CONTROL OF DISCRETE LINEAR REPETITIVE PROCESSES WITH APPLICATION TO A MATERIAL ROLLING PROCESS

B. Sulikowski[1], **K. Galkowski**[1] **E. Rogers**[2], **D. H. Owens**[3]

[1] Institute of Control and Computation Engineering, University of Zielona Góra, Poland
[2] Department of Electronics and Computer Science,
University of Southampton, Southampton SO17 1BJ, UK, etar@ecs.soton.ac.uk
[3] Department of Automatic Control and Systems Engineering, University of Sheffield, UK.

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Abstract

Repetitive processes are a distinct class of 2D systems of both systems theoretic and applications interest. They cannot be controlled by direct extension of existing techniques from either standard or 2D systems theory. Here we give new results on the design of physically based feedback control laws. These results relate to design for performance and are illustrated on data for a model which arises in the modelling of a physical process.

1 Introduction

Linear repetitive processes are a distinct class of 2D systems of both system theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the new pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(p),\ 0 \leq p \leq \alpha$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the new pass profile $y_{k+1}(p),\ 0 \leq p < \alpha,\ k \geq 0$. The fact that the pass length is finite (and hence information propagation in this direction only occurs over a finite duration) is the key difference with other classes of 2D linear systems, such as those with discrete dynamics described by well known and extensively studied state space models such as that due to Roesser [5].

Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations (see, for example, [2]). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications of repetitive processes include classes of iterative learning control (ILC) schemes [1] and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle [4]. Attempts to control these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure. In particular, such an approach ignores the fact that information propagation occurs from pass-to-pass and along a given pass, and that the pass initial conditions are reset before the start of each new pass.

A rigorous stability theory for linear repetitive processes has been developed. This theory [6] is based on an abstract model in a Banach space setting which includes all such processes as special cases. Also the results of applying this theory to a wide range of special cases have been reported, including the one considered here. This has resulted in stability tests that can be implemented by direct application of well known 1D linear systems tests. In the case of ILC for the linear dynamics case, the stability theory for so-called differential and discrete linear repetitive processes is the essential basis for a rigorous stability/convergence analysis of such algorithms.

One unique feature of repetitive processes is that it is possible define physically meaningful control laws. For example, in the ILC application, one such family of control laws is composed of (state or output based) feedback control action on the current pass combined with information 'feedforward' from the previous pass (or trial in the ILC context) which, of course, has already been generated and is therefore available for use. Also it is already known that an LMI setting can be used to design such control laws for stability along the pass of differential and discrete linear repetitive processes under the action of such control laws. (In the case of discrete processes see, for example, [3]).

In this paper, we continue the development of this family of control laws for discrete processes by considering design for stability and performance closed loop. The next section gives the required background material, including the model of the material rolling process example used to illustrate the controller design algorithms developed.

2 Background

Following [6] the state-space model of a discrete linear repetitive process has the following form over $0 \leq p \leq \alpha,\ k \geq 0$

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_p(p) \end{aligned} \quad (1)$$

Here on pass k, $x_k(p)$ is the $n \times 1$ state vector, $y_k(p)$ is the $m \times 1$ pass profile vector, and $u_k(p)$ is the $l \times 1$ vector of control inputs. To complete the process description, it is necessary to specify the 'initial conditions' — termed the boundary conditions here, i.e. the state initial vector on each pass and the initial pass profile. The simplest possible form for these is

$$\begin{aligned} x_{k+1}(0) &= d_{k+1},\ k \geq 0 \\ y_0(p) &= f_0(p) \end{aligned} \quad (2)$$

where the $n \times 1$ vector d_{k+1} has known constant entries and the entries in the $m \times 1$ vector $f_0(p)$ are known functions of p. Next we introduce the example which we use in this paper to illustrate our new controller design results.

Material rolling is an extremely common industrial process where, in essence, deformation of the workpiece takes place between two rolls with parallel axes revolving in opposite directions. Figure 1 is a schematic diagram of the process where one approach is to pass the stock (i.e. the material to be rolled to a pre-specified thickness (also termed the gauge or shape)) through a series of rolls for successive reductions which can be 'costly' in terms of the equipment required. A more economic route is to use a single two high stand, where this process is often termed 'clogging'.

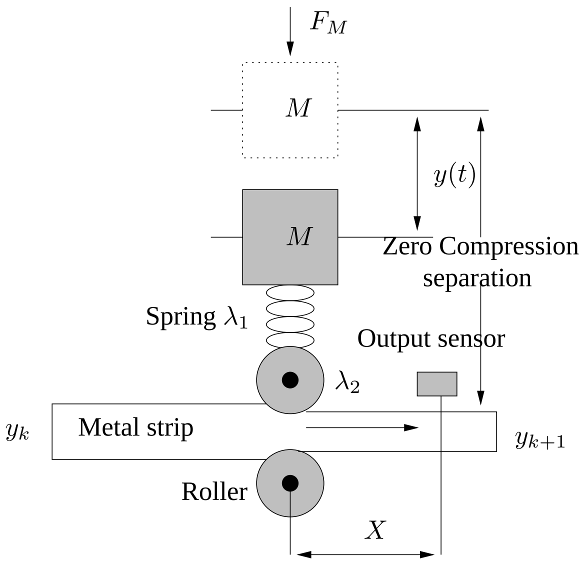


Figure 1: Material Rolling Process

In practice, a number of models of this process can be developed based on the assumptions made concerning the dynamics describing the particular mode (or modes) of operation under consideration. Here, however, it will suffice to develop a linearised model of the dynamics of the (idealised) case shown in Figure 1.

The particular task considered is the development of a simplified (but practically feasible) model relating the gauge on the current and previous passes through the rolls. These are denoted here by $y_{k+1}(t)$ and $y_k(t)$ respectively and the other process variables and physical constants are defined as follows:
F_M is the force developed by the motor;
F_s is the force developed by the spring;
M is the lumped mass of the roll-gap adjusting mechanism;
λ_1 is the stiffness of the adjustment mechanism spring;
λ_2 is the hardness of the metal strip;
$\lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ is the composite stiffness of the metal strip and the roll mechanism.

To model the basic process dynamics, refer again to Figure 1 and following [7] first note that the force developed by the motor is

$$F_M = F_s + M\ddot{y}(t) \quad (3)$$

and the force developed by the spring is given by

$$F_s = \lambda_1[y(t) + y_{k+1}(t)] \quad (4)$$

This last force is also applied to the metal strip by the rolls and hence

$$F_s = \lambda_2[y_k(t) - y_{k+1}(t)] \quad (5)$$

Hence the following linear differential equation models the relationship between $y_{k+1}(t)$ and $y_k(t)$ under the above assumptions

$$\ddot{y}_{k+1}(t) + \frac{\lambda}{M} y_{k+1}(t) = \frac{\lambda}{\lambda_1}\ddot{y}_k(t) + \frac{\lambda}{M} y_k(t) - \frac{\lambda}{M\lambda_2} F_M \quad (6)$$

Suppose now that differentiation in (6) is approximated by backward difference with sampling period T. Then the resulting difference-domain approximation is

$$\begin{aligned} y_{k+1}(t) =& a_1 y_{k+1}(t-T) + a_2 y_{k+1}(t-2T) + a_3 y_k(t) \\ &+ a_4 y_k(t-T) + a_5 y_k(t-2T) + bF_M \end{aligned} \quad (7)$$

where

$$a_1 = \frac{2M}{\lambda T^2 + M},\ a_2 = \frac{-M}{\lambda^2 T + M},\ a_3 = \frac{\lambda}{\lambda T^2 + M}\left(T^2 + \frac{M}{\lambda_1}\right)$$

$$a_4 = \frac{-2\lambda M}{\lambda_1(\lambda T^2 + M)},\ a_5 = \frac{\lambda M}{\lambda_1(\lambda T^2 + M)},\ b = \frac{-\lambda T^2}{\lambda_2(\lambda T^2 + M)}$$

Now set $t = pT$ and $y_{k+1}(p) = y_{k+1}(pT)$. Then (7) can be written as

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned}$$

where

$$\begin{aligned} x_{k+1}(p) = [\ & y_{k+1}(p-1) \quad y_{k+1}(p-2) \quad \ldots \\ & \ldots \quad y_k(p-1) \quad y_k(p-2)\ ]^T,\ u_{k+1}(p) = F_M \end{aligned}$$

$$A = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \end{bmatrix},\ B_0 = \begin{bmatrix} a_3 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \end{bmatrix},\ D = b,\ D_0 = a_3$$

This last state space model is clearly a special case of (1) and in this paper we use the numerical data $\lambda_1 = 600$, $\lambda_2 = 2000$, $M = 100$ and $T = 0.1$. This yields $\lambda = 461.54$ and

$$\begin{aligned} A &= \begin{bmatrix} 1.9118 & -0.0047 & -1.4706 & 0.7353 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} -2.2059 \times 10^{-5} \\ 0 \\ 0 \\ 0 \end{bmatrix} B_0 = \begin{bmatrix} 0.7794 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 1.9118 & -0.0047 & -1.4706 & 0.7353 \end{bmatrix} \\ D &= \begin{bmatrix} 2.2059 \times 10^{-5} \end{bmatrix},\ D_0 = \begin{bmatrix} 0.7794 \end{bmatrix} \end{aligned} \tag{8}$$

The stability theory [6] for constant pass length linear repetitive processes is based on an abstract model in a Banach space setting which includes all such processes as special cases and consists of two distinct concepts termed asymptotic stability and stability along the pass respectively. In effect, asymptotic stability demands that, for admissible inputs, the output sequence of pass profiles converge strongly as $k \to \infty$ to a so-called limit profile which for the processes considered here is described by a 1D discrete linear systems state space model. This stability definition is of the bounded-input bounded-output type (over the finite and fixed pass length where a bounded signal is defined in terms of the norm on the underlying function space) and the necessary and sufficient condition for it to hold is $r(D_0) < 1$, where $r(\cdot)$ denotes the spectral radius. Moreover, the state matrix in this limit profile state space model is (setting $D = 0$ in (1) incurs no loss of generality) $A_{lp} := A + B_0(I_m - D_0)^{-1}C$, and it is possible that an example can be asymptotically stable but have a limit profile which is 'unstable along the pass', i.e. $r(A_{lp}) \geq 1$.

This undesirable property is due to the finite pass length and stability along the pass prevents it from happening by, in effect, demanding that the bounded-input bounded-output property holds uniformly, i.e. independent of the pass length. Asymptotic stability is a necessary condition for stability along the pass and several sets of conditions for this latter property are known. Here, however, we use a sufficient condition expressed in LMI terms as summarised next.

Define the following matrices from the state space model (1)

$$\widehat{A}_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix},\quad \widehat{A}_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix} \tag{9}$$

Then we have the following sufficient condition for stability along the pass of processes described by (1) (see, for example, [3]).

Theorem 1 *A discrete linear repetitive processes described by (1) is stable along the pass if* $\exists$ *matrices* $P = P^T > 0$ *and* $Q = Q^T > 0$ *satisfying the following LMI*

$$\begin{bmatrix} \widehat{A}_1^T P \widehat{A}_1 + Q - P & \widehat{A}_1^T P \widehat{A}_2 \\ \widehat{A}_2^T P \widehat{A}_1 & \widehat{A}_2^T P \widehat{A}_2 - Q \end{bmatrix} < 0 \tag{10}$$

Even though this condition is sufficient but necessary, previous work has concluded that this is offset by the fact that it immediately leads, see the next section, to a systematic and easily implemented method for designing a powerful class of control laws for stability along the pass under control action. Moreover, the new results in this paper show that this design setting can also be extended in a straightforward way to deal with control law design for stability coupled with well defined performance requirements.

3 LMI based Controller Design for Stability

The design of control laws for 2D discrete linear systems described by, for example the Roesser [5] state space model has received very considerable attention in the literature over the years. A valid criticism of such work, however, is that the structure of the control laws used is not well founded physically due to the fact that, for example, the concept of a state of these systems is not uniquely defined. For example, it is possible to define a state feedback law based on the local or global state vectors. Also in the absence of generalizations of well defined and understood 1D concepts, e. g. the pole assignment problem and error actuated output feedback control action, it has not been really possible to formulate a control design problem beyond that of obtaining conditions for stabilization under the control action.

The first difficulty above does not arise with linear repetitive processes. For example, it is physically meaningful to define the current pass error as the difference, at each point along the pass, between a specified reference trajectory for that pass (which in most cases will be the same on each pass) and the actual pass profile produced. Then one can define a so-called current pass error actuated controller which uses the generated error vector to construct the current pass control input vector. In which context, preliminary work (see, for example, [2]) has shown that, except in a few very restrictive special cases, the controller used must be actuated by a combination of current pass information and 'feedforward' information from the previous pass to guarantee even stability along the pass closed loop. (Note that in the ILC application area the previous trial output vector is an obvious signal to use as feedforward action.)

As the first attempt at removing the second difficulty outlined above, previous work (see, for example, [3]) has considered a control law of the following form over $0 \le p \le \alpha,\ k \ge 0$

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p) := K \begin{bmatrix} x_{k+1}(p) \\ y_k(p) \end{bmatrix} \quad (11)$$

where K_1 and K_2 are appropriately dimensioned matrices to be designed. In effect, this control law uses feedback of the current pass state vector (which is assumed to be available for use here) and 'feedforward' of the previous pass profile vector. (Note that in repetitive processes the term feedforward is used to describe the case where (state or pass profile) information from the previous pass (or passes) is used as (part of) the input to a control law applied on the current pass, i.e. to information which is propagated in the pass-to-pass (k) direction.)

This control law has clear physical meaning for practical applications of discrete linear repetitive processes and the following result uses the LMI setting to give a controller design algorithm which can be easily implemented.

Theorem 2 *[3] Suppose that a discrete linear repetitive process of the form described by (1) is subjected to a control law of the form (11). Then the closed loop system is stable along the pass if there exists matrices $Y = Y^T > 0,\ Z = Z^T > 0$, and N such that the following LMI holds*

$$\begin{bmatrix} Z - Y & 0 & Y\widehat{A}_1^T + N^T\widehat{B}_1^T \\ 0 & -Z & Y\widehat{A}_2^T + N^T\widehat{B}_2^T \\ \widehat{A}_1 Y + \widehat{B}_1 N & \widehat{A}_2 Y + \widehat{B}_2 N & -Y \end{bmatrix} < 0, \quad (12)$$

where

$$\widehat{B}_1 = \begin{bmatrix} B \\ 0 \end{bmatrix},\ \widehat{B}_2 = \begin{bmatrix} 0 \\ D \end{bmatrix} \quad (13)$$

If (12) holds, then a stabilizing K in the control law (11) is given by

$$K = NY^{-1} \quad (14)$$

4 Model Matching Based Design

The controller design procedure outlined in the previous section guarantees closed loop stability along the pass but not resulting closed loop performance. In this and the next section we give new results which address the currently open question of how to design a control law for discrete linear repetitive processes for both closed loop stability along the pass and performance.

Model following control is a long standing technique in standard (or 1D) systems theory and there has also been some work on this problem for 2D discrete linear systems described by the Roesser and Fornasini Marchesini state space models, see, for example, [7]. Below, we give some new results which provide a possible starting point for the development of a 'mature' model following control theory for discrete linear repetitive processes.

First note that the state space quadruple $\{A, B_0, C, D_0\}$ describes the contribution of the previous pass profile to the current one. Also under the action of the control law (11) this quadruple is 'mapped' as follows

$$\begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix} \rightarrow \left[\begin{array}{c|c} A + BK_1 & B_0 + BK_2 \\ \hline C + DK_1 & D_0 + DK_2 \end{array}\right]$$

Suppose also that we want to assign the closed loop matrices here to $\{\mathcal{A},\ \mathcal{B}_0,\ \mathcal{C},\ \mathcal{D}_0\}$, where these matrices are selected to give a state space model whose behavior the controlled process is required to follow (in terms of the contribution of the previous pass profile to the current one). Then the following result is relevant.

Theorem 3 *Suppose that a discrete linear repetitive process of the form described by (1) is subjected to a control law of the form (11). Then the resulting closed loop process is stable along the pass and reaches the required form $\{\mathcal{A},\ \mathcal{B}_0,\ \mathcal{C},\ \mathcal{D}_0\}$ if $\exists$ matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that*

$$\begin{bmatrix} Z - Y & 0 & Y\widetilde{A}_1^T + N^T\hat{B}_1^T \\ 0 & -Z & Y\widetilde{A}_2^T + N^T\hat{B}_2^T \\ \widetilde{A}_1 Y + \hat{B}_1 N & \widetilde{A}_2 Y + \hat{B}_2 N & -Y \end{bmatrix} < 0 \quad (15)$$

where

$$\widetilde{A}_1 = \begin{bmatrix} A - \mathcal{A} & B_0 - \mathcal{B}_0 \\ 0 & 0 \end{bmatrix},\ \widetilde{A}_2 = \begin{bmatrix} 0 & 0 \\ C - \mathcal{C} & D_0 - \mathcal{D}_0 \end{bmatrix}$$

and the other matrices are the same as before. If this last condition holds, the required control law matrices K_1 and K_2 are computed using (14).

Proof First note again that if the LMI (12) holds then the control law matrix $K = [K_1\, K_2]$ is given by (14). Also it is a standard fact that it is possible to obtain from the LMI solver a matrix K such that

$$\left[\begin{array}{c|c} A - \mathcal{A} & B_0 - \mathcal{B}_0 \\ \hline C - \mathcal{C} & D_0 - \mathcal{D}_0 \end{array}\right] + \left[\begin{array}{c} B \\ \hline D \end{array}\right] [K_1\, K_2] = 0 \quad (16)$$

holds. In which case, the closed loop system matrices are such that

$$\left[\begin{array}{c|c} \tilde{A} & \tilde{B}_0 \\ \hline \tilde{C} & \tilde{D}_0 \end{array}\right] := \left[\begin{array}{c|c} A + BK_1 & B_0 + BK_2 \\ \hline C + DK_1 & D_0 + DK_2 \end{array}\right] = \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B}_0 \\ \hline \mathcal{C} & \mathcal{D}_0 \end{array}\right] \quad (17)$$

which completes the proof.

It is essential to note here that it is impossible to obtain an arbitrarily specified set $\{\mathcal{A},\ \mathcal{B}_0,\ \mathcal{C},\ \mathcal{D}_0\}$ starting from a given set $\{A,\ B_0,\ C,\ D_0\}$. However, conditions under which (16) has a solution can be characterized easily using, for example, Cramer's rule for linear vector equations and the matrix Kronecker matrix product. This is clearly an area for further research and in the remainder of this paper another approach to control law design for stability along the pass and desired performance is developed.

5 Design for Performance

In common with 1D linear systems, a natural approach to the control of repetitive processes is to specify a reference signal as desired performance on each pass and then attempt to use a control law to achieve this goal. Here we consider first the case when the term $K_3 r_{k+1}(p),\ 0 \leq p \leq \alpha - 1$ is added to the control law where $r_{k+1}(p)$ is an $m \times 1$ column vector representing desired behavior on pass $k+1,\ k \geq 0$, and K_3 is an $r \times m$ controller matrix to be selected. This results in the closed loop process state space model

$$\begin{aligned} x_{k+1}(p+1) &= \mathcal{A}x_{k+1}(p) + BK_3 r_{k+1}(p) + \mathcal{B}_0 y_k(p) \\ y_{k+1}(p) &= \mathcal{C}x_{k+1}(p) + DK_3 r_{k+1}(p) + \mathcal{D}_0 y_k(p) \end{aligned} \quad (18)$$

Obvious questions which now arise are: (i) what is a suitable choice for $r_{k+1}(p)$?; and (ii) how can we design the control law to give stability along the pass plus 'acceptable' (in an appropriate sense) performance?

To illustrate what can be achieved here, we focus on the single-input single-output case and use the material rolling problem data given earlier in this paper. In this application, an appropriate choice for the current pass reference signal is $r_{k+1}(p) = -1,\ 0 \leq p \leq \alpha - 1,\ k \geq 0$, i.e the objective is to reduce the material thickness by one unit which is modeled by a downward unit step applied at $p = 0$ on each pass. (Since the process is linear, any target reduction by a constant amount can be studied by simple scaling of the output pass profiles to a unit step demand.)

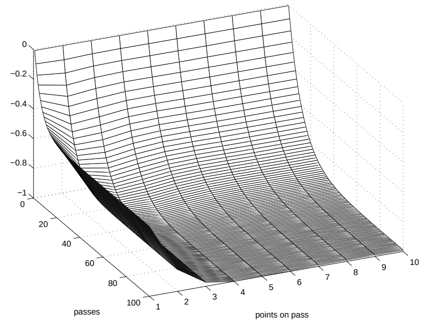
One possible way of designing the control law is to note that K_3 does not influence stability along the pass. Hence we can execute the LMI design of Section 3 to obtain control law matrices K_1 and K_2 which ensure closed loop stability along the pass and then attempt to select a suitable K_3 to meet the performance requirements by 'tuning' the response of the resulting closed loop process model. In the case of the given numerical data, it is easily checked that this model is unstable along the pass and the stabilization procedure of Theorem 3 provides the control law matrices K_1 and K_2 as

$$\begin{aligned} K_1 &= \left[\begin{array}{cccc} 83203.2 & 4226.3129 & -67555.2 & 29928.8 \end{array}\right], \\ K_2 &= \left[\begin{array}{c} -2882.67 \end{array}\right] \end{aligned} \quad (19)$$

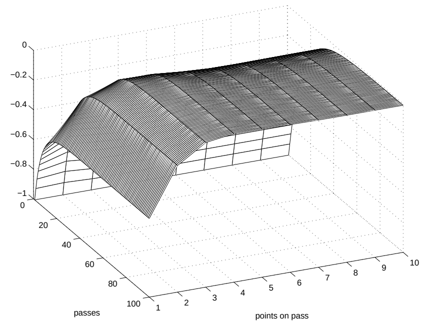
and in the resulting stable along the pass closed loop process (see (17))

$$\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B}_0 \\ \hline \mathcal{C} & \mathcal{D}_0 \end{array}\right] = \left[\begin{array}{cccc|c} 0.0764 & -0.0979 & 0.0196 & 0.0751 & 0.8430 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ \hline 0.0764 & -0.0979 & 0.0196 & 0.0751 & 0.8430 \end{array}\right] \quad (20)$$

Figure 2 shows the sequence of pass profiles and the tracking error on each pass for the case when $K_3 = -3765.8$ where this value was arrived at by repeated numerical experimentation with the objective of obtaining the smallest error between



Pass Profiles



Tracking Error
Figure 2

$r_{k+1}(p)$ and $y_k(p)$ anywhere in the domain of operation. This is an acceptable design, especially as it does not contain any oscillations in the transients along any pass (not a desirable feature in material rolling). The empirical nature of the above approach, however, means that it is clearly not feasible in the general case. Next we develop a systematic method for the task under consideration and again illustrate it on the material rolling problem data.

This new approach is based on a (simple structure) reformulation of the problem, starting from the fact that the control task here is to drive the process pass profiles to some prescribed reference signal $y_{ref}(p)$ (which in the material rolling case) is a constant positive thickness after the rolling operation is complete, i.e. $y_{ref}(p) \equiv y_{ref},\ 0 \leq p \leq \alpha - 1$. It is an immediate consequence (see Section 2 in this paper) of the stability theory that if asymptotic stability holds then the pass profile sequence converges to a steady, or so-called limit, profile described for discrete linear repetitive processes by a 1D linear systems state space model. Here what we are aiming to do is to specify this limit profile as y_{ref}.

To solve this last problem introduce a new, modified output vector variable termed the incremental pass profile vector as

$$\chi_k(p) := y_k(p) - y_{ref} \quad (21)$$

Then it is clear that the design requirement here requires that

$$\chi_k(p) \to 0,\ 0 \leq p \leq \alpha - 1,\ k \to \infty \quad (22)$$

Now replace the process state space model (1) by the following one obtained from it by substitution using (21)

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0\chi_k(p) \\ \chi_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0\chi_k(p) \end{aligned} \quad (23)$$

and apply to it the following control law (which is clearly of the form (14), i.e. current pass state feedback augmented in this case by feedforward of the difference between $y_k(p)$ and y_{ref})

$$\begin{aligned} u_{k+1}(p) &= K_1x_{k+1}(p) + K_2\chi_k(p) \\ &= K_1x_{k+1}(p) + K_2(y_k(p) - y_{ref}) \end{aligned} \quad (24)$$

Also choose this control law to transform the process of (23) into the form of those for which Theorem 3 holds, i.e. $\{\mathcal{A}, \mathcal{B}_0, \mathcal{C}, \mathcal{D}_0\}$. Then it follows immediately that this resulting closed loop model must be stable along the pass and also (22) holds. Moreover,

$$x_k(p) \to 0,\ 0 \le p \le \alpha - 1, k \to \infty \quad (25)$$

which is a natural, and most frequently obtained, result when using the LMI based approach to controller design. Also, no oscillations can occur in the resulting pass profiles (which is clearly a required feature in the specific material rolling example considered here).

Given this designed feedback law and converting back to the original pass profile vector $y_k(p)$ we obtain the resulting closed loop state space model

$$\begin{aligned} x_{k+1}(p+1) =& \mathcal{A}x_{k+1}(p) + \mathcal{B}_0(y_k(p) - y_{ref}) \\ y_{k+1}(p) =& \mathcal{C}x_{k+1}(p) + \mathcal{D}_0y_k(p) + (I - \mathcal{D}_0)y_{ref} \end{aligned} \quad (26)$$

which is stable along the pass and whose limit pass profile, due to (22) and (25), is clearly equal to y_{ref}, i.e. the control design task has been exactly achieved.

To illustrate this approach, return to the open loop model data here and execute this design for $y_{ref}(p) = -1,\ 0 \le p \le \alpha - 1,\ k \ge 0$. This produces the simulation results of Figure 3 for the resulting closed loop process in the case of zero boundary conditions and these confirm that the design objective has indeed been achieved.

6 Conclusions

Previous work has shown that an LMI setting an be used to design control laws to ensure closed loop stability along the pass of discrete linear repetitive processes. These control laws are based on an additive combination of current pass state feedback and feedforward action based on the previous pass profile. This paper has developed, and illustrated using of a physical example, new results which show that this LMI setting can be extended/augmented to enable the design of a class of physically based control laws for these processes which also meet specified performance objectives. Detailed investigation (and extension) of these results is currently under way and results from this work will be reported in due course.

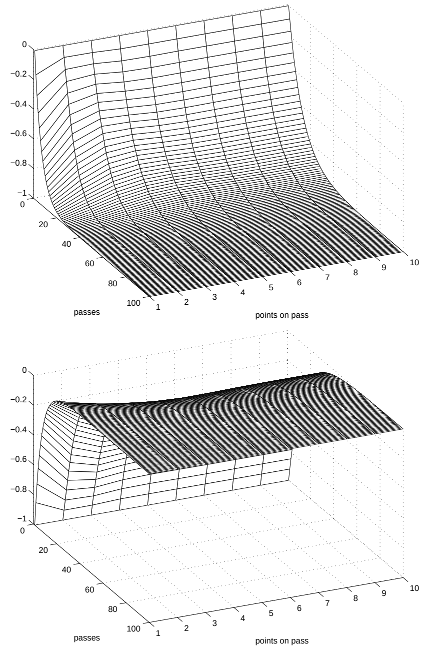


Figure 3: Pass profiles (top), tracking error (bottom)

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