ROBUSTNESS BOUNDS FOR MATRIX $\mathcal{D}_U$-STABILITY

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Abstract

This paper addresses the problem of robust matrix root-clustering analysis. The considered matrices are complex and subject to both polytopic and parameter-dependent norm-bounded uncertainties. The clustering regions are unions of convex and possibly disjoint and nonsymmetric subregions of the complex plane. The proposed clustering conditions are formulated through an approach based upon Linear Matrix Inequalities. These numerical tools enable to easiliy compute Lyapunov matrices (possibly parameter-dependent) that ensure the clustering property.

1 Introduction

Linear state-space model is a very usual way to describe the behaviour of a plant around a setting point. It particularly makes a suitable control law easier to determine. While designing this control law, it matters to reach some desired performances for the closed-loop systems or at least to check them *a posteriori*. For this reason, it is important to define criteria that allow to evaluate the performances especially the transient ones. These ones are strongly influenced by the location of the state matrix spectrum in the complex plane, in terms of settling time, damping ratio... Hence it can be useful to check if the closed-loop eigenvalues lie inside a region $\mathcal{D}$. $\mathcal{D}$ must be discerningly specified to guarantee satisfactory transient behaviour. Such a property is referred to as "'matrix $\mathcal{D}$-stability". In the present paper, $\mathcal{EEMI}$-regions ($\mathcal{E}$xtended $\mathcal{E}$llipsoidal $\mathcal{M}$atrix $\mathcal{I}$nequality) are considered [1, 2]. This class of regions enable to hande unions of several possible disjoint and non symmetric subregions. If $\mathcal{D}$ is an open region (*i.e.* not including its boundary), this concept encompasses those of Hurwitz and Schur stabilities.

Many authors proposed interesting necessary and sufficient conditions for matrix $\mathcal{D}$-stability, obviously depending on the chosen description of $\mathcal{D}$ [5, 11]. These conditions are very tractable from a computational point of view as far as nominal matrix $\mathcal{D}$-stability is concerned. However, a higher challenge consists in obtaining sufficient conditions for robust matrix $\mathcal{D}$-stability that are not too pessimistic (*i.e.* as close as possible to a necessary condition). Indeed, in practice, state matrices are not precisely known. Some model uncertainties exist, due, for example, to neglected dynamics, approximation in the linearization step, physical parameter deflections (parametric/structured uncertainty)... These uncertainties can generate unexpected pole migration in the complex plane. When stability is concerned, a solution to analyze robust stability is to find a bound on an additive uncertainty (on the 2-norm of a matrix for an unstructured uncertainty or on the modulus of the parameter variations for the structured uncertainty) such that stability is ensured. Such a bound is called a robust stability bound and is conservative most of the time. The first work is probably the one of [16], starting from the Lyapunov equation and later improved in [14, 21, 25, 26]. This approach will be referred to, in the sequel, as Lyapunov approach. For extension to $\mathcal{D}$-stability, see [3, 7, 18, 24].

In the present paper, the considered uncertainty is both polytopic and parameter-dependent norm-bounded. Conditions for robust $\mathcal{D}$-stability are expressed in terms of $\mathcal{LMI}$ ($\mathcal{L}$inear $\mathcal{M}$atrix $\mathcal{I}$nequality) involving parameter-dependent "Lyapunov matrices" (PDLM), therefore reducing the conservatism induced by quadratic $\mathcal{D}$-stability (non dependent-Lyapunov matrices proving $\mathcal{D}$-stability over the uncertainty domain).

The paper is organized as follows: after this introduction, the second section is dedicated to the problem introducing the descriptions of the clustering regions, of the uncertainties and proposing some vocabulary helping comprehension. In the third one, a robust $\mathcal{D}$-stability bound is computed thanks to $\mathcal{LMI}$ conditions implicitly involving PDLM. A numerical illustration is proposed in the fourth section before concluding in the fifth one.

Notations : We denote by M', the transpose conjugate of M, by $\mathcal{H}(M)$ the Hermitian expression $M + M'$. The Kronecker product is denoted by $\otimes$. $||M||_2$ is the 2-norm of matrix M induced by the Euclidean vector-norm, *i.e.* the maximal singular value of M. $\mathbb{I}_n$ is the identity matrix of order n, $\mathbb{O}$ is a null matrix of suitable dimension and $\mathbb{1}_{u,v}$ is a matrix of dimension $u \times v$ whose all entries equal 1. Matrix inequalities are considered in the sense of Löewner *i.e.* "$\prec 0$" ("$\preceq 0$") means negative (semi-)definite and "$\succ 0$" ("$\succeq 0$") positive (semi-)definite. HPD stands for Hermitian Positive Definite. Small letters are used for scalar numbers and vectors while capital letters denote matrices or sets. Matrix $[f_k]_{k=1}^m$ denotes $[f_1 \dots f_m]$. At last, $\mathbf{i}$ denotes the imaginary unit and Re(z) is the real part of complex number z.

2 Problem Statement

In this section, the problem is stated. First, the uncertain matrix is presented. Then, the description for clustering regions is given with associated conditions for nominal matrix root-clustering. At last, the precise goal is formulated.

2.1 The uncertain matrix

Consider a complex uncertain matrix $\mathbf{A} \in \mathbb{C}^{n\times n}$ defined by:

$$\mathbf{A} = A + \mathbf{E} \quad \text{with} \quad \mathbf{E} = JEL \tag{1}$$

Matrices A and $\mathbf{E}$ are both uncertain. $\mathbf{E}$ is an additve uncertainty in which $E \in \mathbb{C}^{q\times r}$ is unknown. E is assumed to belong to $\mathcal{B}(\rho)$, the ball of $(q \times r)$ complex matrices checking $||E||_2 \leq \rho$. Define matrix M by:

$$M = \begin{bmatrix} A & J \\ L & \mathbb{O} \end{bmatrix} \tag{2}$$

Matrix M is assumed to belong to a polytope of matrices $\mathcal{M}$ defined by:

$$\mathcal{M} = \Big\{ M = M(\theta) \in \mathbb{C}^{(n+r)\times(n+q)} \;|\; M(\theta) = \sum_{i=1}^{N} (\theta_i M_i) \,;\, \theta \in \Theta \Big\} \tag{3}$$

where Θ is the set of all barycentric coordinates:

$$\Theta = \left\{ \theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix} \in \{\mathbb{R}^+\}^N \;|\; \sum_{i=1}^{N} \theta_i = 1 \right\} \tag{4}$$

Extreme matrices M_i, $i = 1, ..., N$ are the vertices of $\mathcal{M}$ and read:

$$M_i = \begin{bmatrix} A_i & J_i \\ L_i & \mathbb{O} \end{bmatrix} \tag{5}$$

It is well known that this formulation encompasses the case where M linearly depends on some parameter deflections around a nominal value. In that case, $\mathcal{M}$ is actually an "orthotope of matrices". If no polytopic dependence is assumed (only one vertex for $\mathcal{M}$) then $\mathbf{A}$ is only subject to a classical norm-bounded uncertainty [19].

2.2 Clustering regions and nominal $\mathcal{D}$-stability

In this part, the reader is reminded of the definition of $\mathcal{EEMI}$-regions. The corresponding condition for nominal matrix root-clustering is also recalled.

2.2.1 $\mathcal{D}_U$-stability

Definition 1 *[1] : Let $\mathcal{R}$ be a set of m Hermitian matrices R_k defined by:*

$$\begin{cases} R_k = R_k' = \begin{bmatrix} R_{k_{00}} & R_{k10} \\ R'_{k_{10}} & R_{k11} \end{bmatrix} \in \mathbb{C}^{2d\times 2d} \\ R_{k_{11}} \succeq 0,\, R_{k_{11}} \in \mathbb{C}^{d\times d} \qquad \forall k \in \{1, ..., m\} \end{cases} \tag{6}$$

The set of points $\mathcal{D}_U$ defined by:

$$\mathcal{D}_U = \bigcup_{k=1}^{m} \mathcal{D}_{R_k} \tag{7}$$

where each subregion $\mathcal{D}_{R_k}$ is an $\mathcal{EMI}$-region ($\mathcal{E}$llipsoidal $\mathcal{M}$atrix $\mathcal{I}$nequality) [17, 18] of degree d_k i.e., $\forall k \in \{1, ..., m\}$*:*

$$\mathcal{D}_{R_k} = \left\{ z \in \mathbb{C} \;|\; \begin{bmatrix} \mathbb{I}_{d_k} & \tilde{z}\mathbb{I}_{d_k} \end{bmatrix} R_k \begin{bmatrix} \mathbb{I}_{d_k} \\ z\mathbb{I}_{d_k} \end{bmatrix} \prec 0 \right\} \tag{8}$$

is called an open $\mathcal{EEMI}$-region.

Remark 1 *: In the sequel, it is assumed that $d_k = d$, $\forall k \in \{1, ..., m\}$ which is not restrictive [1].*

An $\mathcal{EEMI}$-region, generically denoted by $\mathcal{D}_U$, is then a nonconnected region that results from the union of possibly complex and disjoint $\mathcal{EMI}$-regions. Such a $\mathcal{EMI}$-region is convex. For a real R, this formulation is equivalent to the one of $\mathcal{LMI}$ regions (see [5, 18] for more details). Here, we allow R to be complex, *i.e.* $\mathcal{D}_R$ to be nonsymmetric. $\mathcal{D}_R$ can be, for instance, a shifted and nonsymmetric half plane, a classical or hyperbolic sector, a vertical or horizontal strip, a disc or the inside of an ellipse the center of which is not necessarily on the real axis... A matrix is said $\mathcal{D}_U$-stable when all its eigenvalues belong to $\mathcal{D}_U$. Some necessary and sufficient condition for this property to hold is now recalled:

Theorem 1 *[1, 2] : Let $\mathcal{D}_U$ be an $\mathcal{EEMI}$-region as introduced in definition 1. A matrix $A \in \mathbb{C}^{n\times n}$ is $\mathcal{D}_U$-stable if and only if there exists a set $\mathcal{P}$ of m HPD matrices $P_k \in \mathbb{C}^{n\times n}$, $k = 1, ..., m$, such that:*

$$\mathbb{U}(A, \mathcal{P}) = Q'(A)\; \mathcal{U}(\mathcal{P})\; Q(A) \prec 0 \tag{9}$$

with:

$$\mathcal{U}(\mathcal{P}) = \begin{bmatrix} \sum_{k=1}^{m} (R_{k_{00}} \otimes P_k) & [R_{k_{10}} \otimes P_k]_{k=1}^{m} \\ ([R_{k_{10}} \otimes P_k]_{k=1}^{m})' & \underset{k=1,\ldots,m}{\text{diag}} \{R_{k_{11}} \otimes P_k\} \end{bmatrix} \tag{10}$$

and

$$Q(A) \;=\; \left[\begin{array}{c}\mathbb{I}_{dn}\\ \hline \mathbb{1}_{m,1}\otimes\mathbb{I}_{d}\otimes A\end{array}\right] \tag{11}$$

The complete proof is proposed in [1] following original arguments borrowed from [4, 5, 11].

2.3 The precise purpose

The aim of this work is to compute a bound on $||E||_2$ such that, once the $\mathcal{EEMI}$-region $\mathcal{D}_U$ is specified, $\mathcal{D}_U$-stability of matrix $\mathbf{A}$ defined in (1) is ensured. In other words, it is aimed to determine, owing to $\mathcal{LMI}$ tools, a bound $\rho^\diamond$, as great as possible, such that for any radius $\rho \le \rho^\diamond$ of the ball $\mathcal{B}(\rho)$, $\mathcal{D}_U$-stability of $\mathbf{A}$ is guaranteed, for any $\theta \in \Theta$ and for any $E \in \mathcal{B}(\rho)$. $\rho^\diamond$ is a robust $\mathcal{D}_U$-stability bound taking both unstructured and parametric uncertainty into account and using a parameter-dependent approach. This is what next section deals with.

3 Parameter-dependent $\mathcal{D}_U$-stability analysis

In this section, it is assumed that M is somewhere in the polytope $\mathcal{M}$ and it matters to take parameter deflections into account. This is why a proposed $\mathcal{LMI}$ condition leading to a robust $\mathcal{D}_U$-stability bound $\rho^\diamond$, will implicitly involve PDLM. The first paragraph tackles the computation of $\rho^\diamond$ while the second one emphasizes the interest of $\rho^\diamond$ in both closed-loop robustness and fragility analysis.

3.1 Parameter-dependent robust $\mathcal{D}_U$-stability bound $\rho^\diamond$

Theorem 2 *Let $\mathbf{A} \in \mathbb{C}^{n\times n}$ be a matrix as defined in (1) and $\mathcal{D}_U$ be an $\mathcal{EEMI}$-region as formulated in definition 1. Matrix $\mathbf{A}$ is robustly $\mathcal{D}_U$-stable with respect to $\mathcal{B}(\rho)$ and $\mathcal{M}$ if there exist N sets $\mathcal{P}_i$, each one made up by m HPD matrices P_{k_i}, $k = 1, ..., m$, as well as a matrix $G_U \in \mathbb{C}^{d((m+1)n+q+r)\times mdn}$ such that:*

$$\mathbb{G}_U(M_i, \mathcal{P}_i, G_U) \;=\;$$

$$\left[\begin{array}{cc|c|c}\multicolumn{2}{c|}{\mathcal{U}(\mathcal{P}_i)} & \mathbb{O} & \begin{array}{c}\mathbb{I}_d\otimes L_i'\\ \hline \mathbb{O}\end{array}\\ \hline \multicolumn{2}{c|}{\mathbb{O}} & -\mathbb{I}_{dq} & \mathbb{O}\\ \hline \mathbb{I}_d\otimes L_i & \mathbb{O} & \mathbb{O} & -\gamma\mathbb{I}_{dr}\end{array}\right] +$$

$$\mathcal{H}\big(\; G_U\; T_i \;\big) \prec 0 \quad \forall i \in \{1, ..., N\} \tag{12}$$

where:

$$\gamma = \rho^{-2} \tag{13}$$

and

$$T_i = \left[\; \mathbb{1}_{m,1}\otimes\mathbb{I}_d\otimes A_i \;\middle|\; -\mathbb{I}_{mdn} \;\middle|\; \mathbb{1}_{m,1}\otimes\mathbb{I}_d\otimes J_i \;\middle|\; \mathbb{O}\;\right] \tag{14}$$

Proof: For any vector of barycentric coordinates $\theta \in \Theta$, consider the convex combination

$$\mathbb{G}_U(M(\theta), \mathcal{P}(\theta), G_U) \;=\; \sum_{i=1}^{N}(\theta_i \mathbb{G}_U(M_i, \mathcal{P}_i, G_U))$$

where the set $\mathcal{P}(\theta)$ is made up by m HPD matrices $P_k(\theta)$ defined by:

$$P_k(\theta) \;=\; \sum_{i=1}^{N}(\theta_i P_{k_i}) \;=\; P_k'(\theta) \succ 0 \tag{15}$$

Since every θ_i is positive, it comes:

$$\mathbb{G}_U(M(\theta), \mathcal{P}(\theta), G_U) \;\prec\; 0 \tag{16}$$

Define matrix $S(M(\theta), E)$ by:

$$S(M(\theta), E) \;=\; \left[\begin{array}{c|c}\mathbb{I}_{dn} & \mathbb{O}\\ \hline \mathbb{1}_{m,1}\otimes\mathbb{I}_d\otimes\mathbf{A} & \mathbb{O}\\ \hline \mathbb{I}_d\otimes(EL) & \mathbb{O}\\ \hline \mathbb{O} & \mathbb{I}_{dr}\end{array}\right] \tag{17}$$

Left and right mutliply (16) respectively by $S'(M(\theta), E)$ and $S(M(\theta), E)$ yields

$$\left[\begin{array}{c|c}\mathbb{U}(\mathbf{A}, \mathcal{P}(\theta)) \;-\; \mathbb{I}_d\otimes(L'E'EL) & \mathbb{I}_d\otimes L'\\ \hline \mathbb{I}_d\otimes L & -\gamma\mathbb{I}_{dr}\end{array}\right] \prec 0$$

By virtue of Schur's lemma and taking the expression of γ into account, it is equivalent to:

$$\mathbb{U}(\mathbf{A}, \mathcal{P}(\theta)) \;\prec\; \mathbb{I}_d\otimes(L'E'EL) \;-\; \rho^2\mathbb{I}_d\otimes(L'L)$$

Since E belongs to $\mathcal{B}(\rho)$, it comes $E'E \preceq \rho^2\mathbb{I}_n$ and the right member of the previous inequality is negative semi-definite. Hence, the left member is negative definite.□

The previous condition can be easily tested from a numerical point of view owing to $\mathcal{LMI}$ softwares. Moreover, the value of γ can be minimized leading to a maximum value of ρ, denoted by $\rho^\diamond$, which is a robust $\mathcal{D}_U$-stability bound.
Note that the Lyapunov matrices are parameter-dependent. For each instance $\theta \in \Theta$, matrices $P_k(\theta)$ is a convex combination of the Lyapunov matrices P_{k_i} associated with each vertex. Each $P_k(\theta)$ therefore describes a polytope of matrices when θ describes Θ. Such a dependence is enabled by the addition of G_U. Rather than keeping $\mathcal{P}$ constant over the whole polytope $\mathcal{M}$, it is better to keep G_U constant and to allow the set $\mathcal{P}(\theta)$ to vary in a polytopic way which can be proved less pessimistic. This is an idea that was originally proposed in [10]. It must be

noted that the use of parameter-dependent Lyapunov functions of matrices was exploited in many works and the reader is invited to see [6] for example.

Remark 2 *It could be shown that the satisfaction of (12) for some full G_U and some $\mathcal{P}_i$ is equivalent to the satisfaction of (12) for some $G_U = [\bar{G}_U \ \mathbb{O}\ ]'$ and the same $\mathcal{P}_i$ with $G_U \in \mathbb{C}^{d((m+1)n+q+r)\times mdn}$ and $\bar{G}_U \in \mathbb{C}^{d(m+1)n\times mdn}$.*

Remark 3 *If no polytopic uncertainty is considered,* i.e. M *is a* known *and* constant *matrix (*i.e. $N = 1$*), (12) is then sufficient for quadratic $\mathcal{D}_U$-stability against $\mathcal{B}(\rho)$, proved by the existence of a single set of Lyapunov matrices $\mathcal{P}$.*
Moreover, the special case where $d = 1$ makes the condition also necessary. Furtheremore, if $m = 1$, it is, on the one hand, equivalent to the condition of the Bounded Real Lemma that enables to compute the $\mathcal{H}_\infty$-norm of transfer function $L(s\mathbb{I} - A)^{-1}J$, and on the other hand, an application of the well known Kalman-Yakubovich-Popov Lemma. In that case, the bound is known to be the complex $\mathcal{D}$-stability radius [13, 15].

3.2 Application to closed-loop robustness and fragility analysis

In this paragraph, a state-space model describing a system behaviour and associated with the triple of matrices $(\tilde{A}; B; C)$ is considered. This triple of matrices is actually subject to parametric uncertainties so that it can be assumed to belong to a polytope of matrices such as the one defined in (2) and (4). A static output feedback control law is applied on this system. It is associated with the feedback matrix K. Because it is impossible in pratice to precisely implement this control law, it is reasonnable to assume that the feedback matrix is actually uncertain and reads $K + \Delta_K$ where K is the nominal term and Δ_K the uncertain one. Thus, the closed-loop uncertain state matrix is given by:

$$\mathbf{A} = \tilde{A} + BKC + B\Delta_K C \qquad (18)$$

Now assume that either B or C is precisely known and that it matters to attest robust $\mathcal{D}_U$-stability of $\mathbf{A}$ for a specified $\mathcal{EEMI}$-region $\mathcal{D}_U$. This problem is the same as the one handled above. It suffices to compute $\rho^\diamond$ by solving $\mathcal{LMI}$ system (12) with $A = \tilde{A} + BKC$, $J = B$ and $L = C$ (considering that $E = \Delta_K$). If this condition holds for some $\mathcal{P}_i$ and for some G_U, then, the closed-loop matrix $\mathbf{A}$ is $\mathcal{D}_U$-stable and $\rho^\diamond$ can be seen as the maximal acceptable deviation (in terms of 2-norm) while implementing K. It is then a fragility criterion.

The case where both B and C are subject to parametric uncertainties is a bit harder to tackle. It requires another formulation of uncertainty (with several polytopes) and might lead to more pessimistic results.

4 Numerical illustration

The numerical illustration is build around a model inspired from an example proposed in [8]. The computations are performed with MATLAB 6.1 and its LMITOOLBOX [9] on a PC Pentium 1700 Mhz.

A satellite is modelled by two masses connected by a spring with torque Γ and viscous damping δ. State and input matrices are:

$$\tilde{A}(\delta) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -10\Gamma & -10\delta & 10\Gamma & 10\delta \\ 0 & 0 & 0 & 1 \\ \Gamma & \delta & -\Gamma & -\delta \end{bmatrix} ; \ B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (19)$$

The torque is assumed to read $\Gamma = 0.35$ and the viscous damping δ is assumed to read $\delta = \phi\psi$ with $\psi = \sqrt{\Gamma/10}$. Function ϕ equals $\phi_0 + \Delta\phi$ where the nominal term is $\phi_0 = 0.15$ and where $\Delta\phi$ is unknown. Thus, it comes:

$$\delta = \phi\psi = (\phi_0 + \Delta\phi)\psi = \phi_0\psi + \Delta\phi\psi = \delta_0 + \Delta\delta \qquad (20)$$

A static state feedback control law associated with matrix

$$K = \begin{bmatrix} -17.499 & -53.326 & -78.501 & -16.691 \end{bmatrix} \qquad (21)$$

assigns an arbitrary spectrum to nominal closed-loop state matrix $A = \tilde{A}(\delta_0) + BK$ inducing a damping ratio $\zeta \simeq 0.7$.

It is assumed that the feedback matrix K is subject to an additive uncertainty Δ_K because of practical problems encountered while implementing the control law. The closed-loop state matrix reads:

$$\mathbf{A} = \underbrace{\tilde{A}(\delta_0) + BK + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \Delta\delta}_{A} + \underbrace{B}_{J}\ \underbrace{\Delta_K}_{E}\ \underbrace{\mathbb{I}_4}_{L} \qquad (22)$$

Is is assumed that $|\Delta\phi| \leq 0.05$ what makes A describe a polytope of matrices. Besides, it is expected to estimate $\rho^\diamond$ such that $\mathcal{D}_U$-stability of $\mathbf{A}$ is guaranteed for any Δ_K such that the damping ratio remains in about $[0.4; 0.9]$. To achieve such an analysis, a clustering region $\mathcal{D}_U$ is specified. It is the union of two discs, $\mathcal{D}_{R_1}$ and $\mathcal{D}_{R_2}$, respectively centred on $-2 - 2\mathbf{i}$ and $-2 + 2\mathbf{i}$ and both of radius 1 and the vertical half plane,

$\mathcal{D}_{R_3}$, defined by $\text{Re}(z) < -3$. $\mathcal{D}_U$ is then symmetric with respect to the real axis. The three subregions are disjoint and the $\mathcal{D}_U$-stability of $\mathbf{A}$ actually clusters the damping ratio in $[0.4114; 0.9114]$. Applying theorems 2 gives:

$$\rho^\diamond = 7.91 \times 10^{-1}$$

This computation was performed in $200s$. On figures 1 and 1, the uncertain matrix roots are plotted for 2000 random values of Δ_K such that $|\Delta_K| \leq \rho^\diamond$. Such a plot is a way to appreciate the shape of the spectral value set [12] that was also studied under the name of pseudospectrum in [22, 23]. The figure 1 corresponds to the practical case where $\Delta\delta$ is real. The figure 2 corresponds to an unrealistic case where $\Delta\delta$ is complex. Such figures enable to appreciate the conservatism induced by the approach.

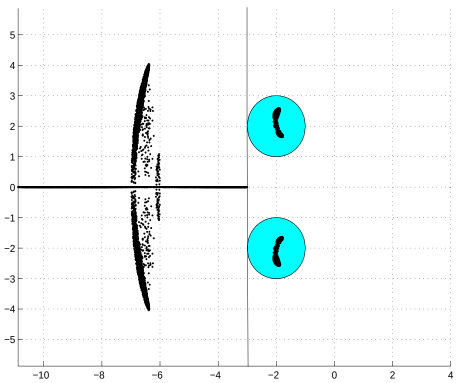

Figure 1: Pole migration with $||\Delta_K||_2 \leq \rho^\diamond$: $\Delta_K \in \mathbb{R}^{1\times 4}$

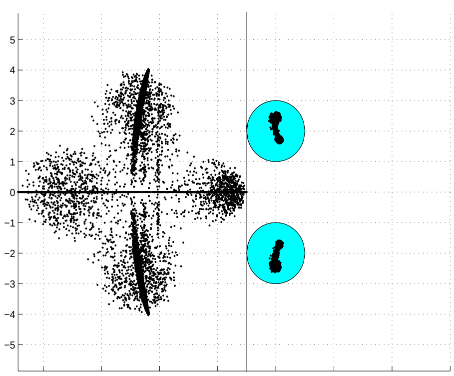

Figure 2: Pole migration with $||\Delta_K||_2 \leq \rho^\diamond$: $\Delta_K \in \mathbb{C}^{1\times 4}$

5 Conclusion

The problem of robust matrix root-clustering was tackled. The uncertainty is both parametric and norm-bounded and the clustering regions denoted by $\mathcal{D}_U$ are referred to as $\mathcal{EEMI}$-regions, *i.e.* unions of possibly non symmetric and disjoint $\mathcal{EMI}$-subregions. This is an original point of the paper. A $\mathcal{LMI}$ technique to compute robust $\mathcal{D}_U$-stability bounds was presented. This method is part of the so-called "Lyapunov framework". It induces a weak conservatism so that the bound might often be very close to the complex $\mathcal{D}_U$-stability radius. When parametric uncertainties are handled, the Lyapunov matrices implicitly involved in the $\mathcal{LMI}$ systems are parameter dependent in order to reduce the pessimism of quadratic $\mathcal{D}_U$-stability.

Some further investigations could concern extensions to the case where the uncertainty is real that is to the research of a good lower bound for the real $\mathcal{D}_U$-stability radius, maybe using the work of [20]. Another challenge is the discerning use of these robust transient performance criteria in a design context.

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