ANTI-WINDUP CIRCUITS IN ADAPTIVE POLE-PLACEMENT CONTROL

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Abstract

Adaptive pole-placement control of linear discrete systems in presence of saturating input is considered. Controllers in two structures with anti-windup compensation are analysed and compared. Simulations of second-order systems with time delay are given.

1 INTRODUCTION

It is well known that the presence of saturating actuators can cause a windup behaviour in the control system. There are many anti-windup compensators (AWCs) known in the corresponding literature. In the context of adaptive control with AWC, an important issue, besides the stability, is the convergence of the control law. Both stability and convergence of adaptive pole-placement control with AWCs have been examined in [1], [2] and [3] for a deterministic discrete-time ARMA system.

In [1] and [2], a conventional RST pole-placement controller with a simple compensator is considered, while in [3], and AWC based on a generalised conditioning technique (GCT) is introduced into the RST controller. In both cases, the recursive least-squares (RLS) algorithm is used to identify the unknown parameters and to show the stability and convergence of the adaptive pole-placement control. A basic idea of GCT is presented in [4].

In this paper, both approaches are comparatively analysed where, in extension to [1] and [3], a time delay is directly introduced to the ARMA model. Moreover, the performance of adaptive pole-placement controller with AWC is analysed with respect to time delay and location of the poles of the system.

As it is shown later in the paper, the performance of the control depends strongly not only on the identification process, rate of convergence of the system output signal to the given reference signal, but also essentially on system's damping factor, connected with the location of the poles on a z plane.

2 CONVENTIONAL RST STRUCTURE

The system is given by the following deterministic discrete-time ARMA model

$$A(q^{-1})y_t = B(q^{-1})u_{t-d}\,, \tag{1}$$

where y_t and u_t denote the output and the constrained input respectively and $d \geq 1$ is a time delay. The polynomials of the unit delay operator q^{-1}

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{nA} q^{-nA}\,, \tag{2}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_{nB} q^{-nB} \tag{3}$$

are relatively prime with unknown parameters $\underline{\theta} = [a_1, \ldots, a_{nA}, b_0, \ldots, b_{nB}]^T$.

The control signal u_t applied to the system is constrained by the saturation function

$$u_t = \mathrm{sat}(v_t;\, \alpha)\,, \tag{4}$$

where v_t is the calculated, i.e. the unconstrained, control signal. The aim of the designed controller is to ensure tracking of a bounded set-point r_t by the system output y_t, given the properties of the closed loop specified by the stable polynomial $A_M(q^{-1})$ and taking into account the possible saturation of the control signal. The unknown system parameters suggest the use of an adaptive controller.

To obtain a control law, a standard diophantine equation has to be solved which for the linear case, i.e. in the absence of the saturation, has the following form

$$A(q^{-1})R(q^{-1}) + q^{-d}B(q^{-1})S(q^{-1}) = A_M(q^{-1})A_0(q^{-1})\,, \tag{5}$$

where $R(q^{-1})$, $S(q^{-1})$ are controller polynomials (of orders $nB + d - 1$, $nA - 1$) and $A_M(q^{-1})$ is a given closed-loop polynomial. In this case, the control signal is calculated as follows

$$R(q^{-1})v_t = -S(q^{-1})y_t + T(q^{-1})r_t\,, \tag{6}$$

where polynomial $T(q^{-1})$ is chosen as

$$T(q^{-1}) = \left[\frac{A_M(1)}{B(1)}\right] A_0(q^{-1})\,. \tag{7}$$

When the constraint gets active and the control system becomes nonlinear, the controller equation takes the form

$$R(q^{-1})u_t = -S(q^{-1})y_t + T(q^{-1})r_t\,. \tag{8}$$

Since the above equation may not have a solution for arbitrary sequences of the output and the reference signals, the constrained control such that the latter equation holds, may not exist. Taking into consideration that $R(q^{-1})$ is monic, the control signal applied to the system is derived according to the following equations

$$\begin{aligned} v_t &= (1-\hat{R}(q^{-1}))u_t - \hat{S}(q^{-1})y_t + \hat{T}(q^{-1})r_t\,, \\ u_t &= \mathrm{sat}(v_t;\,\alpha)\,, \end{aligned} \tag{9}$$

which represent an indirect adaptive control law. In the case of saturation, i.e. when equation (8) may not have a solution, one requires a definition of a new modified reference signal r_t^r which is obtained by solving (9) for u_t

$$r_t^r = \frac{1}{\hat{t}_0}\left((\hat{t}_0 - \hat{T}(q^{-1}))r_t^r + \hat{R}(q^{-1})u_t + \hat{S}(q^{-1})y_t\right)\,. \tag{10}$$

The signal r_t^r may be interpreted as being the closest to r_t such that the controller can move the output in the presence of saturation. The constrained control signal is then generated according to the following equation

$$\begin{aligned} v_t &= (1-\hat{R}(q^{-1}))u_t - \hat{S}(q^{-1})y_t + \hat{t}_0 r_t + (\hat{T}(q^{-1}) - \hat{t}_0)r_t^r\,, \\ u_t &= \mathrm{sat}(v_t;\,\alpha)\,, \end{aligned} \tag{11}$$

which ensures that equation (8) has a solution.

The adaptive control law gives a BIBO stability of the closed-loop system provided that the system is stable, polynomials $\hat{A}(q^{-1})$, $\hat{B}(q^{-1})$ are coprime and polynomial $\hat{T}(q^{-1})$ is stable [1].

In the case of a tracking error feedback controller, the polynomial $T(q^{-1})$ is chosen as follows

$$T(q^{-1}) = S(q^{-1})\,, \tag{12}$$

and the adaptive controller equation is

$$\begin{aligned} v_t &= (1-\Delta(q^{-1})\hat{R}(q^{-1}))u_t - \hat{S}(q^{-1})e_t\,, \\ u_t &= \mathrm{sat}(v_t;\,\alpha)\,, \end{aligned} \tag{13}$$

where $e_t = y_t - r_t$. The diophantine equation in this case takes the form

$$\begin{aligned} &\Delta(q^{-1})A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1})q^{-d-1} = \\ &\qquad = A_M(q^{-1})A_0(q^{-1})\,, \end{aligned} \tag{14}$$

where $\Delta(q^{-1}) = 1 - q^{-1}$ is introduced to guarantee the asymptotic tracking property. The stability and convergence analysis of the considered indirect adaptive control is given in [2].

3 GENERALISED CONDITIONING TECHNIQUE

Consider the ARMA model (1), the constraint (4) and the controller (6). Let the desired closed-loop system be

$$y_t = \frac{B_M(q^{-1})}{A_M(q^{-1})} r_{t-d}\,. \tag{15}$$

For the purpose of controller design the following factorisations are performed

$$B(q^{-1}) = B^+(q^{-1})B^-(q^{-1})\,, \tag{16}$$

$$B_M(q^{-1}) = KB'_M(q^{-1})B^-(q^{-1})\,, \tag{17}$$

where $B^-(q^{-1})$ is a polynomial comprising stable zeros of the system that are not cancelled in the closed-loop system and $B'_M(q^{-1})$ is a monic polynomial with the new zeros that are introduced to the closed-loop system. The gain K is chosen to satisfy the equation

$$\frac{B_M(1)}{A_M(1)} = \frac{KB'_M(1)B^-(1)}{A_M(1)} = 1\,. \tag{18}$$

The controller polynomials $R(q^{-1})$, $S(q^{-1})$ are obtained by solving the following diophantine equation

$$\begin{aligned} &A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1})q^{-d} = \\ &\qquad = A_M(q^{-1})A_0(q^{-1})B^+(q^{-1})\,, \end{aligned} \tag{19}$$

where $A_0(q^{-1})$ is the observer polynomial. To obtain a unique solution of the latter equation, polynomials $A(q^{-1})$ and $B(q^{-1})$ must be coprime and $nR < nA$. The controller polynomial $T(q^{-1})$ is given by

$$T(q^{-1}) = KB'_M(q^{-1})A_0(q^{-1})\,. \tag{20}$$

In the case of a saturated control the controller internal states may be subjected to the windup phenomenon resulting in performance detoriation, thus the AWC has to be used to prevent the latter. Below, the AWC based on GCT [3] is used, where polynomial $T(q^{-1})$ is factorised as follows

$$T(q^{-1}) = T_2(q^{-1})T_1(q^{-1})\,, \tag{21}$$

and $T_2(q^{-1})$ is stable. A filtered setpoint is introduced

$$r_{f,t} = \frac{T_1(q^{-1})Q(q^{-1})}{L(q^{-1})} r_t \tag{22}$$

with stable polynomials $Q(q^{-1})$ and $L(q^{-1})$. Using (22) as a setpoint, the controller (6) can then be rewritten as follows

$$\begin{aligned} Q(q^{-1})R(q^{-1})v_t &= -Q(q^{-1})S(q^{-1})y_t + \\ &\quad + T_2(q^{-1})L(q^{-1})r_{f,t}\,. \end{aligned} \tag{23}$$

When the control signal saturates then the internal states of the controller are not consistent with the constrained control input applied to the system. In order to restore the consistency, a modified setpoint $r_{f,t}^r$ is defined in such a way that using it in the controller (23) instead of a filtered reference signal causes the controller output v_t to be equal to u_t.

Assuming that $r^r_{f,t}$ has been introduced for all past time instants such that for all past values the controller output is not saturated, the present control signal can be calculated as follows

$$\begin{aligned} v_t \;=&\; (1 - Q'(q^{-1})R(q^{-1}))u_t + \frac{t_{2,0}}{q_0}\, r_{f,t} + \\ &-Q'(q^{-1})S(q^{-1})y_t + \frac{1}{q_0}\,(T_2(q^{-1})L(q^{-1}) - t_{2,0})r^r_{f,t}\,, \end{aligned} \tag{24}$$

where $Q'(q^{-1}) = \frac{Q(q^{-1})}{q_0}$.

Since the modified filtered setpoint is chosen so that $u_t = v_t$, from the latter equation one obtains

$$\begin{aligned} u_t \;=&\; (1 - Q'(q^{-1})R(q^{-1}))u_t + \frac{t_{2,0}}{q_0}\, r_{f,t} + \\ &-Q'(q^{-1})S(q^{-1})y_t + \frac{1}{q_0}\,(T_2(q^{-1})L(q^{-1}) - t_{2,0})r^r_{f,t}\,, \end{aligned} \tag{25}$$

that is

$$\begin{aligned} Q(q^{-1})R(q^{-1})u_t =&\; -Q(q^{-1})S(q^{-1})y_t + \\ &+T_2(q^{-1})L(q^{-1})r^r_{f,t}\,. \end{aligned} \tag{26}$$

By comparing (24) and (26) a modified filtered reference signal is obtained

$$r^r_{f,t} = r_{f,t} + \frac{q_0(u_t - v_t)}{t_{2,0}}\,. \tag{27}$$

Using (25) and (27) the final adaptive control law can be written

$$\begin{aligned} v_t \;=&\; (1 - Q'(q^{-1})\hat{R}(q^{-1}))u_t - Q'(q^{-1})\hat{S}(q^{-1})y_t + \\ &+\frac{\hat{t}_{2,0}}{q_0}\, r_{f,t} + (\hat{T}_2(q^{-1})L(q^{-1}) - \hat{t}_{2,0})r^r_{f,t}\,, \qquad (28) \\ u_t \;=&\; \mathrm{sat}(v_t;\,\alpha)\,. \end{aligned}$$

The adaptive control law gives a BIBO stability of the closed-loop system provided that the system is stable, polynomials $\hat{A}(q^{-1})$, $\hat{B}(q^{-1})$ are coprime and polynomial $\hat{T}_2(q^{-1})L(q^{-1})$ is stable [3].

4 SIMULATION RESULTS (A)

Two second order systems are considered in the simulations

a) $A(q^{-1}) = 1 - 1.8q^{-1} + 0.9q^{-2}$, $B(q^{-1}) = 1 + 0.5q^{-1}$ (poles $p_1 = 0.9 + 0.3i$, $p_2 = 0.9 - 0.3i$);

b) $A(q^{-1}) = 1 - 1.6q^{-1} + 0.64q^{-2}$, $B(q^{-1}) = 1 + 0.5q^{-1}$ (poles $p_1 = 0.8$, $p_2 = 0.8$).

The initial values of the estimates of the parameters are taken as half of their true values, the initial value of matrix P is taken as $P(0) = 10 \cdot I$ for implementation of RLS algorithm. The closed-loop characteristic polynomial is chosen to be $A_M(q^{-1}) = 1 - 0.5q^{-1} + 0.06q^{-2}$ (poles $p_1 = 0.2$, $p_2 = 0.3$), $A_0(q^{-1}) = 1$ and $Q(q^{-1}) = 1 - 0.15q^{-1}$, $L(q^{-1}) = 1 - 0.05q^{-1}$, $B^+(q^{-1}) = 1 + \frac{b_1}{b_0}$, $B^-(q^{-1}) = b_0$, $B'_M(q^{-1}) = 1 + 0.5q^{-1}$, $T_2(q^{-1}) = K$. The reference signal filter parameters are chosen to have fixed values to emphasize the need of tuning them in order to obtain good performance.

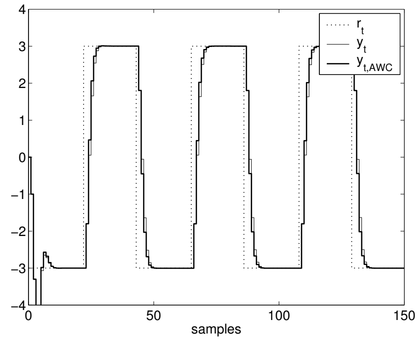

Fig. 1: Output signals with and without AWC (RST), $d = 1$

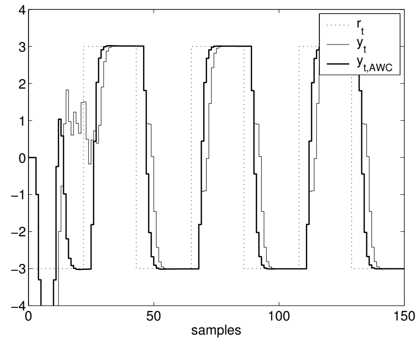

Fig. 2: Output signals with and without AWC (RST), $d = 3$

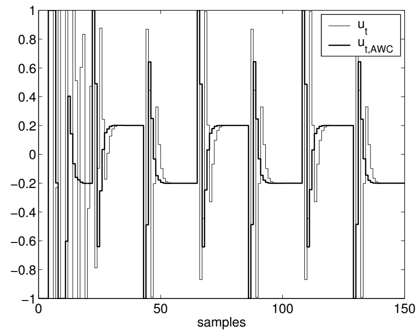

Fig. 3: Control signals with and without AWC (RST), $d = 3$

5 CONCLUSIONS (A)

Figure 1 depicts the performance of RST-controlled system (a) with $d = 1$, $\alpha = 1$ showing the advantage of the controller comprising an AWC in faster error regulation. The latter is absent when GCT controller is considered (Figure 4) for the same parameters due to intendedly improperly adjusted parameters of the filter (27). For $d = 3$ with $\alpha = 1$ an increase in

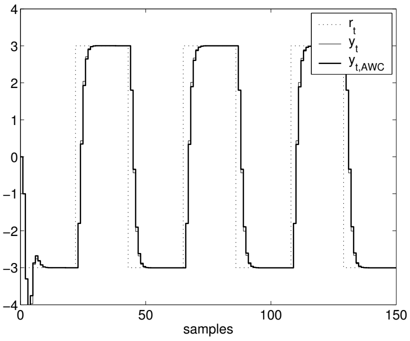

Fig. 4: Output signals with and without AWC (GCT), $d = 1$

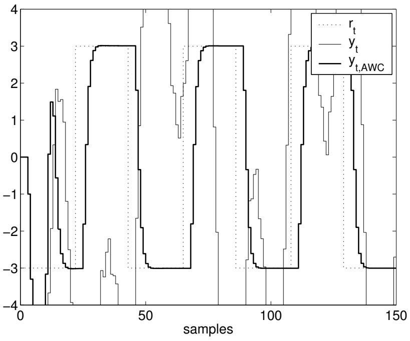

Fig. 5: Output signals with and without AWC (GCT), $d = 3$

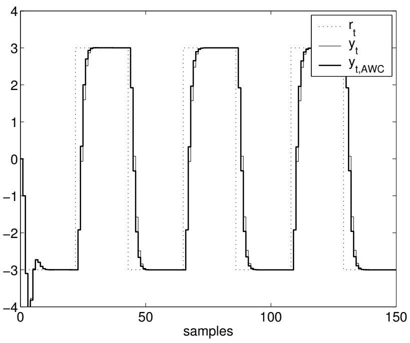

Fig. 6: Output signals with and without AWC (RST), $d = 1$

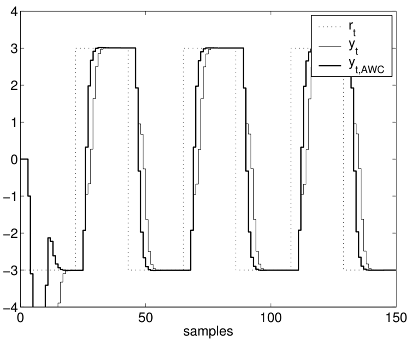

Fig. 7: Output signals with and without AWC (RST), $d = 3$

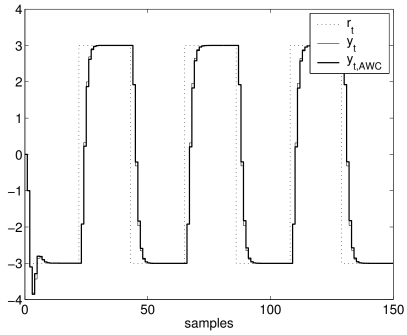

Fig. 8: Output signals with and without AWC (GCT), $d = 1$

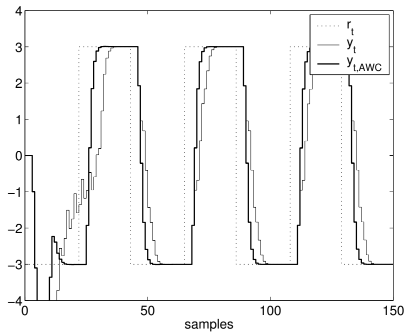

Fig. 9: Output signals with and without AWC (GCT), $d = 3$

the performance can be readily observed when AWC is applied (Figures 2, 3 and 5). Furthermore, in GCT case (Figure 5) any tracking is impossible for the system without windup compensation, whereas in the uncompensated RST case a rate of transients is aimlessly reduced. As it can be seen in the Figure 3, AWC causes the constrained control signal not only to desaturate, but prevents it from resaturating improving the adaptation conditions (for remaining cases v_t and u_t are not presented). For system (b) (Figures 6–9) the conclusions are similar, but in contrast to oscillatory system, in the case of $d = 3$ tracking is possible, but the resulting closed-loop system is slower than the chosen closed-loop model.

6 SIMULATION RESULTS (B)

For comparative purposes additional simulations have been performed for non-adaptive case, where the point of interest are sums of absolute errors given as

$$E_{AE} = \sum_{t=1}^{150} |e_t| = \sum_{t=1}^{150} |r_t - y_t| \tag{29}$$

for the system without AWC and for the system with AWC as

$$E_{AE,AWC} = \sum_{t=1}^{150} |e_{t,AWC}| = \sum_{t=1}^{150} |r_t - y_{t,AWC}| \,. \tag{30}$$

In order to evaluate the performance, a difference in quality of control $\Delta_{AE} = E_{AE} - E_{AE,AWC}$ is introduced, which is greater than zero if AWC improves the performance and less then zero if otherwise.

Two different sets of simulations have been performed, with $B(q^{-1})$ given as in Section 4 and $\alpha_{\min} = 3\frac{A(1)}{B(1)}$, aiming to analyse the influence on the performance of the location of:

- complex poles

  where polynomial $A(z^{-1})$ represented in z domain takes the form

  $$A(z^{-1}) = (1 - z^{-1}(\sigma + \omega i))(1 - z^{-1}(\sigma - \omega i))\,, \quad (31)$$

  and the real part of the poles, i.e. σ, changes from 0 to 1, and ω changes from 0 to 1. Simulations have been run only if the absolute value of the conjugate poles is less than one (the zero-value plane presented in the pictures is shown as a reference for Δ surfaces).

- real poles

  where polynomial $A(z^{-1})$ takes the form

  $$A(z^{-1}) = (1 - z^{-1}z_1)(1 - z^{-1}z_2)\,, \quad (32)$$

  and the both poles change from 0 to 1.

The GCT method enables additional tuning of the performance by reference signal filter design. Because its parameters should correspond to model parameters, saturation level and setpoint values, a special choice of parameters of the filter (22) for minimumphase second-order model is proposed. Let p_1 and p_2 denote poles of stable $A(z^{-1})$, then

$$\varrho = \max(|p_1|,\, |p_2|)\,, \quad (33)$$
$$Q(q^{-1}) = 1 + \left((1-\varrho)^{\xi} - 1\right) q^{-1}\,, \quad (34)$$
$$L(q^{-1}) = 1 - (1-\varrho)^{\xi} q^{-1}\,, \quad (35)$$

where $0 < \xi \leq 1$ is the damping factor obtained from classical root locus theory for the second-order systems. The suggested filter (33–35) takes into consideration model parameters and setpoint values only, forcing the initial values of the filtered reference signal for slow models and reducing the amplitude and rate of transients for oscillatory ones.

7 CONCLUSIONS (B)

The application of an AWC into RST controller results in performance improvement for complex-poled systems with slow modes (Figures 10 and 11), but with presence of performance degradation for some cases of $A(z^{-1})$, which is reduced for $d = 3$. For real-poled systems the improvement caused by the introduction of the AWC is clearly visible (Figures 12 and 13), even for systems slower than the chosen closed-loop model.

In the case of GCT AWC with tuned reference signal filter (Figures 33–35) there is no difference in between performance for $d = 1$ or $d = 3$ for both types of systems, i.e. complex-poled (Figures 14–16) and real-poled (Figures 17–19), which would not hold if the reference signal had fixed parameters. For the introduced cases it can be stated that the greater the improvement in performance visible is, the less tight constraint is. Most of the negative values of the Δ_{AE} correspond to the lack of tracking with output revolving around zero, which disorts the information carried by the performance index.

It is to be mentioned that further performance improvement for the GCT case can be made by incorporating additional rules for the reference signal filter in order to make its parameters depend on the $\frac{\alpha}{\alpha_{\min}}$ ratio, which has been omitted in the paper.

In general, the AWC circuit incorporated into the pole–(zero)–placement controller enables one to assure tracking even for $\alpha = \alpha_{\min}$ for large values of d, but at the cost of the increase in complexity of the controller, as in the GCT case. Furthermore, occasionally the improvement may not be worth introducing the compensation into the controller, which could be read as a drawback of the applied AWCs.

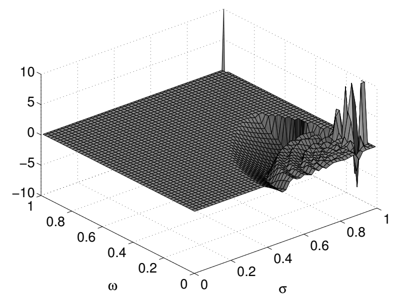

Fig. 10: Δ_{AE} (RST), $d = 1$, $\alpha = 2\alpha_{\min}$, (a)

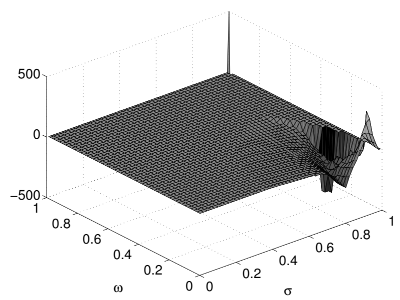

Fig. 11: Δ_{AE} (RST), $d = 3$, $\alpha = 2\alpha_{\min}$, (a)

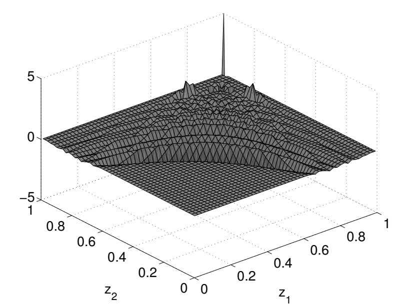

Fig. 12: Δ_{AE} (RST), $d = 3$, $\alpha = 2\alpha_{\min}$, (b)

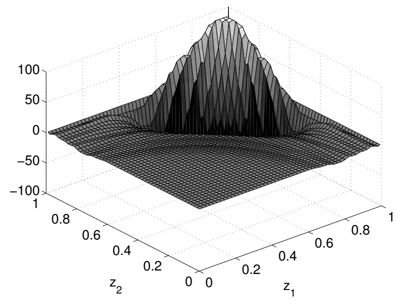

Fig. 13: Δ_{AE} (RST), $d = 3$, $\alpha = 2\alpha_{\min}$, (b)

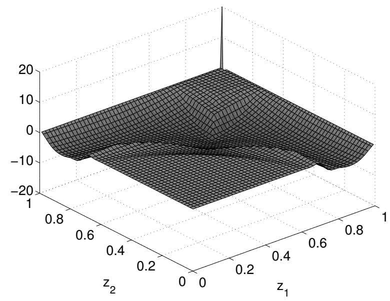

Fig. 17: Δ_{AE} (GCT), $\alpha = 2\alpha_{\min}$

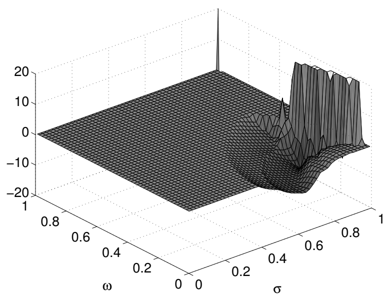

Fig. 14: Δ_{AE} (GCT), $\alpha = 2\alpha_{\min}$

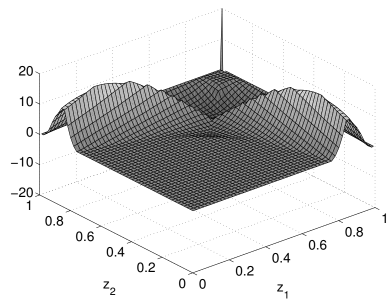

Fig. 18: Δ_{AE} (GCT), $\alpha = 4\alpha_{\min}$

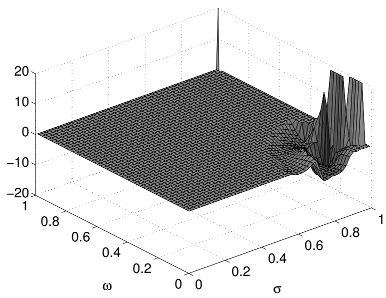

Fig. 15: Δ_{AE} (GCT), $\alpha = 5\alpha_{\min}$

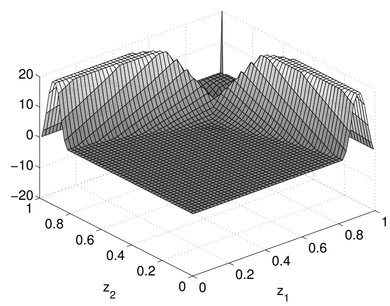

Fig. 19: Δ_{AE} (GCT), $\alpha = 6\alpha_{\min}$

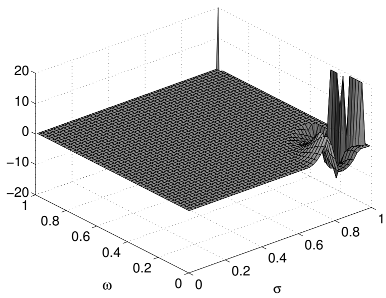

Fig. 16: Δ_{AE} (GCT), $\alpha = 8\alpha_{\min}$

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