ROBUST STABILIZATION OF NONLINEAR PLANTS WITH UNCERTAIN HYSTERESIS-LIKE ACTUATOR NONLINEARITIES

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Abstract

This paper addresses the stabilization problem of an uncertain intrinsically nonlinear SISO plant containing non-smooth nonlinearities (dead-zone, backlash, hysteresis) in the actuator device. A unified framework for its solution is here proposed, assuming that the parameters of the nonlinearities are uncertain as well. To this purpose, the hysteresis model used in [8] has been modified into an "extended" one, and a robust control law ensuring asymptotic stabilization has been synthesized using it. The resulting controller has been shown to be a full generalization of previous results (it includes, as particular cases, control laws [3] previously developed for backlash and dead-zone), ensuring also that the inner "forbidden" part of nonlinearity characteristics is never entered, even in the presence of uncertainties. Theoretical results have been validated by simulation on a simple mechanical system.

1 Introduction

Non-smooth nonlinearities such as hysteresis, backlash, dead-zone, are always present in real control plants. Industrial engineering components always show non-smooth behavior with memory effect and their main undesired effects (from an engineering point of view) are some power loss, proportional to the hysteresis loop area, and a time delay. Due to the non differentiability and memory effect of these nonlinearities, industrial control techniques usually ignore them in control design. On the other side it is well known from a theoretical point of view that, if unaccommodated, non-smooth dynamics with hysteresis loop can produce severe loss of control authority and even more instability. Even for systems that show thin hysteresis loop linear control methods can be ineffective due to phase shifts and unmodeled energy loss. It can be claimed that "Actuator and sensor nonlinearities are among the key factors limiting both static and dynamic performance of feedback control systems" [7].

A number of different models are available in literature (for a complete survey see f.i. [2]). In most cases, however, rigorous mathematical models of hysteresis tend to be very complicated and are hardly suited for controller design. A well assessed hysteresis model, capturing most of its characteristics, still retaining some simplicity being piecewise linear, has been proposed in [8] [7]. Even in simple cases, however, traditional control methods fail and new approaches claim for being developed [6].

An important research thrust, dealing with unknown nonlinearities cascaded to linear plants, is based on adaptive control [7]. It is worth noting, however, that when the adaptive inverse is used for control, the effect of hysteresis may not be completely cancelled [8].

Variable structure control (VSC) techniques have been used as well [1], [3], [9]. Indeed, the well known robustness features and the discontinuous character of sliding mode control [12] appears particularly well suited for handling intrinsically nonlinear and uncertain SISO plants containing non-smooth nonlinearities.

With the aim of attaining a controller general enough to include backlash and dead-zone, the hysteresis model used in [8] has been modified into an "extended" one. Using it, a robust control law ensuring asymptotic stabilization has been synthesized.

2 System model and problem statement

Consider an uncertain single-input intrinsically nonlinear system:

$$\dot{\mathbf{x}} = \mathbf{h}(\mathbf{x}) + \Delta\mathbf{h}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x}) \quad (1)$$
$$u = f(v) \quad (2)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the *state vector* at time t , $u(t) \in \mathbb{R}$ is the *input*, $\mathbf{g}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the smooth state-input map, $\mathbf{h}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a smooth function describing the known plant dynamics, and finally $\Delta\mathbf{h}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $\mathbf{d}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ account for parameter variations and exogenous disturbances respectively.

The nonlinear system is supposed to be preceded by the actuating device affected by a hysteresis-like nonlinearity $u = f(v)$ (see Fig.1), u being the plant input not available for control.

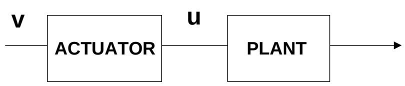

Fig.1 - Block scheme of a plant driven by the actuator.

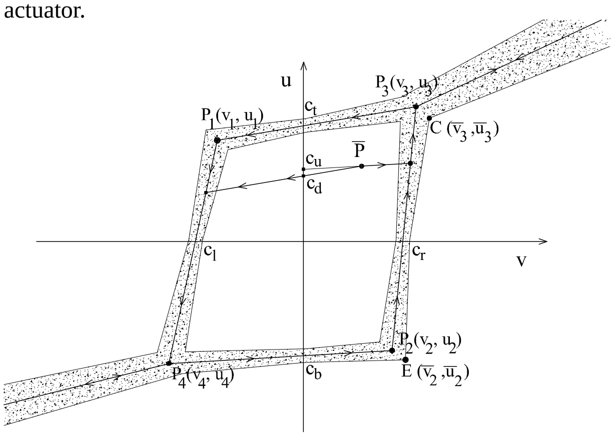

Fig.2 - Sketch of the "extended" hysteresis nonlinearity.

2.1 The "extended" hysteresis model.

Consider the input/output characteristic in the plane (v,u) reported in Fig.2, and define:

$$\begin{array}{ll} r_r(v,u) := u - m_r(v - c_r), & r_t(v,u) := u - m_t v - c_t, \\ r_s(v,u) := u - m_s v - c_s, & r_l(v,u) := u - m_l(v - c_l), \\ r_b(v,u) := u - m_b v - c_b, & r_i(v,u) := u - m_i v - c_i, \end{array} \tag{3}$$

where $m_r, m_t, m_s, m_l, m_b, m_i$ are assumed to be nonnegative real numbers, $c_r, c_t, c_l, c_b \in \mathbb{R}$ and c_s, c_i satisfy: $c_s = u_3 - m_s v_3 \quad c_i = u_4 - m_i v_4$ v_1,v_2,v_3,v_4 being the values of v at the upper-left (point P_1 in Fig.2), lower-right (point P_2), upper-right (point P_3) and lower-left (point P_4) corners of the quadrilateral.

Define the following regions in $\mathbb{R}^2$: $\Lambda := \{(v,u) \in \mathbb{R}^2 \mid r_r(v,u) > 0,\ r_l(v,u) < 0,\ r_t(v,u) < 0,\ r_b(v,u) > 0\}$, $\Gamma_1 := \{(v,u) \in \mathbb{R}^2 \mid r_s(v,u) = 0,\ v \geq v_3\}$, $\Gamma_2 := \{(v,u) \in \mathbb{R}^2 \mid r_i(v,u) = 0,\ v \leq v_4\}$, $\Gamma := \Gamma_1 \cup \Gamma_2$.

Define Σ_B as the set of the states of the hysteresis model [5], i.e. the set of all the points in $\mathbb{R}^2$ lying inside or upon the hysteresis characteristic. It is easily verified that $\Sigma_B = \overline{\Lambda} \cup \Gamma$, the bar denoting the closure with respect to the usual topology in $\mathbb{R}^2$.

For an analytic description of the nonlinearity, the following two functions $f_i(\cdot, v(t_k), u(t_k), t_k) : [v(t_k), \infty) \to [u(t_k), \infty)$ and $f_d(\cdot, v(t_k), u(t_k), t_k) : (-\infty, v(t_k)] \to (-\infty, u(t_k)]$ need to be first introduced for any state $(v(t_k), u(t_k)) \in \overline{\Lambda}$, $t > t_k$:

$$f_i := \begin{cases} m_b v + c_u & \text{if } v_u > v_3 \text{ and } v \in [\, v(t_k), v_g\,) \\ m_t v + c_t & \text{if } v_u > v_3 \text{ and } v \in [\, v_g, v_3) \\ m_b v + c_u & \text{if } v_u < v_3 \text{ and } v \in [\, v(t_k), v_u\,) \\ m_r(v - c_r) & \text{if } v_u < v_3 \text{ and } v \in [\, v_u, v_3) \\ m_s v + c_s & \text{if } v \in [\, v_3, +\infty) \end{cases}$$

$$f_d := \begin{cases} m_t v + c_d & \text{if } v_d < v_4 \text{ and } v \in (\, v_h, v(t_k)\,] \\ m_b v + c_b & \text{if } v_d < v_4 \text{ and } v \in (\, v_4, v_h\,] \\ m_t v + c_d & \text{if } v_d > v_4 \text{ and } v \in (\, v_d, v(t_k)\,] \\ m_l(v - c_l) & \text{if } v_d > v_4 \text{ and } v \in (\, v_4, v_d] \\ m_i v + c_i & \text{if } v \in (-\infty, v_4] \end{cases}$$

where, following [7], v_d, v_u, c_d, c_u are such that [1]: $v_d = \dfrac{m_l c_l + c_d}{m_l - m_t}$; $v_u = \dfrac{m_r c_r + c_u}{m_r - m_b}$; $v_g = \dfrac{c_t - c_u}{m_b - m_t}$; $v_h = \dfrac{c_b - c_d}{m_b - m_t}$ and $c_d = u(t_k) - m_t v(t_k)$; $c_u = u(t_k) - m_b v(t_k)$. Finally, the input-output relation Φ_h of the hysteresis is defined as follows: for any state $\bar{P} = (v(t_k), u(t_k)) \in \Sigma_B$ and for any input $v(\cdot)$ monotone over $[t_k, t_{k+1}]$, the hysteresis output $u(t)$ is given $\forall t \in [t_k, t_{k+1}]$ by:

$$u(t) = \Phi_h(v(t)) = \begin{cases} F_i(v(t), v(t_k), u(t_k), t_k) \\ \qquad \text{if } v(\cdot) \text{ is increasing over } [t_k, t_{k+1}] \\ F_d(v(t), v(t_k), u(t_k), t_k) \\ \qquad \text{if } v(\cdot) \text{ is decreasing over } [t_k, t_{k+1}] \end{cases} \tag{4}$$

with:

$$F_i := \begin{cases} m_s v(t) + c_s & \text{if } \bar{p} \in \Gamma_1 \\ f_i(v(t), v(t_k), u(t_k)) & \text{if } \bar{p} \in \overline{\Lambda} \\ m_i v(t) + c_i & \text{if } (\bar{p} \in \Gamma_2) \wedge (v(t_k) < v(t) \leq v_4) \\ f_i(v(t), v_4, u_4) & \text{if } (\bar{p} \in \Gamma_2) \wedge (v(t) > v_4) \end{cases} \tag{5}$$

$$F_d := \begin{cases} m_i v(t) + c_i & \text{if } \bar{p} \in \Gamma_2 \\ f_d(v(t), v(t_k), u(t_k)) & \text{if } \bar{p} \in \overline{\Lambda} \\ m_s v(t) + c_s & \text{if } (\bar{p} \in \Gamma_1) \wedge (v_3 \leq v(t) < v(t_k)) \\ f_d(v(t), v_3, u_3) & \text{if } (\bar{p} \in \Gamma_1) \wedge (v(t) < v_3) \end{cases} \tag{6}$$

Remark 2.1 *It is easily verified that backlash and dead zone are special cases of this model of hysteresis. As a matter of fact backlash can be obtained with $m_t = 0$, $m_b = 0$, $m_r = m_l$ and sufficiently large c_t and c_b. On the other side dead zone is obtained imposing $c_t = c_b = m_t = m_b = 0$. A variety of other least common nonlinearities and a wider variety than using an usual piecewise linear model [7] can be obtained with other values of these parameters.*

[1] To ease the understanding of notation, it may be useful to notice that, in the expression of $f_i(v(t), v(t_k), u(t_k), t_k)$, the condition $v_u > v_3$ implies that $m_t < m_b$ and simultaneously that the point $(v(t_k), u(t_k))$ is close enough to the top segment of the hysteresis.

With reference to the described hysteresis model, the following Assumption is introduced.

Assumption 2.1 *Coefficients describing the hysteresis nonlinearity are uncertain with bounded uncertainties, i.e.* $m_j = \hat{m}_j + \Delta m_j \quad |\Delta m_j| \le \rho_{mj},\ j \in \{t,l,r,b,s,i\},\ c_j = \hat{c}_j + \Delta c_j \quad |\Delta c_j| \le \rho_{cj},\ j \in \{t,l,r,b\}$. *Hysteresis loop slopes* $m_j\ j \in \{t,l,r,b\}$ *are assumed nonnegative, whilst* m_s *and* m_i *are assumed strictly positive.*

Referring to Fig. 2 in the perturbed condition, we will denote by $\bar{v}_2$ (the abscissa of point E in figure) (and resp. $\underline{v}_2$) the largest (resp. smallest) abscissa of the intersection between the bottom and the right segments taken "in the worst case", i.e. for m_b, c_b, m_r, c_r varying within their uncertainty interval. Analogous notation will be used for $\bar{v}_1$, $\bar{v}_3$, $\bar{v}_4$, $\underline{v}_1$, $\underline{v}_3$, $\underline{v}_4$.

2.2 Problem statement

The following assumptions are supposed to hold:

Assumption 2.2 *There exists a smooth function:* $s(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ *such that the achievement of a sliding motion on the surface:*

$$s(\mathbf{x}) = 0 \tag{7}$$

ensures the asymptotic stabilization of system (1). It is assumed that $\mathbf{g}(\mathbf{x})$ *and* $s(\mathbf{x})$ *satisfy* $\dfrac{\partial s(\mathbf{x})}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x}) \ne 0,\ \forall \mathbf{x}$. *By the smoothness of* $s(x)$ *and* $g(x)$ *and this last Assumption, it can be assumed without loss of generality that:* $\dfrac{\partial s(\mathbf{x})}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x}) > 0\ \forall \mathbf{x}$. *Moreover plant uncertainties satisfy* $\left|\dfrac{\partial s(\mathbf{x})}{\partial \mathbf{x}}[\Delta\mathbf{h}(\mathbf{x}) + \mathbf{d}(\mathbf{x})]\right| \le \rho(\mathbf{x}), \forall \mathbf{x}$.

By Assumption 2.1 it is always possible to find a value of the available input v such that any initial point lying inside Σ_B is forced towards the boundary of the hysteresis loop, even in presence of uncertainties. In the sequel we will denote by v_r^{max} (resp. v_l^{min}) the largest (resp. smallest) value of v, taken on the increasing (resp. decreasing) boundary of the hysteresis loop, corresponding to the current working point with the worst choice of m_r, c_r, m_b, c_b, m_s, c_s, m_i, c_i (resp. m_l, c_l, m_t, c_t, m_s, c_s, m_i, c_i).

Problem 1 *The addressed problem, provided that Assumptions are satisfied, is finding a feedback controller guaranteeing the robust stabilization of the system (1) containing the hysteresis nonlinearity in the actuator device.*

3 Notation and results

In order to concisely state the main result, some definitions will be given in the following. They formalize some useful relationships between the sliding surface, system dynamics and actuator characteristics. Define first the following functions, linking together the system dynamics and the sliding surface:

$$w(\mathbf{x}) \ := \ \frac{\partial s(\mathbf{x})}{\partial \mathbf{x}}\mathbf{h}(\mathbf{x}) \qquad r(\mathbf{x}) \ := \ \frac{\partial s(\mathbf{x})}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x})$$

$$\delta(\mathbf{x}) \ := \ \frac{\partial s(\mathbf{x})}{\partial \mathbf{x}}[\Delta\mathbf{h}(\mathbf{x}) + \mathbf{d}(\mathbf{x})] \ .$$

Note that $r(\mathbf{x}) > 0\ \forall \mathbf{x} \in \mathbb{R}^n$ due to Assumption 2.2. Next, it is useful to define the functions below, built considering both system and actuator uncertainties:

$$\bar{f}_j(\mathbf{x}) := \frac{\rho(\mathbf{x}) + r(\mathbf{x})\rho_{cj} + r(\mathbf{x})\rho_{mj}\left|v_e^{(j)}\right|}{r(\mathbf{x})(\hat{m}_j - \rho_{mj})}$$

$$\bar{f}_k(\mathbf{x}) := \frac{\rho(\mathbf{x}) + r(\mathbf{x})(\hat{m}_k + \rho_{mk})\rho_{ck} + r(\mathbf{x})\rho_{mk}\left|\hat{c}_k + \rho_{ck} - v_e^{(k)}\right|}{r(\mathbf{x})(\hat{m}_k - \rho_{mk})}$$

for $j = b,t,s,i,\ k = r,l,$ where $v_e^{(j)},\ j = b,t,s,i,r,l$ are suitable functions built using quantities describing the nominal system:

$$\begin{aligned} v_e^{(j)} &:= -\frac{\hat{c}_j}{\hat{m}_j} - \frac{w(\mathbf{x})}{r(\mathbf{x})\hat{m}_j} \qquad j = s,i,b,t \\ v_e^{(j)} &:= \hat{c}_j - \frac{w(\mathbf{x})}{r(\mathbf{x})\hat{m}_j} \qquad j = l,r \end{aligned}$$

Remark 3.1 *Under the assumption of strict positiveness of hysteresis loop slopes, and due to Assumption 2.1, the above functions exist finite for all* $\mathbf{x} \in \mathbb{R}^n$.

Before giving the main result, it is helpful to state the following Lemma.

Lemma 3.1 *For the system described by (1) and (2), under Assumptions 2.1-2.2, the following control law:*

$$v = \begin{cases} v_e^{(s)} + v_n^{(s)} = -\dfrac{\hat{c}_s}{\hat{m}_s} - \dfrac{w(\mathbf{x})}{r(\mathbf{x})\hat{m}_s} + \Theta_s \bar{f}_s(\mathbf{x}) & \text{if} \quad s(\mathbf{x}) < 0 \\ v_e^{(i)} + v_n^{(i)} = -\dfrac{\hat{c}_i}{\hat{m}_i} - \dfrac{w(\mathbf{x})}{r(\mathbf{x})\hat{m}_i} - \Theta_i \bar{f}_i(\mathbf{x}) & \text{if} \quad s(\mathbf{x}) > 0 \end{cases} \tag{9}$$

and Θ_i *chosen as:*

$$\begin{aligned} \Theta_s &> \max\left\{1, \frac{v_r^{max} - v_e^{(s)}}{\bar{f}_s(\mathbf{x})}, \frac{\bar{v}_3 - v_e^{(s)}}{\bar{f}_s(\mathbf{x})}\right\}; \\ \Theta_i &> \max\left\{1, \frac{v_e^{(i)} - v_l^{min}}{\bar{f}_i(\mathbf{x})}, \frac{v_e^{(i)} - \underline{v}_4}{\bar{f}_i(\mathbf{x})}\right\} \end{aligned} \tag{10}$$

ensures the achievement of a sliding motion on $s(\mathbf{x}) = 0$*, hence asymptotic stabilization.*

Remark 3.2 *The theoretical result of Lemma 3.1 simply states that a stabilizing control input can always be found working outside the hysteresis loop, i.e. choosing the available input* v

in correspondence of the two half lines of the hysteresis characteristics. This result, although interesting, can hardly be effectively applied in practice. In fact, it would imply the systematic avoidance of some points of the input variable domain, for example the origin in case the hysteresis characteristics is centered around it. Therefore, constraining the controller to work always outside the hysteresis could produce persistent chattering for stabilization points contained inside the hysteresis itself.

3.1 Main result

In order to overcome the drawbacks of Lemma 3.1 discussed in Remark 3.2, the main result is now introduced.

Theorem 3.1 *For the system described by (1) and (2) under Assumptions II.1-II.4, the achievement of a sliding motion on $s(x) = 0$, hence asymptotic stabilization, is guaranteed by the following control law*

$$v = \begin{cases} \begin{cases} v_e^{(i)} + \Theta_i \bar{f}_i(\mathbf{x}) & \text{if} \quad \underline{v}_4 > \max\{v_r^{max}, \bar{f}_i(\mathbf{x}) + v_e^{(i)}\} \\ v_e^{(b)} + \Theta_b \bar{f}_b(\mathbf{x}) & \text{if} \quad \underline{v}_2 > \max\{v_r^{max}, \bar{f}_b(\mathbf{x}) + v_e^{(b)}\} \\ v_e^{(r)} + \Theta_r \bar{f}_r(\mathbf{x}) & \text{if} \quad \underline{v}_3 > \max\{v_r^{max}, \bar{f}_r(\mathbf{x}) + v_e^{(r)}\} \\ v_e^{(s)} + \Theta_s \bar{f}_s(\mathbf{x}) & \text{otherwise} \end{cases} \\ \quad \text{if} \quad s(x) < 0 \\ \\ \begin{cases} v_e^{(s)} - \Theta_s \bar{f}_s(\mathbf{x}) & \text{if} \quad \bar{v}_3 < \min\{v_l^{min}, v_e^{(s)} - \bar{f}_s(\mathbf{x})\} \\ v_e^{(t)} - \Theta_t \bar{f}_t(\mathbf{x}) & \text{if} \quad \bar{v}_1 < \min\{v_l^{min}, v_e^{(t)} - \bar{f}_t(\mathbf{x})\} \\ v_e^{(l)} - \Theta_l \bar{f}_l(\mathbf{x}) & \text{if} \quad \bar{v}_4 < \min\{v_l^{min}, v_e^{(l)} - \bar{f}_l(\mathbf{x})\} \\ v_e^{(i)} - \Theta_i \bar{f}_i(\mathbf{x}) & \text{otherwise.} \end{cases} \\ \quad \text{if} \quad s(x) > 0 \end{cases}$$

with $v_e^{(j)}$, $j = i, r, t, s, b, l$, chosen according to (8), and Θ_j, $j = i, r, t, s, b, l$, selected as:

$$\text{if} \quad s(x) < 0$$

$$\max\left\{1, \frac{v_r^{max} - v_e^{(i)}}{\bar{f}_i(\mathbf{x})}\right\} < \Theta_i < \frac{\underline{v}_4 - v_e^{(i)}}{\bar{f}_i(\mathbf{x})}$$
$$\max\left\{1, \frac{v_r^{max} - v_e^{(b)}}{\bar{f}_b(\mathbf{x})}, \frac{\bar{v}_4 - v_e^{(b)}}{\bar{f}_b(\mathbf{x})}\right\} < \Theta_b < \frac{\underline{v}_2 - v_e^{(b)}}{\bar{f}_b(\mathbf{x})}$$
$$\max\left\{1, \frac{v_r^{max} - v_e^{(r)}}{\bar{f}_r(\mathbf{x})}, \frac{\bar{v}_2 - v_e^{(r)}}{\bar{f}_r(\mathbf{x})}\right\} < \Theta_r < \frac{\underline{v}_3 - v_e^{(r)}}{\bar{f}_r(\mathbf{x})}$$
$$\Theta_s > \max\left\{1, \frac{v_r^{max} - v_e^{(s)}}{\bar{f}_s(\mathbf{x})}, \frac{\bar{v}_3 - v_e^{(s)}}{\bar{f}_s(\mathbf{x})}\right\}$$

$$\text{if} \quad s(x) > 0$$

$$\max\left\{1, \frac{v_e^{(s)} - v_l^{min}}{\bar{f}_s(\mathbf{x})}\right\} < \Theta_s < \frac{v_e^{(s)} - \bar{v}_3}{\bar{f}_s(\mathbf{x})}$$
$$\max\left\{1, \frac{v_e^{(t)} - v_l^{min}}{\bar{f}_t(\mathbf{x})}, \frac{v_e^{(t)} - \underline{v}_3}{\bar{f}_t(\mathbf{x})}\right\} < \Theta_t < \frac{v_e^{(t)} - \bar{v}_1}{\bar{f}_t(\mathbf{x})}$$
$$\max\left\{1, \frac{v_e^{(l)} - v_l^{min}}{\bar{f}_l(\mathbf{x})}, \frac{v_e^{(l)} - \underline{v}_1}{\bar{f}_l(\mathbf{x})}\right\} < \Theta_l < \frac{v_e^{(l)} - \bar{v}_4}{\bar{f}_l(\mathbf{x})}$$
$$\Theta_i > \max\left\{1, \frac{v_e^{(i)} - v_l^{min}}{\bar{f}_i(\mathbf{x})}, \frac{v_e^{(i)} - \underline{v}_4}{\bar{f}_i(\mathbf{x})}\right\}$$

The proofs are technical and are omitted for sake of brevity. The interested reader can find proofs and more details in [4].

4 Some special cases

In the previous section, the hysteresis characteristics has been assumed with strictly positive loop slopes (Assumption 2.1). This limitation is mainly due to some technicalities; indeed, if not satisfied, $\bar{f}_j(\mathbf{x})$ and $v_e^{(j)}$ could not exist for all $\mathbf{x} \in \mathbb{R}^n$.

The implications associated to the condition $m_t = 0$ will be now discussed. To this purpose, let $s(\mathbf{x}) > 0$, and recall that the existence of a feasible control within a given range requires that: $i)$ a sliding motion on (7) is achieved, $ii)$ the control input belongs to the specified characteristics range, $iii)$ the inner part of the hysteresis characteristics is avoided. Being the considered characteristics interval bounded (superiorly and inferiorly), it is easy to realize that relations $i)$ and $ii)$ cannot be simultaneously satisfied as m_t tends to zero. It follows that, in the case $m_t = 0$, the stabilizing control law should never be sought in the top segment of hysteresis. As discussed in Remark 2.1, the "extended" hysteresis model described in Section 2.1 reduces to backlash with the choice $m_t = 0$, $m_b = 0$, $m_r = m_l$ and sufficiently large c_t and c_b. It has been argued before that no feasible control law can be found in the "top" and "bottom" parts of the hysteresis characteristics, since $m_t = 0$, $m_b = 0$. Moreover, choosing large c_t and c_b means translating, from a graphical viewpoint, the segments "t" and "b" in the upward and downward direction respectively of a sufficient amount (i.e. larger than the usual operating range of the hysteresis). As a consequence, v_4 moves to the left (and v_3 to the right), hence one has that $v_4 << 0$ and $v_3 >> 0$. This implies that the conditions associated to the use of the "s" and "i" segments of the hysteresis are necessarily violated, while a feasible solution can always be found using the "l" and "r" segments. Therefore, in the backlash case the control law of Theorem 3.1 reduces to a form fully equivalent to the result reported in [3].

Fully analogous arguments hold in the case of dead-zone.

5 Simulation Results

In order to validate previous theoretical results, the proposed control approach has been applied by simulation on the mechanical system, proposed in [10], representing a robot-like system with one link. Due to the large presence of mechanical components, indeed, robotics can be considered a key field where the robust compensation of actuator nonlinearities should be pursued. The system under study is described by the following model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha_1 x_2 + \alpha_2 x_2^2 \cos(x_1) - \alpha_3 \sin(x_1) + u \end{cases} \tag{11}$$

where $\alpha_i = \hat{\alpha}_i + \Delta\alpha_i$, $i = 1, \ldots, 3$ are uncertain parameters whose nominal values are given by $\hat{\alpha}_1 = \dfrac{1}{T}$, $\hat{\alpha}_2 = \bar{m}a$, $\hat{\alpha}_3 = \bar{m}ga$, being $\bar{m}$ the load mass, T the motor time constant, a the length and g the gravitational constant. As in [10] the following nominal values have been used: $T = 1$ s, $\bar{m} = 1$ kg, $a = 3.5$ m. Hysteresis parameters have been chosen as follows: $\hat{m}_r = 5$, $\hat{m}_l = 8$, $\hat{m}_t = 0.7$, $\hat{m}_b = 0.5$, $\hat{c}_r = \hat{c}_t = 1$, $\hat{c}_l = \hat{c}_b = -1$.

A 25% variation has been applied to system dynamical parameters α_i, while hysteresis parameters have been varied of 35% and 15% for c_j and m_j respectively. The system is supposed to be driven by an actuator containing an extended hysteresis nonlinearity (as in Fig.1). The following standard sliding surface has been chosen $s(\mathbf{x}) = x_2 + \lambda x_1 = 0$.

The obtained simulation results are shown in Fig. 3 for the hysteresis nonlinearity, where the first panel (a)) shows the state variables, the second (b)) and third (c)) the control variables v and u respectively, and the fourth one (d)) displays the points (marked) of the nonlinearity characteristics used by the controller. The results of Fig.3 have been obtained setting $\lambda = 7$, and using a a boundary layer [11] of width $\epsilon = 0.05$. Further results for the hysteresis with $\lambda = 5$ have been reported in Fig.4 in order to further demonstrate that the proposed controller tends to avoid the internal part of the hysteresis characteristics, trying to move over the (increasing or decreasing) boundary of Σ_B. This behavior is particularly evident when no boundary layer is present (Fig. 4-d), while the use of a boundary layer produces a ”smoothing” of this effect, as shown in Fig. 4-c.

6 Conclusions

The problem of the presence of non-smooth uncertain nonlinearities in the actuator devices has been addressed in this study. Control design techniques usually applied in practice do often ignore the presence of such nonlinearities in system components due to the lack of results in this field, and this approach may produce catastrophic effects on the control effectiveness, manly due to the time delay the hysteresis introduces.

Simulation results addressing the stabilization problem have been included to show the effectiveness of the proposed controller. The regulation problem is straightforward since Theorem 3.1 still holds if error variables instead of the plant state variables are considered.

No specific dependence on the initial conditions have been found while testing the algorithm, as theoretically expected. Control performances can be tuned by the user suitably setting the design parameters i.e. the system response can be f.i. speeded using a ’quick’ sliding surface (high values of λ), or using stronger control inputs by selecting θ_i, $i = 1, 2$ greater than the smallest limit values.

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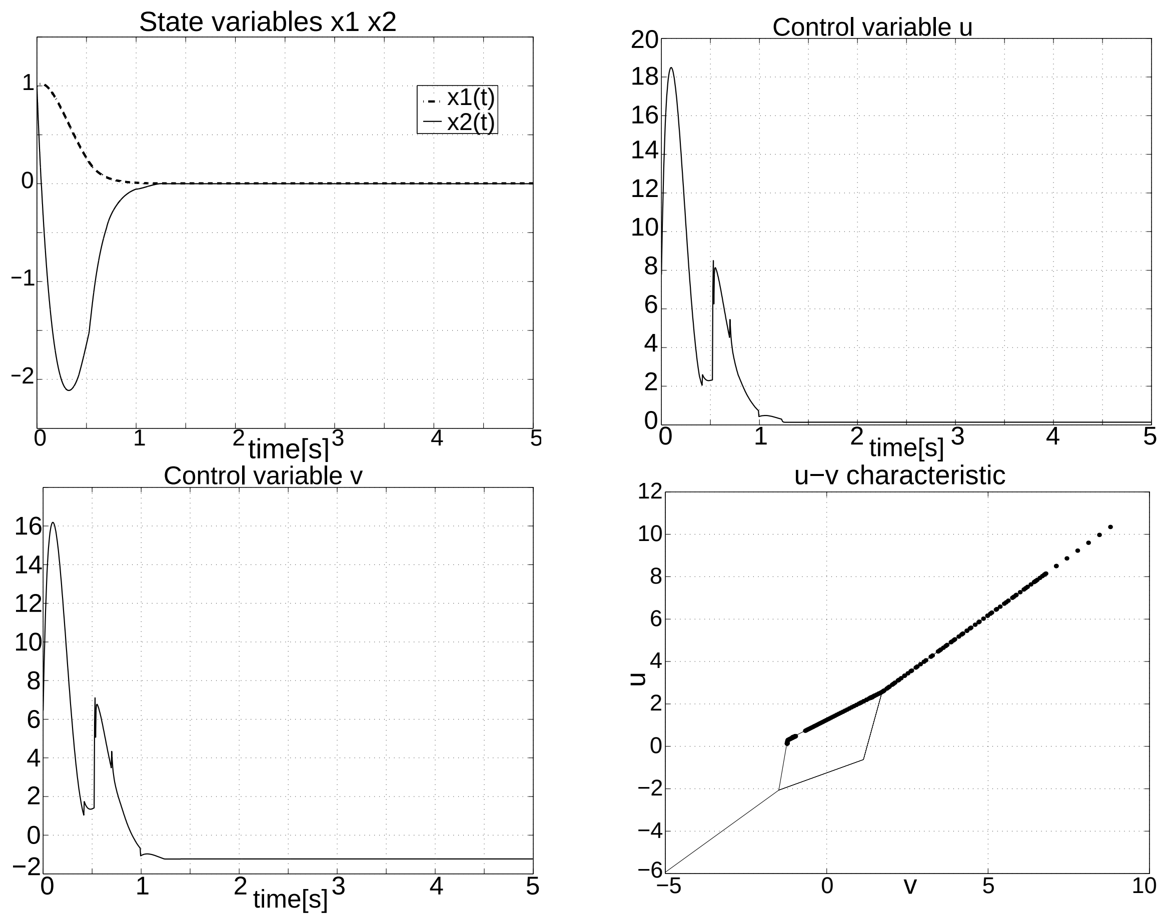

Fig.3 a),b)c)d) - Simulation results: extended hysteresis ($\lambda = 7$, $\epsilon = 0.05$)