PERSISTENT BOUNDED DISTURBANCE REJECTION FOR IMPULSIVE SYSTEMS WITH POLYTOPIC UNCERTAINTIES

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Abstract

The problem of bounded disturbance rejection for linear impulsive systems with polytopic uncertainties is considered in this paper. By using the Lyapunov function and positively invariant set method, a sufficient condition for robustly internal stability and L_1-performance of the impulsive systems is obtained in terms of linear matrix inequalities. A simple algebraic approach to the design of a linear state-feedback controller that robustly stabilizes the system and achieves a desired level of disturbance attenuation is proposed. Furthermore, since the Lyapunov function matrix is decoupled from coefficient matrices in the newly obtained sufficient criterion, it is convenient to study the robustness problem for impulsive systems with respect to polytopic uncertainty. A numerical example is worked out to illustrate the efficiency of the proposed approach and less conservatism of the newly obtained results.

1 Introduction

Impulsive systems arise in areas such as neural networks, communication, rhythm in medicine and biology, optimal control in economics and so on (see, e.g., [1]–[6] and the references therein). Recently, the issue of analysis and stabilization of disturbed impulsive systems has attracted much attention [4], [6]. The problem of persistent bounded disturbance rejection is of considerable practical importance because it is concerned with minimizing the maximum magnitude of the system error [7]. Particularly, this problem for linear systems without impulsive effects has been extensively studied in recent years (see, e.g., [8, 9] and the references therein). However, there have been few results concerning the same problem for uncertain impulsive systems (even for impulsive systems without uncertainty) so far (see [10]). The aim of this work is to provide some further results in this direction.

To be specific, we investigate the robust stability (with respect to polytopic uncertainty) and performance of linear impulsive systems subject to persistent bounded disturbances. By using positively invariant set analysis and Lyapunov function method, we establish a sufficient condition for existence of a state-feedback controller that ensures the internal stability and the desired level of bounded disturbance attenuation for impulsive systems in terms of linear matrix inequalities (LMIs). Furthermore, we obtain the new results (Theorems 3, 4) by decoupling Lyapunov matrix from all coefficient matrices of the system. The obtained result does not require the existence of a common positive definite matrix solution to all vertex systems. Hence our proposed method is easy to use and has less conservatism than that of general quadratic stability and performance. Moreover, the present results on nonstrict proper case (for the channel from disturbance to the regulated output) with polytopic uncertainty extend the results of [10]. Finally, we also give a numerical example to illustrate the efficiency and less conservatism of the theoretical results.

This paper is organized as follows. Some preliminaries and supporting results are presented in the next section. The main results on stability and performance are given in Section 3. A numerical example illustrating the efficiency of the proposed approach is given in Section 4. We provide the conclusions in the last Section.

2 Preliminaries

In this paper, $\mathbf{R}$ is the set of all real numbers. $\mathbf{R}^n$ is the set of all n-tuples of real numbers, and $\mathbf{R}^{m\times n}$ the set of all real matrices with m rows and n columns. $\mathbf{BR}^p = \{w \in \mathbf{R}^p : ||w||_2 \le 1\}$ denotes the closed unit ball in the space $\mathbf{R}^p$. Denote by A^T and A^{-1} the transpose and the inverse of a matrix A (if it is invertible), and by I the unit matrix of appropriate dimensions. Recall that Schur complement formula (see [12], [13] for more details), namely

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} < 0 \ (\le 0)$$

if and only if one of the following conditions holds.

1) $R_{22} < 0$ and $R_{11} - R_{12}R_{22}^{-1}R_{12}^T < 0 \ (\le 0)$;

2) $R_{11} < 0$ and $R_{22} - R_{12}^T R_{11}^{-1} R_{12} < 0 \ (\le 0)$.

Consider the following impulsive system (denoted by Σ):

$$\begin{cases} \dot{x} = Ax + Bu + \bar{B}w, t \neq t_k \\ \Delta x(t) = Ex(t_k) + \hat{B}u, t = t_k \\ z = Cx + Dw \\ x(t_0) = x(0) = 0 \end{cases} \tag{1}$$

where $x(\cdot) : \mathbf{R} \to \mathbf{R}^n$, $u(\cdot) : \mathbf{R} \to \mathbf{R}^m$, $w(\cdot) : \mathbf{R} \to \mathbf{R}^p$ and $z(\cdot) : \mathbf{R} \to \mathbf{R}^p$ are the state, the input, the external disturbance, and the regulated output vectors, respectively. A, B, $\bar{B}$, $\hat{B}$, C, D, E are known real constant matrices of appropriate dimensions. $\Delta x(t) = x(t_k^+) - x(t_k^-)$, $\lim_{h\to 0^+} x(t_k - h) = x(t_k^-)$, $\lim_{h\to 0^+} x(t_k + h) = x(t_k^+)$. $0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, and $t_k \to \infty$ as $k \to \infty$. Assume $\lim_{h\to 0^+} x(t_k - h) = x(t_k^-) = x(t_k)$, that is, the solution $x(t)$ of system (1) is right continuous at t_k. Also assume that the admissible disturbance set is $\mathcal{W} := \{w : \mathbf{R} \to \mathbf{B}\mathbf{R}^p,\ w$ is measurable $\}$, the L_∞ norm is defined by $\|w\|_\infty =: \sup_t \|w(t)\|_2$.

Recall that a set Ω is said to be *positively invariant* for a dynamical system, if $x(0) \in \Omega$, then the trajectory $x(t)$ of the system remains in Ω for all $t > 0$. Similarly, we can define the robust positive invariant set for an uncertain system. The *origin-reachable set* $(R_\infty(0))$ of a system is the set that the state of the system can reach from the origin for all admissible disturbances. It is the minimal robust closed positive invariant set containing the origin. The induced L_∞ norm of the system is $\|\Sigma\|_{i\infty} = \sup_{R_\infty(0)} \|z\|_\infty$.

For a given scalar $\rho > 0$, consider the uncontrolled system

$$\begin{cases} \dot{x} = Ax + \bar{B}w,\ t \neq t_k \\ \Delta x(t) = x(t_k^+) - x(t_k^-) = Ex(t_k),\ t = t_k \\ z = Cx + Dw \\ x(t_0) = x(0) = 0 \end{cases} \tag{2}$$

The system with initial state $x(0) = 0$ is said to have ρ-*performance* if $\|z\|_\infty \leq \rho$ for all $w \in \mathcal{W}$. Define the performance set

$$\Omega(\rho) = \{x :\ \|z\|_\infty = \|Cx + Dw\|_\infty \leq \rho,\ \forall w \in \mathcal{W}\}.$$

Thus, if $R_\infty(0) \subset \Omega(\rho)$, then the *system has ρ-performance.*

The main objective of this paper is to find for system (1) a state-feedback controller $u = Kx$ with $K \in \mathbf{R}^{m\times n}$ a constant matrix, such that the resulting closed-loop system

$$\begin{cases} \dot{x} = (A + BK)x + \bar{B}w,\ t \neq t_k \\ \Delta x(t) = (E + \hat{B}K)x(t_k),\ t = t_k \\ z = Cx + Dw \\ x(t_0) = x(0) = 0 \end{cases} \tag{3}$$

satisfies the following conditions:

(i) The system is internally stable, namely, the system without external disturbance (i.e., $w = 0$) is asymptotically stable; and

(ii) For a given scalar $\rho > 0$, the system has ρ-performance.

In dealing with such a problem, we will make use of the concept of robust attractor of a disturbed dynamical system. A set Ω is said to be a *robust attractor* of system (2) (or (3)) with respect to (w.r.t.) $w \in \mathcal{W}$, if all the state trajectories of the system initiating from the exterior of Ω eventually enter and remain in Ω for all $w \in \mathcal{W}$. Obviously, a robust attractor is robustly positively invariant.

The following inequality will be useful in our discussion.

Lemma 1. ([11]) Let P, $\bar{B}$ be matrices of appropriate dimensions, then for any scalar $\alpha > 0$, it follows that

$$2x^T P\bar{B}w \leq \frac{1}{\alpha} x^T P\bar{B}\bar{B}^T Px + \alpha w^T w.$$

3 Main results

3.1 The case of systems without uncertainty

For a positive definite matrix P, denote the ellipsoid $\Omega_P = \{x : x^T Px \leq 1\}$. We first consider the uncontrolled system (2).

Theorem 1. For a given scalar $\rho > 0$, if there exist a positive definite matrix P and a scalar $\alpha > 0$ such that the following conditions hold:

$$\begin{bmatrix} PA + A^T P + \alpha P & P\bar{B} \\ \bar{B}^T P & -\alpha I \end{bmatrix} < 0, \tag{4}$$

$$\begin{bmatrix} -P & (I+E)^T P \\ P(I+E) & -P \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} \alpha P & 0 & C^T \\ 0 & (\rho^2 - \alpha)I & D^T \\ C & D & I \end{bmatrix} > 0. \tag{6}$$

Then system (2) is internally stable and Ω_P is a robust attractor of it w.r.t. $w \in \mathcal{W}$. Moreover, $\Omega_P \subset \Omega(\rho)$ and hence system (2) has ρ-performance.

Proof. To prove that Ω_P is a robust attractor of system (2) w.r.t. $w \in \mathcal{W}$, we only need to show that the time derivative of $V(x) = x^T Px$ along the solution of the system is negative for any $x \notin \Omega_P$ (see [10] for more details, it is obtained by using Lemma 1).

Next, we show that the system has ρ-performance. In fact, by Schur complement formula, condition (6) is equivalent to

$$\begin{bmatrix} \alpha P - C^T C & -C^T D \\ -D^T C & (\rho^2 - \alpha)I - D^T D \end{bmatrix} > 0,$$

which implies $0 < \alpha < \rho^2$. This is equivalent to

$$\alpha x^T Px + (\rho^2 - \alpha)w^T w - \|Cx + Dw\|^2 > 0.$$

From this and $0 < \alpha < \rho^2$, it is clear that if $x^T Px \leq 1$ and $w^T w \leq 1$, then $\|Cx + Dw\| < \rho$, which implies $\Omega_P \subset \Omega(\rho)$, and hence $R_\infty(0) \subset \Omega_P \subset \Omega(\rho)$. This completes the proof. ■

Remark 1. (see [10]) For a fixed $\alpha > 0$, condition (4) in Theorem 1 is an LMI in P. Since A is stable, let $\lambda_m = -2\max(\mathrm{Re}\lambda(A))$ (where $\mathrm{Re}\lambda(A)$ denotes the real part of eigenvalues of matrix A), then there exists an $\alpha \in (0, \lambda_m)$ such that $A + \frac{\alpha}{2}I =: \hat{A}$ is stable. For a small $\epsilon > 0$, let $\tilde{B} = \frac{1}{\sqrt{\alpha}}\bar{B}$, $\hat{C} = \epsilon I$. Consider the system Σ_1:

$$\begin{cases} \dot{x} = \hat{A}x + \tilde{B}u \\ y = \hat{C}x \end{cases}.$$

By bounded real lemma (see, e.g., [12] [13]), that the H_∞ norm of the transfer function $H(s) = \hat{C}(sI - \hat{A})^{-1}\tilde{B}$ of system Σ_1 is less than 1, i.e., $H^*(s)H(s) \leq I$ for Re$s \geq 0$, is equivalent to the existence of a positive definite matrix P such that

$$\begin{aligned} & PA + A^TP + \frac{1}{\alpha}P\bar{B}\bar{B}^TP + \alpha P + \epsilon^2 I \\ = \;& P\hat{A} + \hat{A}^TP + P\tilde{B}\tilde{B}^TP + \hat{C}^T\hat{C} \\ < \;& 0. \end{aligned}$$

This implies $PA + A^TP + \frac{1}{\alpha}P\bar{B}\bar{B}^TP + \alpha P < 0$, which is in turn equivalent to (4) by Schur complement. This provides a frequency-domain explanation of condition (4).

We are now able to give the following result.

Theorem 2. For system (1) and a given performance level $\rho > 0$, if there exist a matrix $M \in \mathbf{R}^{m\times n}$, a positive definite matrix Q, and a scalar $\alpha > 0$, satisfying the following conditions:

$$\begin{bmatrix} AQ + QA^T + \alpha Q + BM + M^TB^T & \bar{B} \\ \bar{B}^T & -\alpha I \end{bmatrix} < 0, \quad (7)$$

$$\begin{bmatrix} -Q & (I+E)Q + \hat{B}M \\ Q(I+E)^T + M^T\hat{B}^T & -Q \end{bmatrix} < 0, \quad (8)$$

$$\begin{bmatrix} -\alpha Q & 0 & QC^T \\ 0 & -(\rho^2 - \alpha)I & D^T \\ CQ & D & -I \end{bmatrix} < 0, \quad (9)$$

then the closed-loop system (3) is internally stable and $\Omega_{Q^{-1}}$ is a robust attractor of system (3) w.r.t. $w \in \mathcal{W}$, where the state-feedback gain matrix

$$K = MQ^{-1}. \quad (10)$$

Moreover, $\Omega_{Q^{-1}} \subset \Omega(\rho)$ and hence the closed-loop system (3) has ρ-performance.

Proof. Take $V(x) = x^TPx$ with $P = Q^{-1}$. Let $u = Kx = MQ^{-1}x$ in system (1). Substituting $K = MP$ into (7), (8) and using congruent transformation, we get the conditions in Theorem 1 for the closed-loop system, hence the desired result follows. ■

3.2 Decoupling Lyapunov function matrix from system coefficient matrices

In what follows, we will discuss the decoupling of Lyapunov function matrix from system coefficient matrices.

Lemma 2. The following conditions are equivalent:

i) There exist $Q > 0$ and $M \in \mathbf{R}^{m\times n}$ such that (8) holds;

ii) There exist $Q > 0$, $Z \in \mathbf{R}^{n\times n}$ and $M \in \mathbf{R}^{m\times n}$ such that the following inequality holds.

$$\begin{bmatrix} -Q & (I+E)Z + \hat{B}M \\ Z^T(I+E)^T + M^T\hat{B}^T & -Z - Z^T + Q \end{bmatrix} < 0. \quad (11)$$

Proof. i)$\Rightarrow$ ii) Obvious.

ii)$\Rightarrow$ i) Since the matrix $[I \quad (I+E)]$ is of full row rank, premultiplying $[I \quad (I+E)]$ and postmultiplying its transpose on both sides of (11), we obtain

$$-Q + (I+E)M^T\hat{B}^T + \hat{B}M(I+E)^T + (I+E)Q(I+E)^T < 0.$$

This is equivalent to (8) by Schur complement. ■

Lemma 3. The following conditions are equivalent:

i) There exists $Q > 0$ such that (9) holds;

ii) There exist $Q > 0$ and $Z \in \mathbf{R}^{n\times n}$ such that the following inequality holds:

$$\begin{bmatrix} -\alpha(Z^T + Z - Q) & 0 & Z^TC^T \\ 0 & -(\rho^2 - \alpha)I & D^T \\ CZ & D & -I \end{bmatrix} < 0. \quad (12)$$

Proof. Obviously, by using Schur complement formula and congruent transformation, (9) is equivalent to

$$\begin{bmatrix} -Q & 0 & QC^T \\ 0 & -\alpha(\rho^2 - \alpha)I & \alpha D^T \\ CQ & \alpha D & -\alpha I \end{bmatrix} < 0.$$

Furthermore, this inequality is equivalent to

$$\begin{bmatrix} -Q & QC^TD \\ D^TCQ & -\alpha[(\rho^2 - \alpha)I + D^TD] \end{bmatrix} < 0.$$

Premultiplying and postmultiplying

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

on both sides of the above inequality, we obtain

$$\begin{bmatrix} -\alpha[(\rho^2 - \alpha)I + D^TD] & D^TCQ \\ QC^TD & -Q \end{bmatrix} < 0.$$

By similar arguments as in Lemma 2, this is equivalent to the existence of $Z \in \mathbf{R}^{n\times n}$ such that

$$\begin{bmatrix} -\alpha[(\rho^2 - \alpha)I + D^TD] & D^TCZ \\ Z^TC^TD & -Z^T - Z + Q \end{bmatrix} < 0.$$

By similar analysis, we can obtain equivalently

$$\begin{bmatrix} -(Z^T + Z - Q) & 0 & Z^TC^T \\ 0 & -\alpha(\rho^2 - \alpha)I & \alpha D^T \\ CZ & \alpha D & -\alpha I \end{bmatrix} < 0.$$

This is equivalent to (12) by similar arguments. ■

Lemma 4. If there exist a positive definite matrix Q and two scalars $\epsilon > 0,\ \alpha > 0$, such that

$$\begin{bmatrix} -\epsilon^{-1}Q + \alpha Q & Q + \epsilon AQ & \bar{B} \\ Q + \epsilon QA^T & -\epsilon Q & 0 \\ \bar{B}^T & 0 & -\alpha I \end{bmatrix} < 0 \qquad (13)$$

then $\epsilon\alpha < 1$ and by Schur complement formula (13) is equivalent to

$$F_{Q,\alpha}(\epsilon) := \begin{bmatrix} AQ + QA^T + \alpha Q + \epsilon AQA^T & \bar{B} \\ \bar{B}^T & -\alpha I \end{bmatrix} < 0.$$

Then, $F_{Q,\alpha}(\epsilon) \geq F_{Q,\alpha}$ for all such ϵ, where

$$F_{Q,\alpha} := \begin{bmatrix} AQ + QA^T + \alpha Q & \bar{B} \\ \bar{B}^T & -\alpha I \end{bmatrix}.$$

Moreover, $F_{Q,\alpha}(\epsilon)$ is monotonical in ϵ and $F_{Q,\alpha}(\epsilon) \to F_{Q,\alpha}$ as $\epsilon \to 0$.

Proof. Obviously, $\epsilon\alpha < 1$ by (13) and $F_{Q,\alpha}(\epsilon)$ is monotonical in ϵ. On the other hand, by the definitions of $F_{Q,\alpha}(\epsilon)$ and $F_{Q,\alpha}$, clearly, $F_{Q,\alpha}(\epsilon) \geq F_{Q,\alpha}$ and $F_{Q,\alpha}(\epsilon) \to F_{Q,\alpha}$ as $\epsilon \to 0$. ■

Remark 2. Similar to Lemmas 3, 4, (13) is equivalent to the existence of positive definite matrix Q, matrix $Z \in \mathbf{R}^{n\times n}$ and scalars $\epsilon > 0,\ \alpha > 0,\ \epsilon\alpha < 1$ such that

$$\begin{bmatrix} -\epsilon^{-1}Q + \alpha Q & Z + \epsilon AZ & \bar{B} \\ Z^T + \epsilon Z^T A^T & -\epsilon(Z + Z^T - Q) & 0 \\ \bar{B}^T & 0 & -\alpha I \end{bmatrix} < 0.$$

This implies (4) by Lemma 4.

Theorem 3. For a given scalar $\rho > 0$, if there exist a positive definite matrix Q, matrix $Z \in \mathbf{R}^{n\times n}$ and scalars $\epsilon > 0,\ \alpha > 0,\ \epsilon\alpha < 1$ such that (12), (13) and the following condition hold:

$$\begin{bmatrix} -Q & (I+E)^T Z^T \\ Z(I+E) & -Z - Z^T + Q \end{bmatrix} < 0, \qquad (14)$$

then system (2) is internally stable and $\Omega_{Q^{-1}}$ is a robust attractor of it w.r.t. $w \in \mathcal{W}$. Moreover, $\Omega_{Q^{-1}} \subset \Omega(\rho)$ and hence system (2) has ρ-performance.

Proof. By Lemmas 2, 3, 4 and Remark 2, the proof can be completed along a similar line of arguments as in the proof of Theorem 1. ■

Theorem 4. For a given scalar $\rho > 0$, if there exist a positive definite matrix Q, matrices $Z \in \mathbf{R}^{n\times n}$, $M \in \mathbf{R}^{m\times n}$ and scalars $\epsilon > 0,\ \alpha > 0,\ \epsilon\alpha < 1$ such that (11), (12) and the following condition hold:

$$\begin{bmatrix} -\epsilon^{-1}Q + \alpha Q & Z + \epsilon AZ + \epsilon BM & \bar{B} \\ Z^T + \epsilon Z^T A^T + \epsilon M^T B^T & -\epsilon(Z + Z^T - Q) & 0 \\ \bar{B}^T & 0 & -\alpha I \end{bmatrix} < 0, \qquad (15)$$

then the controller $u = Kx$ with gain matrix

$$K = MZ^{-1} \qquad (16)$$

can internally stabilized the controlled system (1) and $\Omega_{Q^{-1}}$ is a robust attractor of it w.r.t. $w \in \mathcal{W}$. Moreover, $\Omega_{Q^{-1}} \subset \Omega(\rho)$ and hence system (1) has ρ-performance.

Proof. By Lemmas 2, 3, 4, Remark 2 and Theorem 3, the proof can be completed by similar arguments as in the proof of Theorem 2. ■

3.3 The case of systems with polytopic uncertainties

As shown above, the Lyapunov function matrix Q can be decoupled from the system matrices in stability and performance conditions. This merit allows for further study of the related robustness issues for the system with uncertain coefficients specified by convex polytopic matrix set as below:

$$[A, B, \bar{B}, E, \hat{B}, C, D] \in \Psi =: \{[A, B, \bar{B}, E, \hat{B}, C, D] = \sum_{i=1}^{\mathcal{L}} \xi_i [A_i, B_i, \bar{B}_i, E_i, \hat{B}_i, C_i, D_i],\ \xi_i \geq 0, \sum_{i=1}^{\mathcal{L}} \xi_i = 1\}.$$

That is, Ψ is a convex polytope with $\mathcal{L}$ vertices $[A_i, B_i, \bar{B}_i, E_i, \hat{B}_i, C, D_i],\ i = 1, \cdots, \mathcal{L}$.

Theorem 5. For a given scalar $\rho > 0$, if there exist a positive definite matrix $Q_i,\ i = 1, 2, \cdots, \mathcal{L}$, matrix $Z \in \mathbf{R}^{n\times n}$ and scalars $\epsilon > 0,\ \alpha > 0,\ \epsilon\alpha < 1$ such that the following conditions hold for $i = 1, 2, \cdots, \mathcal{L}$,

$$\begin{bmatrix} -\epsilon^{-1}Q_i + \alpha Q_i & Z + \epsilon A_i Z & \bar{B}_i \\ Z^T + \epsilon Z^T A_i^T & -\epsilon(Z + Z^T - Q_i) & 0 \\ \bar{B}_i^T & 0 & -\alpha I \end{bmatrix} < 0, \qquad (17)$$

$$\begin{bmatrix} -Q_i & (I+E_i)^T Z^T \\ Z(I+E_i) & -Z - Z^T + Q_i \end{bmatrix} < 0, \qquad (18)$$

$$\begin{bmatrix} -\alpha(Z^T + Z - Q_i) & 0 & Z^T C_i^T \\ 0 & -(\rho^2 - \alpha)I & D_i^T \\ C_i Z & D_i & -I \end{bmatrix} < 0, \qquad (19)$$

then system (2) is robustly internally stable w.r.t. uncertainty Ψ, and $\Omega_{Q^{-1}}$ is a robust attractor of it w.r.t. $w \in \mathcal{W}$ and Ψ, where $Q(\xi) = \sum_{i=1}^{\mathcal{L}} \xi_i Q_i$. Moreover, $\Omega_{Q^{-1}} \subset \Omega(\rho)$ and hence system (2) has robust ρ-performance w.r.t. uncertainty Ψ.

Proof. The result can be easily proved by Theorem 3 and the parameter dependent Lyapunov function $V(x(t)) = x^T(t)Q(\xi)x(t)$, where $Q(\xi) = \sum_{i=1}^{\mathcal{L}} \xi_i Q_i$, and ξ are the same as in Ψ. ■

Similarly, considering system (1) with the uncertainty Ψ, we can establish the result for designing a full state-feedback controller as follows.

Theorem 6. For a given scalar $\rho > 0$, if there exist positive definite matrices $Q_i,\ i = 1, 2, \cdots, \mathcal{L}$, matrix $Z \in \mathbf{R}^{n\times n}$, matrix $M \in \mathbf{R}^{m\times n}$ and scalars $\epsilon > 0,\ \alpha > 0,\ \epsilon\alpha < 1$ such that (19) and the following conditions hold for $i = 1, 2, \cdots, \mathcal{L}$,

$$\begin{bmatrix} -Q_i & (I+E_i)Z + \hat{B}_i M \\ * & -Z - Z^T + Q_i \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} -\epsilon^{-1}Q_i + \alpha Q_i & Z + \epsilon A_i Z + \epsilon B_i M & \bar{B}_i \\ * & -\epsilon(Z + Z^T - Q_i) & 0 \\ * & * & -\alpha I \end{bmatrix} < 0, \tag{21}$$

where $*$ denotes the block symmetrical matrix, then system (1) is robustly internally stabilized by state-feedback controller $u = Kx$ w.r.t. uncertainty Ψ, and $\Omega_{Q^{-1}}$ is a robust attractor of it w.r.t. $w \in \mathcal{W}$ and Ψ, where the gain matrix K is described by (16) and $Q(\xi) = \sum_{i=1}^{\mathcal{L}} \xi_i Q_i$. Moreover, $\Omega_{Q^{-1}} \subset \Omega(\rho)$ and hence the closed-loop system of system (1) with $u = Kx$ has robust ρ-performance w.r.t. uncertainty Ψ. ■

Remark 3. Notice that the above results don't require the $\mathcal{L}$ vertex systems to have a common Lyapunov function for obtaining robust stability of the polytopic uncertain systems, hence are less conservative than those based on common Lyapunov functions. This advantage will be further demonstrated with a numerical example in the next section. Moreover, Theorems 5 and 6 can be easily extended to multilinear uncertainty case, that is, the uncertainty can be described by the following polytope with $\mathcal{I} \times \mathcal{J} \times \mathcal{K} \times \mathcal{L} \times \mathcal{P} \times \mathcal{Q} \times \mathcal{R}$ vertices.

$$\begin{aligned}\Psi =: \{\ & [A, B, \bar{B}, E, \hat{B}, C, D] = \\ & \textstyle\sum_{i=1}^{\mathcal{I}} \alpha_i \sum_{j=1}^{\mathcal{J}} \beta_j \sum_{k=1}^{\mathcal{K}} \gamma_k \sum_{l=1}^{\mathcal{L}} \zeta_l \sum_{p=1}^{\mathcal{P}} \delta_p \\ & \textstyle\cdot \sum_{q=1}^{\mathcal{Q}} \eta_q \sum_{r=1}^{\mathcal{R}} \theta_r [A_i, B_j, \bar{B}_k, E_l, \hat{B}_p, C_q, D_r] : \\ & \textstyle\alpha_i, \beta_j, \gamma_k, \zeta_l, \delta_p, \eta_q, \theta_r \geq 0, \sum_{i=1}^{\mathcal{I}} \alpha_i = 1, \\ & \textstyle\sum_{j=1}^{\mathcal{J}} \beta_j = 1, \sum_{k=1}^{\mathcal{K}} \gamma_k = 1, \sum_{l=1}^{\mathcal{L}} \zeta_l = 1, \\ & \textstyle\sum_{p=1}^{\mathcal{P}} \delta_p = 1, \sum_{q=1}^{\mathcal{Q}} \eta_q = 1, \sum_{r=1}^{\mathcal{R}} \theta_r = 1\}.\end{aligned}$$

4 Numerical example

Now consider the following impulsive system with polytopic uncertainty,

$$\begin{cases} \dot{x} = A_i x + B_i u + \bar{B}_i w,\ t \neq t_k \\ \Delta x = E_i x(t_k) + \hat{B}_i u,\ t = t_k \\ z = C_i x + D_i w \\ x(0) = 0 \end{cases} \tag{22}$$

i.e., the vertex system with coefficient matrices as follows.

$$A_1 = \begin{bmatrix} -1 & 0.5 & 0 \\ 0 & -1 & 0 \\ 0.1 & 0 & -1 \end{bmatrix}; A_2 = \begin{bmatrix} -2 & 0.3 & 0 \\ 0.1 & -2 & 0 \\ 0.1 & 0 & -1 \end{bmatrix};$$

$$B_1 = \begin{bmatrix} 0.1 & 0.1 \\ -0.2 & 0 \\ 0.3 & 0.12 \end{bmatrix}; B_2 = \begin{bmatrix} 0 & 0.3 \\ 0.5 & 0.1 \\ -0.1 & 0.2 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0.1 & 0.12 & 0.3 \end{bmatrix}; C_2 = \begin{bmatrix} 0.2 & 0 & 0 \\ -0.1 & 0.1 & 0 \\ 0 & 0 & 0.12 \end{bmatrix};$$

$$D_1 = \begin{bmatrix} 0 & 0.1 & 0 \\ -0.2 & 0 & 0.1 \\ 0 & 0 & -0.5 \end{bmatrix}; D_2 = \begin{bmatrix} 0.1 & 0.2 & 0 \\ 0.3 & 0.1 & 0 \\ 0 & 0 & 0.3 \end{bmatrix};$$

$$E_1 = \begin{bmatrix} 1 & 0.2 & 0 \\ 0 & 0 & 0.1 \\ 0.1 & 0.2 & 0.01 \end{bmatrix}; E_2 = \begin{bmatrix} 0.2 & 0 & 0 \\ -0.3 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \end{bmatrix};$$

$$\hat{B}_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \\ 0 & 0.3 \end{bmatrix}; \hat{B}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0.2 & 0.12 \end{bmatrix};$$

$$\bar{B}_1 = \begin{bmatrix} 0.1 & -0.5 & 0 \\ 0 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \end{bmatrix}; \bar{B}_2 = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0 & 0.1 & 0.1 \\ 0.2 & 0 & -0.1 \end{bmatrix}.$$

Solving inequalities (19)–(21) for $\alpha = 0.2$ and $\rho = 1$, by taking $\epsilon = 0.1$, we obtain

$$Q_1 = \begin{bmatrix} 2.1215 & 0.2758 & 1.6745 \\ 0.2758 & 0.1740 & 0.4889 \\ 1.6745 & 0.4889 & 2.0577 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 1.6232 & 0.0655 & 0.9790 \\ 0.0655 & 0.3004 & 0.6929 \\ 0.9790 & 0.6929 & 2.2632 \end{bmatrix}.$$

$$Z = \begin{bmatrix} 1.4469 & -0.1487 & 0.3624 \\ 0.0745 & 0.2127 & 0.4793 \\ 0.9933 & 0.4493 & 1.6521 \end{bmatrix},$$

$$M = \begin{bmatrix} -8.7014 & -2.0457 & -9.3040 \\ -0.6810 & -3.0302 & -7.3994 \end{bmatrix}.$$

$$K = \begin{bmatrix} -3.1487 & -3.5680 & -3.9057 \\ 1.9188 & -6.5973 & -2.9856 \end{bmatrix}.$$

Let $K = MQ^{-1}$, then by Theorem 6, the closed-loop system is robustly internally stable and has ρ-performance. For any $\alpha > 0$ and $\rho = 1$, by the well-known results (for example, see [13]) on quadratic stability, there dose not exist a common positive definite matrix solution to these matrix inequalities. Particularly, the closed-loop system of the midpoint system (i.e., taking $\xi_i = 0.5,\ i = 1, 2$) with $u = Kx$ is robustly internally stable and has ρ-performance. Now consider an external disturbance of the form $w = \frac{1}{\sqrt{1.8^2+2^2+1.5^2}}[1.8\sin(\pi t + 1)\ \ 2\sin(2\pi t + 1)\ \ 1.5\sin(\pi t + 1)]^T$ and take the impulsive time step as one second. The numerical simulation of the state response of the impulsive system affected by the disturbances is shown in Fig.1; the corresponding state response of the system without disturbances is shown in Fig. 2.

Remark 4. In this example, the vertex systems do not have a common Lyapunov function for quadratic stability, but the system with polytopic uncertainty is indeed robustly stable. Hence the proposed approach has less conservatism.

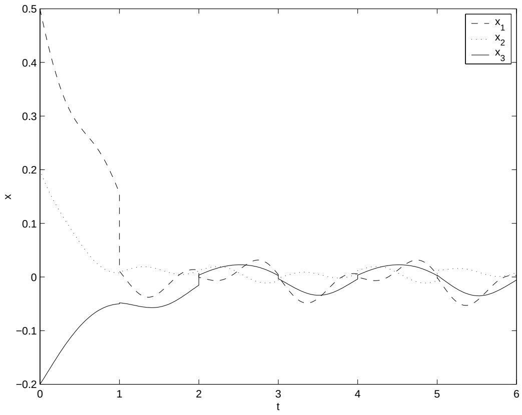

Fig 1. The state response for the closed-loop system with the specified external disturbance w

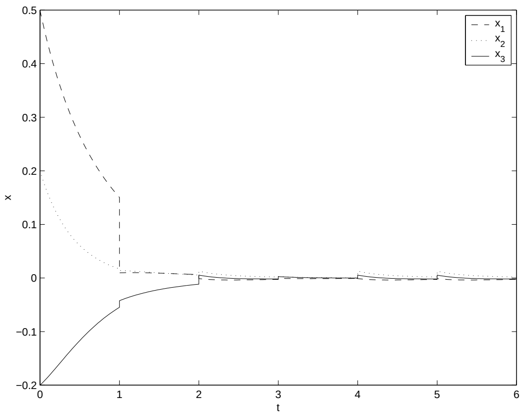

Fig 2. The state response for the closed-loop system without external disturbance w

5 Conclusions

We have discussed the problems of persistent bounded disturbance rejection for impulsive systems with polytopic uncertainty by using positive invariant set analysis and Lyapunov function method. Some sufficient conditions that ensure internal stability and the desired performance level of bounded disturbances for the impulsive systems have been derived in terms of linear matrix inequalities. Based on these results, a simple approach to the design of a linear state-feedback controller has been presented to achieve both robust internal stability and a desired level of disturbance rejection performance for a disturbed impulsive system. Since the obtained Lyapunov function matrix is independent of all coefficient matrices in these results, we do not need to require the existence of a common positive definite solution for vertex systems when dealing with robust stability of uncertain polytopic systems. A numerical example shows the efficiency and less conservatism of the proposed method.

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