SOLVING WEIGHTED MIXED SENSITIVITY H_∞ PROBLEM BY DECENTRALISED CONTROL FEEDBACK

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Abstract

This paper considers the problem of achieving stability and certain H_∞ performances for a large-scale system by a decentralised control feedback law. The performance problem is formulated as a standard weighted mixed sensitivity H_∞ problem. Then, to solve the proposed problem a modification of the original weighting functions is presented. Some sufficient conditions are introduced to ensure the overall stability and performance of the large-scale system. Finally, an example is used to show the effectiveness of the proposed methodology.

1 Introduction

The problem of designing a decentralised control for large-scale interconnected systems has attracted a great amount of interest in recent years, since many interconnected systems can be decomposed into several lower-order subsystems and therefore the design and implementation of each subsystem can proceed independently [1,2,4].

The mixed sensitivity approach for the robust control system design is a direct and effective way of achieving multivariable loop shaping [3,5]. In this approach, transfer function shaping problems in which the sensitivity function is shaped along with one or more other closed-loop transfer functions such as KS or the complementary sensitivity function is adopted [3,5].

In this paper a method for solving the mixed sensitivity H_∞ problem by a decentralised control is proposed. It is shown how by appropriately modifying the weighting functions in the original mixed sensitivity problem the overall stability and performance can be achieved by a decentralised feedback control.

This paper is organised as follows. In section 2, the problem of finding suitable decentralised dynamical controllers for the subsystems of a linear large-scale system is formulated. In section 3, new sufficient conditions for the stability and performance of the system are given. The conditions of section 3 can be satisfied by appropriately modifying the weighting functions. In section 4 an example is carried out to show the effectiveness of the proposed methodology. The results clearly show the achievement of desirable robust performance by decentralised proposed design.

2 Problem Formulation

Consider a large-scale system $G(s)$, with the following state-space equations

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, and composed of N linear time-invariant subsystems $G_i(s)$, described by

$$\begin{aligned}\dot{x}_i(t) &= A_{ii}x_i(t) + B_{ii}u(t)_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}x_j(t) \\ y_i(t) &= C_{ii}x_i(t)\end{aligned} \tag{2}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, $y_i \in R^{P_i}$, $A_{ii} \in R^{n_i \times n_i}$, $B_{ii} \in R^{n_i \times m_i}$ and $C_{ii} \in R^{p_i \times n_i}$. It is assumed that all (A_{ii}, B_{ii}) are controllable and (A_{ii}, C_{ii}) are observable and that all B_{ii}'s and C_{ii}'s are full rank. The term $\sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j$ is due to the interactions of the other subsystems.

The objective is to design a local output feedback dynamical controller

$$U_i(s) = K_i(s)(R_i(s) - Y_i(s)), \tag{3}$$

where R_i is the i-th reference input vector for each subsystem, to achieve desired disturbance rejection performance for the large-scale system $G(s)$. That is to design a decentralised controller

$$K(s) = diag\{K_i(s)\}, \tag{4}$$

such that

$$\left\| \begin{bmatrix} W_1 S \\ W_3 T \end{bmatrix} \right\|_\infty < 1, \tag{5}$$

where S and T are sensitivity and complementary sensitivity functions of the large-scale system respectively. In this paper the goal is to reduce the initial high dimensional H_∞ control problem to N local low dimensional H_∞ problems where each local controller is as follows

$$\left\| \begin{bmatrix} \overline{W}_{1i} S_i \\ \overline{W}_{3i} T_i \end{bmatrix} \right\|_\infty < \mathbf{1} \tag{6}$$

where

$$S_i = (I + G_{di} K_i)^{-1}, \tag{7}$$

$$T_i = I - S_i = (I + G_{di} K_i)^{-1} G_{di} K_i, \tag{8}$$

and

$$G_{di}(s): \begin{cases} \dot{x}_i = A_{ii} x_i + B_{ii} u_i \\ y_i = C_{ii} x_i \end{cases} \tag{9}$$

3 Stability Condition and Performance Achievement via Output Feedback

Consider a large-scale system with state space equations (1) composed of N subsystems given by the equations (2). Applying the decentralised controller $K = diag\{K_i\}$, the next theorem on the overall stability can be proved.

Theorem 3.1 *Assuming the decentralized controller $K(s) = diag\{K_i(s)\}$ stabilizes the diagonal system $G_d(s)$, where $G_d(s) = diag\{G_{di}(s)\}$, then $K(s)$ stabilizes $G(s)$ if*

$$\max_i \{ \| (sI - A_{ii} + B_{ii} K_i C_{ii})^{-1} \|_\infty \} < \mu^{-1}(H) \tag{10}$$

where

$H = A - diag\{A_{ii}\}$, $\|.\|_\infty$, is the maximum singular value of (.), and

$$\mu(H) = \begin{cases} \mathbf{0} & \text{if no } \Delta \text{ solves } \det(I - \Delta H) = \mathbf{0} \\ (\min_\Delta \{ \overline{\sigma}(\Delta) | \det(I - \Delta H) = \mathbf{0} \})^{-\mathbf{1}} & \text{otherwise} \end{cases}$$

Proof: Defining

$$P = (sI - A_d + BKC)^{-1} \tag{11}$$

where $A_d = diag\{A_{ii}\}$ the overall closed-loop system under decentralised control has the following transfer function

$$T(s) = (I - PH)^{-1} PBK \tag{12}$$

Since P is stabilised from the equation (12), the stability of $(I - PH)^{-1}$ results in the overall stability. The transfer function P has no unstable pole, and then the closed-loop system is stable if and only if the Nyquist plot of $\det(I - PH)$ does not encircle the origin. Hence, if

$$\|PH\|_\infty < 1 \tag{13}$$

the overall stability is guaranteed. Since

$$P = diag\{P_i\}, \tag{14}$$

and

$$P_i = (sI - A_{ii} + B_{ii} K_{ii})^{-1} \tag{15}$$

the overall stability is assured if the conditions (10) are satisfied, and the proof is complete.

The goal is to reduce the initial high dimension H_∞ control problem to N local low-dimension H_∞ control problems where each local control problem is given by the equation (6). This is accomplished by choosing appropriate weighting matrices $\overline{W}_{1i}$, and $\overline{W}_{2i}$. Then, the desired performance and stability of the overall system is achieved, by solving local H_∞ control problems. It should be mentioned in an H_∞ control problem, choosing the weighting functions are mostly based on the characteristics of interest in low and high frequencies [3,5]. The following Theorem is proved regarding the performance of the closed-loop system under decentralised control.

Theorem 3.2: *The performance objective of the form given by the equation (6) for a large-scale system given by the equations (1) is satisfied if*

$$\left\| \begin{bmatrix} \overline{W}_{1i} S_i \\ \overline{W}_{3i} T_i \end{bmatrix} \right\|_\infty < 1, \qquad i = 1,\dots,N \qquad (16)$$

where $\overline{W}_{1i} = \overline{W} W_{1i}$, *and* $\overline{W}_{3i} = W_{3i}$, *the transfer functions* W_{1i} *and* W_{3i} *are the i-th elements of diagonal weighting matrices* $W_1 = diag\{W_{1i}\}$ *and* $W_3 = diag\{W_{3i}\}$ *and* $\overline{W}$, *the scalar function is an upper bound of*

$$\left\| (I - C(sI - A_d)^{-1} HC^+ \right\|_\infty \le \left\| \overline{W} \right\|_\infty \qquad (17)$$

where

$$C^+ = C^T (CC^T)^{-1} \qquad (18)$$

Proof: Defining

$$P = (sI - A_d + BK(s)C)^{-1} \qquad (19)$$

the overall closed-loop system has the following transfer function

$$T(s) = C(I - PH)^{-1} PBK(s) \qquad (20)$$

while the matrices C, P, B, and $K(s)$ are diagonal, the matrix H (describing the interconnections) is not. The equation (20) can be written as

$$\begin{aligned} T(s) &= CPH(I - PH)^{-1} PBK(s) + CPBK(s) = \\ & CPH(sI - A - BK(s)C)^{-1} BK(s) + CPBK(s) \qquad (21) \\ &= CPHC^+ T(s) + CPBK(s) \end{aligned}$$

Hence, the closed-loop system under decentralised control is as follow.

$$T(s) = (I - CPHC^+)^{-1} CPBK(s)\,. \qquad (22)$$

Since

$$CPHC^+ = S_d C(sI - A_d)^{-1} HC^+ \qquad (23)$$

at low frequencies S_d and at high frequencies $C(sI - A_d)^{-1} HC^+$ approach zero respectively. Therefore at both low and high frequencies $(I - CPHC^+)^{-1}$ approaches to I, the identity matrix. In the equation (22), therefore at low frequencies, we have

$$T(s) \cong CPBK(s) = T_d(s) \qquad (24)$$

From the equation (21), and the above discussion we have

$$S(s) \cong S_d (I - C(sI - A_d)^{-1} HC^+)^{-1} \qquad (25)$$

At high frequencies $C(sI - A_d)^{-1} HC^+$ approaches zero, therefore it can be concluded that it suffices to modify the weighting function only at low frequencies as given by the equation (17), and the proof is complete.

From the above theorem it can be deduced by choosing

$$\overline{W}(s)\big|_{s=0} = \left\| I - C(sI - A_d)^{-1} HC^+ \right\|_\infty \qquad (26)$$

and

$$\overline{W}(s)\big|_{s=\infty} = 1 \qquad (27)$$

the overall performance will be achieved by solving the modified local H_∞ problems.

4 Illustrative Example

In this section, a robust decentralised controller is designed and applied to a linearised model of a *Three Tank System*. The system consists of three cylindrical tanks with the same diameter, which are connected by two pipes. The aim is to control the water levels in the tanks by adjusting flows of the pumps.

The linear model describing the system around the operating point $h_1 = 0.248m,\ h_2 = 0.2m,$ and $h_3 = 0.3m$, where h_1, h_2, and h_3 are the water levels of the tanks, is given by the following matrices [6].

$$A = \begin{bmatrix} -0.0146 & 0.0103 & 0 \\ 0.0103 & -0.0222 & 0.0071 \\ 0 & 0.0071 & -0.0111 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.7579e-4 & 0 \\ 0 & 0 \\ 0 & 0.7579e-4 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The desired performance is a mixed sensitivity problem such that it has attenuation at least –40dB at low frequencies and closed-loop system has a bandwidth of 0.02rad/sec. The appropriate weighting functions are $W_1 = \frac{(0.01s + 0.02)}{(s + 0.0002)}$, and $W_3 = 50s + 0.01$. Since $\left\| I - C(sI - A_d)^{-1} HC^{+} \right\|_{\infty} = 1.182$, we select $\overline{W} = \frac{0.01s + 0.025}{0.01s + 0.02}$. Figure 1 shows the modified weighting functions. Solving the two local H_{∞} problems the decentralised controller is as follows

$$A_C = \begin{bmatrix} -0.0222 & 0.0003 & -0.0002 & 0 & 0 \\ 0.0001 & -0.0002 & 0.0005 & 0 & 0 \\ 0.0002 & 0.0131 & -1.2971 & 0 & 0 \\ 0 & 0 & 0 & -0.0002 & -0.0005 \\ 0 & 0 & 0 & -0.0131 & -1.2971 \end{bmatrix},$$

$$B_C = \begin{bmatrix} 0.0051 & 0 \\ 1.1838 & 0 \\ 1.4405 & 0 \\ 0 & -1.1838 \\ 0 & 1.4405 \end{bmatrix}$$

$$C_C = \begin{bmatrix} -135.9229 & 1.4968 & 261.4501 & 0 & 0 \\ 0 & 0 & 0 & -0.0402 & 262.1696 \end{bmatrix},$$

$$D_C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Applying the designed controller to the system, the singular values of the sensitivity and complementary sensitivity functions of the overall system with the decentralised controller are plotted in Figure 2, which verifies that the design objectives have been satisfied.

5 Conclusion

In this paper, a decentralised local feedback law is designed for achieving stability and certain H_{∞} performance for a large-scale system. To solve the performance problem, which is formulated as the standard weighted mixed sensitivity H_{∞} problem, modification of the original weighting functions is proposed. Some sufficient conditions are presented which ensure the overall stability and performance of the large-scale systems.

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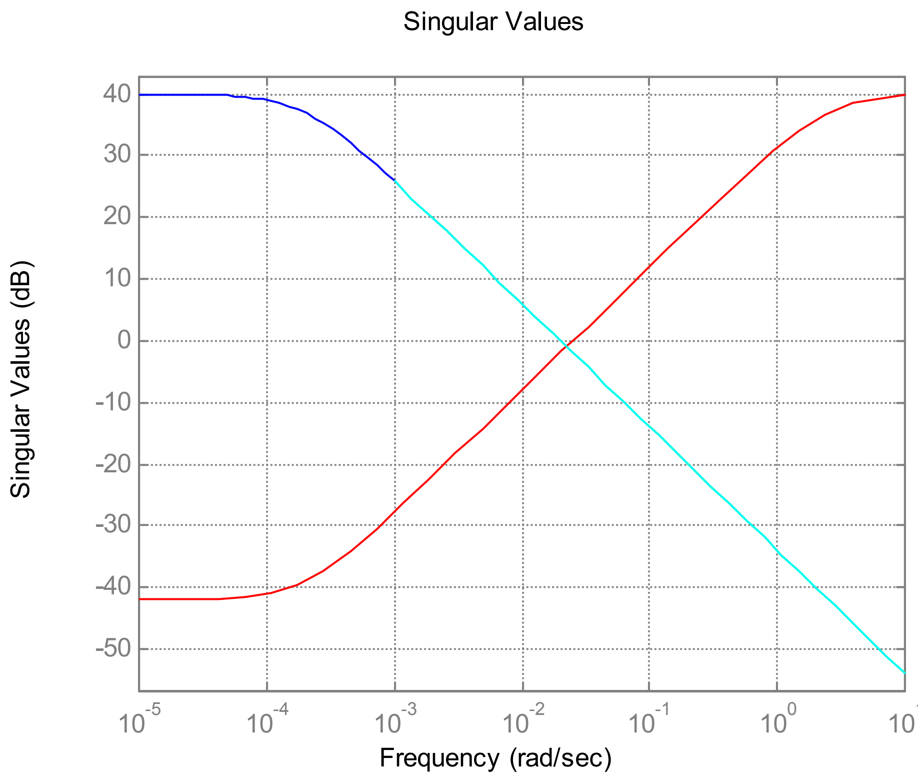

Figure 1 Singular values of the modified weighting functions ($\frac{1}{W_1}, \frac{1}{W_3}$).

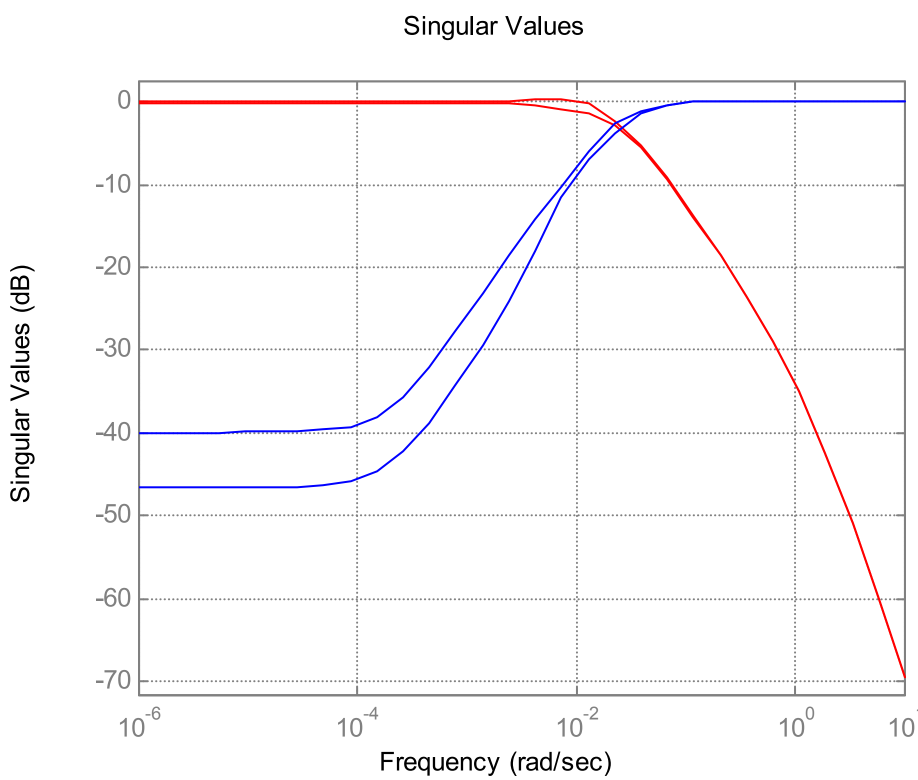

Figure 2 singular values of the sensitivity and complementary Sensitivity functions of the overall system with the decentralised control.