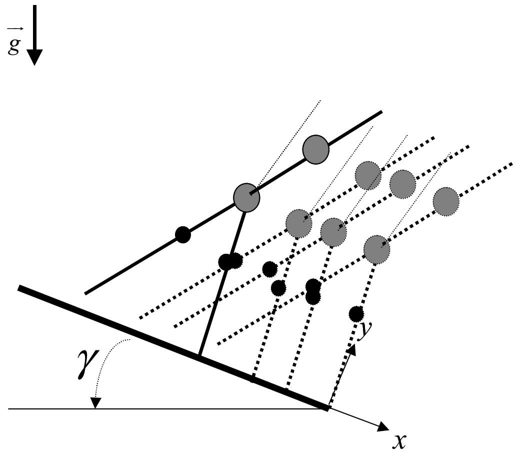

Figure 7: Stick diagram of the walking behavior

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Appendix

In this appendix we give the equations of the biped model (2), in the following :

$$\begin{aligned} \omega &:= \dot{\theta} \\ s_{nsj} &:= \sin(\theta_{ns} - \theta j), j \in \{s, t\} \\ c_{nsj} &:= \cos(\theta_{ns} - \theta j), j \in \{s, t\} \end{aligned}$$

The mechanical model

$$M = \begin{bmatrix} (\frac{5}{4}m + M_h + M_t)r^2 & \frac{-1}{2}mr^2 c_{nss} & M_t r l c_{nst} \\ \frac{-1}{2}mr^2 c_{nss} & \frac{1}{4}mr^2 & 0 \\ M_t r l c_{nst} & 0 & M_t l^2 \end{bmatrix} \tag{25}$$

$$C = \begin{bmatrix} 0 & \frac{-1}{2}mr^2 s_{nss}\omega_s & M_t r l s_{nst}\omega_t \\ \frac{-1}{2}mr^2 s_{nss}\omega_{ns} & 0 & 0 \\ M_t r l c_{nst} & 0 & 0 \end{bmatrix} \tag{26}$$

$$G = \begin{bmatrix} \frac{-1}{2}g(2M_h + 3m + 2M_t)r\sin(\theta_s) \\ \frac{-1}{2}gmr\sin(\theta_{ns}) \\ -gM_t l\sin(\theta_t) \end{bmatrix} \tag{27}$$

$$B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} \tag{28}$$