ADAPTIVE STABILIZATION OF NONLINEAR SYSTEM WITH FUNCTIONAL UNCERTAINTY

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Abstract

Adaptive control schemes are developed, which provide global asymptotic stability property with respect to state space vector of the plant with functional uncertainty. In the presence of disturbance these schemes ensure boundedness of all trajectories of the system. A switching modification of proposed schemes is considered.

1. Introduction

The adaptive stabilisation of nonlinear determined systems is one of the most popular and complicated problems of the modern theory of control. Presented work is devoted to a well known problem of stabilisation of systems with input uncertainty [9, 15, 20, 21, 32] as well as more complicated class of systems with functional uncertainty [10, 14, 17]. The class of uncertainties, which is considered here, includes external not determined perturbation and functional indeterminacies. The same problem was investigated in many works (see works listed above and references therein). In paper [20] the control law ensuring asymptotic tracking with compensating of perturbations was synthesized. However, class of nonlinear plants, for which indicated problem was solved, is limited by models reduced to the so-called normal form [13] and for systems without functional indeterminacy. The monographies [9, 15, 21] and papers [10, 17, 32] concern to the problem of nonlinear control of systems with functional uncertainty, but, unfortunately the influence of input disturbances was not investigated in these works. And what is more, only convergence of all trajectories of a nonlinear system to some small neighbourhood of the origin was proven in the most of above cited works. Adaptive stabilisation of systems with functional uncertainty in the presence of disturbance was investigated by Kosmatopoulus and Ioannou [14], but in that paper it was supposed that all uncertainties are linearly included in equations of the plant.

Opposite to previous works, in the present work the control algorithm is developed for stabilizable nonlinear affine system with functional uncertainty, which provides asymptotic stability in the case of absence of disturbances and asymptotic boundedness of all trajectories of the system for L_∞ bounded disturbances. In the second section the main definitions and statements are presented. The non adaptive and adaptive control algorithms are described in the 3 section, in 4 section some important tasks, which can be solved in line of proposed control algorithms, are presented, the conclusion finishes the paper.

2. Definitions and statements

Let us consider nonlinear dynamic system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\omega}(\mathbf{x},t)) + \mathbf{G}(\mathbf{x})[\mathbf{u} + \mathbf{w}_1(t)], \qquad (1)$$

where $\mathbf{x} \in R^n$ is state vector, $\mathbf{u} \in R^m$ is control; $\mathbf{f}(\mathbf{x}, \boldsymbol{\omega})$ and the columns of $\mathbf{G}(\mathbf{x})$ are continuous and locally Lipschitz vector fields on R^n, $\mathbf{f}(0, \boldsymbol{\omega}) = 0$ for any $\boldsymbol{\omega} \in R^p$; $\boldsymbol{\omega}(\mathbf{x},t)$ is unknown vector function representing functional uncertainty of system; $\mathbf{w}_1 : R_{\geq 0} \to R^p$ is external disturbance, that is Lebesgue measurable and essentially bounded function of time. Let us introduce main restrictions on properties of the system.

A s s u m p t i o n 1. *There are an unknown constant* $m \in [0, +\infty)$, *unknown Lebesgue measurable and essentially bounded signal* $\mathbf{w}_2 : R_{\geq 0} \to R_{\geq 0}$ *and a known function* $r : R^n \to R_{\geq 0}$, *such, that*

$$|\boldsymbol{\omega}(\mathbf{x},t)| \leq m\, r(\mathbf{x}) + \mathbf{w}_2(t),\ \mathbf{x} \in R^n,\ t \geq 0,\ r(0) = 0. \qquad \blacksquare$$

Assumption 1 does not imply boundedness of function $\boldsymbol{\omega}(\mathbf{x},t)$ for variable $\mathbf{x}$. The presence of unknown parameter m leads us to necessity of adaptive controller construction. Opposite to classical adaptive control theories [8, 9, 15] here there is no assumption on compactness of admissible values set of unknown parameter m. Such complication allows to consider more wide class of tasks, for example, in this way we can take into account the presence of unmodeled dynamic, that is unknown dynamic system with *bounded* solution. Then $\boldsymbol{\omega}(\mathbf{x},t)=y(t)r(\mathbf{x})+\mathbf{w}_2(t)$, where signal $y(t)$ is output of unmodeled dynamic system, hence, constant m can reflect the influence of unknown initial condition of unmodeled dynamic system. The robust and adaptive control algorithms for systems with input unmodeled dynamics were investigated in many works [3, 11], here we will mainly focus our attention on the problem of nonlinear joining of uncertainty $\boldsymbol{\omega}(\mathbf{x},t)$ in equation (1).

Further, for system (1) we can consider some differentiable Lyapunov or storage function $V:R^n\to R_{\geq 0}$, then

$$\dot V = L_{\mathbf{f}(\mathbf{x},\mathbf{d})}V(\mathbf{x})+L_{\mathbf{G}}V(\mathbf{x})[\mathbf{u}+\mathbf{w}_1(t)],$$

where

$$L_{\mathbf{f}}V(\mathbf{x})=\partial V(\mathbf{x})/\partial\mathbf{x}\,\mathbf{f}(\mathbf{x},\mathbf{d}),\ L_{\mathbf{G}}V(\mathbf{x})=\partial V(\mathbf{x})/\partial\mathbf{x}\,\mathbf{G}(\mathbf{x}).$$

According to Lemma 2.1 in [17], Lemma 9 in [26] or discussion in section 4 (formula (10)) of paper [16], the first term of above expression can be majorized as follows

$$L_{\mathbf{f}(\mathbf{x},\mathbf{d})}V(\mathbf{x})\leq a(\mathbf{x})+l(\mathbf{d}),$$

where a and l are some continuous function, $a(0)=0$, $l(0)=0$. Continuing in this line and combining with Assumption 1 we obtain:

$$l(\boldsymbol{\omega}(\mathbf{x},t))\leq \mu\,\rho(\mathbf{x})+\sigma(|\mathbf{w}_2(t)|),$$

where μ is a new unknown constant and ρ is some new known continuous function ($\rho(0)=0$), which are dependent on constant m and function r from Assumption 1 and from functions $\mathbf{f}$ and V; σ is some function from class $\mathcal{K}$. The definition of classes $\mathcal{K}$ and $\mathcal{K}_\infty$ are common [23]. Hence inequality for time derivative of V takes form:

$$\dot V\leq a(\mathbf{x})+\mu\,\rho(\mathbf{x})+\sigma(|\mathbf{w}_2(t)|)+L_{\mathbf{G}}V(\mathbf{x})[\mathbf{u}+\mathbf{w}_1(t)],$$

or, finally, using Yang inequality

$$\begin{aligned}\dot V\leq\ & a(\mathbf{x})+\mu\,\rho(\mathbf{x})+0.5|L_{\mathbf{G}}V(\mathbf{x})|^2+\\ &+L_{\mathbf{G}}V(\mathbf{x})\mathbf{u}+\delta(|\mathbf{w}(t)|),\end{aligned}\tag{2}$$

where $\mathbf{w}(t)=col[\mathbf{w}_1(t),\mathbf{w}_2(t)]$ is composed vector of external disturbances and $\delta(s)=\max(\sigma(s),0.5s^2)$.

A s s u m p t i o n 2. *There exists a differentiable Lyapunov (storage) function* $V(\mathbf{x})$, *such, that*

$$\alpha_1(|\mathbf{x}|)\leq V(\mathbf{x})\leq\alpha_2(|\mathbf{x}|)$$

for some functions $\alpha_1,\alpha_2\in\mathcal{K}_\infty$, *and inequality (2) holds for all* $\mathbf{x}\neq 0$ *and any* $\mu\geq 0$ *with properties:*

1. $|L_{\mathbf{G}}V(\mathbf{x})|\equiv 0\ \Rightarrow\ a(\mathbf{x})+\mu\rho(\mathbf{x})<0$;
2. $\left|a(\mathbf{x})+\mu\rho(\mathbf{x})+(1-\sqrt{0.5})|L_{\mathbf{G}}V|^2\right|\geq\alpha(|\mathbf{x}|)/\sqrt{0.5}$, $\alpha\in\mathcal{K}_\infty$. ■

It is worth to note, that the first property of Assumption 2 supposes, that on the set where control can not affect on the dynamic of system (1) (i.e. on set, where $|L_{\mathbf{G}}V(\mathbf{x})|\equiv 0$) this system is asymptotically stable for any admissible value of μ. An example of class of systems which possess conditions of this assumption is the following one with *input* functional uncertainty:

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x})+\mathbf{G}(\mathbf{x})[\boldsymbol{\omega}(\mathbf{x},t)+\mathbf{u}+\mathbf{w}_1(t)].\tag{3}$$

For system (3) property 1 of Assumption 2 takes form

$$|L_{\mathbf{G}}V(\mathbf{x})|\equiv 0\ \Rightarrow\ L_{\mathbf{f}}V(\mathbf{x})=a(\mathbf{x})<0\ \text{ for all }\ \mathbf{x}\neq 0,$$

the last fact simply means that system (3) can be asymptotically stabilised by (continuous) state feedback if functional uncertainty $\boldsymbol{\omega}$ and external disturbance $\mathbf{w}$ are missing (for details see [24], another condition for system (3) to be asymptotically stabilised by state feedback was presented in work [6]). The second property of Assumption 2 can be satisfied by proper choice of function V (see paper [23] for details). Both properties hold, for example, if function ρ contains multiplicative factor, which is dependent from $L_{\mathbf{G}}V(\mathbf{x})$, i.e. $\rho(\mathbf{x})=\rho_1(L_{\mathbf{G}}V(\mathbf{x}))\rho_2(\mathbf{x})$ and $\rho_1(0)=0$.

According to these statements, the solving problem consists in development of adaptive controller, which provides for any initial condition $\mathbf{x}_0\in R^n$ and constant $m\geq 0$:

– asymptotic stability of (1) then $\mathbf{w}(t)\equiv 0$, $t\geq 0$;

– forward completeness of system [1] and asymptotic boundedness of $\mathbf{x}$ for any L_∞ bounded $\mathbf{w}(t)$.

Further, the solution of these tasks will be presented.

3. Main results

For this purpose we will use the theory of input-to-state stable systems [23, 25, 26, 27, 28]. It is worth to stress, that input-to-state stable (ISS) system has global asymptotic stability property for vanishing input (0-GAS) and for any essential bounded inputs its trajectories stay asymptotically bounded by L_∞ norm of input. The generalisation of this property is input-to-output stability property (IOS for short), i.e. system with some continuous output function $\mathbf{y}=\mathbf{h}(\mathbf{x})$ is IOS, if it is forward complete, 0-GAS with respect to output $\mathbf{y}$ and possesses output trajectory boundedness property for L_∞ bounded inputs [22, 29, 30]. So, main characteristics of ISS/IOS properties coincide with requirements of control goal, which were formulated above. Hence, it is possible to base solution of the task on ISS theory using.

3.1. Non adaptive control

At first, for the sake of simplicity, let us assume that constant m from Assumption 1 is known. It is required to design a control law ensuring a input-to-state stability of a system (1) for any function $\boldsymbol{\omega}$ that satisfies Assumption 1. It is well known "universal" control [16, 33], which provides for any function ρ global asymptotic stability property of system (1) (robust stability with specified stability margin ρ):

$$\mathbf{u}=-\kappa_1(\psi(\mathbf{x}),\beta(\mathbf{x}))L_{\mathbf{G}}V(\mathbf{x})^T,\tag{4}$$

where $\psi(\mathbf{x})=a(\mathbf{x})+\mu\rho(\mathbf{x})$, $\beta(\mathbf{x})=|L_{\mathbf{G}}V(\mathbf{x})|$,

$$\kappa_1(r,s)=\begin{cases}\dfrac{r+\sqrt{r^2+s^4}}{s^2}, \text{if } s\neq 0;\\ 0, \text{if } s=0.\end{cases}$$

For the task of global asymptotic stabilisation of system (1) while disturbance $\mathbf{w}$ is missing such controls were formulated in papers [24, 33]. The extension of these results on the problem IOS stabilisation was proposed in [5].

R e m a r k 1 . Note, that control (4) is continuous and locally Lipschitz [24, 33], if function V admits so-called Small Control Property (SCP), that is

$$\lim_{|\mathbf{x}|\to 0}\sup\frac{L_{\mathbf{f}}V(\mathbf{x})}{|L_{\mathbf{G}}V(\mathbf{x})|}\leq 0 .$$

Due to Assumption 2, for $|L_{\mathbf{G}}V(\mathbf{x})|=0$ inequality $L_{\mathbf{f}}V(\mathbf{x})\leq a(\mathbf{x})+\mu\rho(\mathbf{x})<0$ holds for all $|\mathbf{x}|\neq 0$ and any $\mu\geq 0$. But under this condition the limit in general can stay positive and to base continuity property of control (4) SCP should be additionally imposed. ■

The ISS property uniformly with respect to uncertain function $\boldsymbol{\omega}$ for system (1), (4) follows by ISS-Lyapunov function candidate V analysis. Substitute in (2) control (4). According to Assumption 2, for $|L_{\mathbf{G}}V(\mathbf{x})|=0$ term $a(\mathbf{x})+\mu\rho(\mathbf{x})$ is negatively defined and radially unbounded. So, let $|L_{\mathbf{G}}V(\mathbf{x})|\neq 0$ and inequality (2) takes form:

$$\dot V\leq -\sqrt{[a(\mathbf{x})+\mu\rho(\mathbf{x})]^2+|L_{\mathbf{G}}V|^4}+0.5|L_{\mathbf{G}}V|^2+\delta(|\mathbf{w}|).$$

The square root function is concave one, and, hence, inequality $\sqrt{a+b}\geq\sqrt{0.5a}+\sqrt{0.5b}$ is satisfied, thus

$$\dot V\leq -\sqrt{0.5}|a(\mathbf{x})+\mu\rho(\mathbf{x})|-\sqrt{0.5}\left(1-\sqrt{0.5}\right)|L_{\mathbf{G}}V|^2+\delta(|\mathbf{w}|).$$

Due to product $\mu\rho(\mathbf{x})$ is always positive semidefinite, rewriting the last expression as follows:

$$|a(\mathbf{x})+\mu\rho(\mathbf{x})|+\left(1-\sqrt{0.5}\right)|L_{\mathbf{G}}V|^2\geq$$
$$\geq\left|a(\mathbf{x})+\mu\rho(\mathbf{x})+\left(1-\sqrt{0.5}\right)|L_{\mathbf{G}}V|^2\right|,$$

and using property 2 of Assumption 2 finally we obtain

$$\dot V\leq -\alpha(|\mathbf{x}|)+\delta(|\mathbf{w}|),$$

that is sufficient to conclude that V is ISS-Lyapunov function and system (1), (4) is ISS [25]. It is necessary to underline, that control (4) depends nonlinearly and convexly from function ρ and unknown parameter μ. Such kind of dependence allows to construct adaptive algorithm using standard methods of the adaptive control theory [8, 9], however, obtained algorithm of adaptation has in a denominator vanishing in the origin function. Further, let us proceed with adaptive versions of (4).

3.2. Adaptive and robust adaptive controls

Suppose, that constant m from Assumption 1 is unknown and, hence, constant μ is unknown too, then for system (1) control laws (4) should be modified as follows:

$$\mathbf{u}=-\kappa_1(\psi(\mathbf{x}),\beta(\mathbf{x}))L_{\mathbf{G}}V(\mathbf{x})^T , \tag{5}$$

where $\psi(\mathbf{x})=a(\mathbf{x})+\hat\mu\,\rho(\mathbf{x})$; $\hat\mu$ is adjusted parameter, estimation of unknown constant μ. Updating algorithm for parameter $\hat\mu$ is selected in the following way:

$$\dot{\hat\mu}=\gamma\,\rho(\mathbf{x}), \tag{6}$$

where $\gamma>0$ is design parameter. The main properties of designed adaptive system (1), (5), (6) will be substantiated in the following theorem, beforehand note, that control (4) ensure for a system (1) ISS property with respect to input $\mathbf{w}$. The adding of extra dynamic systems (6) means repeating of this problem research in adaptive systems.

R e m a r k 2 . The right hand sides of the equations (11) and (12), (13) vanish then $\mathbf{x}=0$. Hence, the set $\mathcal{A}=\{\tilde{\mathbf{x}}:\mathbf{x}=0\}$ is invariant in extended space of adaptive system $\tilde{\mathbf{x}}=(\mathbf{x},\hat\mu)$. ■

T h e o r e m 1 . L*et Assumptions 1 and 2 be true. Then system (1), (5), (6) possesses the following properties uniformly with respect to functional uncertainty* $\boldsymbol{\omega}$ *satisfied Assumption 1:*

1. *Forward completeness for any essentially bounded and Lebesgue measurable disturbance* $\mathbf{w}$.
2. *Asymptotic gain property with respect to* $\mathbf{x}$ *:*

$$\sup\lim_{t\to+\infty}|\mathbf{x}(t)|\leq\gamma(\|\mathbf{w}\|),$$

where function $\gamma(s)=\alpha^{-1}\circ\delta(s)$ *belongs to class* $\mathcal{K}$, $\|\mathbf{w}\|=\operatorname{ess\,sup}_t\{|\mathbf{w}(t)|\}$. ■

All proofs in this paper are omitted due to economy of space and can be found in [4].

R e m a r k 3 . If the vector $\mathbf{x}$ is separated from zero, variable $\hat\mu$ can increase infinitely (more precisely if the function ρ is separated from zero). Therefore, the control (5) improves its robust property. This increasing is the cost of disturbance attenuation in adaptive system. Indeed, in classical adaptive theory robustification of adaptation algorithms leads to static error in the system [8] even in the case of disturbance is missing. In this approach we recover unboundedness of $\hat\mu$. ■

As mentioned in remark 3, proposed adaptive control scheme (5), (6) does not guarantee boundedness of adjusted parameter $\hat\mu$. To compensate this shortage we need to introduce additional negative feedback in parameter updating law (6):

$$\dot{\hat\mu}=\gamma\rho(\mathbf{x})-\gamma k\hat\mu , \tag{9}$$

where $k>0$. In such a way Theorem 1 can be developed to the following one.

T h e o r e m 2 . L*et assumptions 1 and 2 be true. Then system (1),(5),(9) is ISS with respect to extended input* $(\mathbf{w}(t),\mu)$. ■

In proposed theorem the standard way of robustification was used [8, 9]. As pointed out in remark 3, in such systems there is a static error provided by constant μ (even then external disturbance is vanishing). Algorithm (6) has not this shortage, but algorithm (9) provides boundedness of all variables $\mathbf{x}$ and $\hat{\mu}$ for any L_∞ bounded disturbance $\mathbf{w}$. It seems interesting to develop a modification of proposed adaptive schemes, which possesses the absence of static error property like in system (5), (6) and boundedness of variables $\mathbf{x}$ and $\hat{\mu}$ like in system (5), (9). An intuitive way to solve this problem consists in developing a switching scheme, that combine advances of both approaches described above. One solution, obtained in this field, will be presented below.

3.3. Switching adaptive control

In Theorem 1 it was established, that there exists a moment of time $T \geq 0$, after which $\hat{\mu}(t) \geq \mu$ and algorithm of adaptation (6) can be switched off. Indeed, subsequent adaptation like (6) would increase value of $\hat{\mu}(t)$ without any needs for task of system (1) stabilisation, because inequality (8) would be already satisfied (that according to [25] is enough to state ISS property of system (1), (5)). Such algorithm of adaptation can be formalised as follows:

$$\dot{\hat{\mu}} = F_i(\mathbf{x}),\ i = 1,2;\ F_1(\mathbf{x}) = \gamma\rho(\mathbf{x}),\ \gamma > 0;\ F_2(\mathbf{x}) = 0. \quad (10)$$

Control system with adaptation algorithm (10) becomes a switching one, where signal $i(t) \in \{1,2\}$ describe a current dynamic of variable $\hat{\mu}(t)$. While $i(t) = 1$ dynamic of system (1), (5), (10) possesses properties, which were established in Theorem 1: forward completeness, global asymptotic stability of variable $\mathbf{x}$ for vanishing disturbance $\mathbf{w}$. For $i(t) = 2$ this system becomes equivalent to non adaptive system (1), (5) with some frozen value of variable $\hat{\mu}(t)$. If it happens, that $i(t) = 2$ but still $\hat{\mu}(t) < \mu$, then the behaviour of the system is unknown and should be investigated. In the case, then $i(t) = 2$ and $\hat{\mu}(t) \geq \mu$, as was discussed earlier, inequality (8) would be true and, like in non adaptive case, system receives ISS property. Hence properties of switching adaptive system in a complicated manner depend on system, that will assign value of signal $i(t)$. Such system in theory of logic-based control systems is called a *supervisor* [2, 12, 18, 19].

So, as it follows from previous discussion, supervision system should detect situation then $\hat{\mu}(t)$ becomes bigger than unknown constant μ to switch signal $i(t)$ into second position and prevent further increasing of $\hat{\mu}(t)$. While $\hat{\mu}(t) \geq \mu$ inequality (8) is satisfied, and, hence, inequality

$$\dot{V}(t) \leq -\alpha(|\mathbf{x}(t)|) + D_{\max} \quad (11)$$

is also holding, where $D_{\max} = \delta(W_{\max})$ and $W_{\max}$ is an upper bound of external disturbance $\mathbf{w}$ and $|\mathbf{w}(t)| \leq W_{\max}$ for almost all $t \geq 0$, such constant $W_{\max}$ possibly unknown always exists according to suppositions posed on signal $\mathbf{w}(t)$. Thus, inequality (11) can help us to design a supervisor. Of course inequality (11) does not mean inequality (8) satisfying, but by itself inequality (11) is enough to prove boundedness of state vector $\mathbf{x}$. Before we proceed with algorithm of supervisor system it is necessary to note, that in switching systems under acting of disturbances a strange behaviour can arise, which is called *chattering regime*. Such chattering regime originates from fast switching that can take place in the system due to disturbance presence. Classical definition of differential equations solution does not suit well for dynamic system in chattering regime [7, 31], hence, some special methods are used in logic-based switching control theory to prevent such regime arising. In this work we will use so-called *dwell time* technique [18], which is traditionally used only in linear switching control systems due to finite time escape phenomena in nonlinear systems. Here to avoid this obstacles in nonlinear systems we will prove forward completeness property of whole system under some mild conditions. So, supervision algorithm can be described as follows:

$$i(t) = \begin{cases} i(t_k) \text{ if } \tau < \tau_D; \\ \begin{cases} 1, \text{ if } \dot{V}(t) > -\alpha(|\mathbf{x}(t)|) + D_{\max}; \\ 2, \text{ if } \dot{V}(t) \leq -\alpha(|\mathbf{x}(t)|) + D_{\max}; \end{cases} \text{if } \tau \geq \tau_D; \end{cases} \quad (12)$$
$$\dot{\tau} = 1,\ \tau(t_k) = 0,$$

where auxiliary variable τ represents internal supervisor timer dynamic, $\tau_D > 0$ is dwell time constant and t_k, $k = 0,1,2,...$ are moments of switching (moments then signal $i(t)$ changes its value), k is number of current switching. The operating of algorithm (12) can be explaining in the following way: after each switching internal timer τ is initialised to zero, while $\tau < \tau_D$ signal $i(t)$ does not change its value. Dwell time presence in algorithm (12) help us to prevent fast switching arising in the system (1), (5), (10), (12). After dwell time signal $i(t)$ can be set up to 1, if inequality (11) does not satisfy and, consequently, variable $\mathbf{x}$ is not bounded; signal $i(t)$ would be set up to 2, if variable $\mathbf{x}$ is bounded. To state properties of adaptive switching system we need another one assumption.

A s s u m p t i o n 3. *There exists a constant* $X \geq 0$, *such, that*

$$\rho(\mathbf{x}) - \alpha(|\mathbf{x}|) \leq V(\mathbf{x})$$

for all $|\mathbf{x}| \geq X$ *and constant* $W_{\max}$ *is given.* ■

The knowledge of constant $W_{\max}$ is supposed in algorithm (12). The inequality introduced in Assumption 3 will be used further to prove forward completeness property of the system. Precise investigation of proposed switched system properties is summarised in the following Theorem.

T h e o r e m 3. L*et assumptions 1, 2 and 3 are true. Then system (1), (5), (10), (12) has*

a) *asymptotically bounded solution* $\tilde{\mathbf{x}} = (\mathbf{x}, \hat{\mu})$: *there exists* $T_1 > 0$, *such, that*

$$\mathbf{x}(t) \in \Theta,\ \Theta = \{\mathbf{x} : |\mathbf{x}| \leq \alpha^{-1}(D_{\max})\},\ \hat{\mu}(t) = const,\ t \geq T_1;$$

b) *if, additionally, for each fixed* $\hat{\mu} < \mu$ *system (1), (5) possesses unbounded solution, i.e. for each* $\varepsilon > 0$ *there exists an* $T_\varepsilon > 0$, *such, that* $|\mathbf{x}(t)| > \varepsilon$ *for* $t > T_\varepsilon$, *then asymptotic gain property with respect to variable* $\mathbf{x}(t)$ *holds for the system :*

$$\sup \lim_{t\to+\infty} |\mathbf{x}(t)| \le \gamma(\|\mathbf{w}\|),\ \gamma\in\mathcal{K}. \quad ■$$

The closely connected approach was used for stabilisation of uncertain nonlinear discrete time system in paper [2]. In that work a kind of inequality (11) was used for cyclic switching among of controller candidates.

Remark 4. Here Assumption 3 was used only to prove forward completeness property of system (1), (5) with fixed $\hat{\mu}$, then value $\hat{\mu}<\mu$. So in practise, any other conditions can be applied to base this necessary property. ■

In this section four variants of solution of proposed problem were presented: non adaptive, adaptive, robust adaptive and switching adaptive. Further we will consider some important task, which can be solvable by approach proposed here.

4. Applications

4.1 System with input uncertainty

Let us analyse system (3), that is a special case of system (1) with functional uncertainty linearly introduced on input of the system. Consequently, all schemes presented in the third section can be applied for system (3).

A classical task, that is considered in theory of adaptive control, is adaptive stabilisation of linearly parameterised nonlinear system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})(\omega(\mathbf{x},t)\boldsymbol{\theta} + \mathbf{u} + \mathbf{w}),$$

where $\mathbf{w}(\mathbf{x},t)$ is known regression function and $\boldsymbol{\theta}\in R^d$ is vector of unknown parameters. Usually suppose, that values of vector $\boldsymbol{\theta}$ belong to some compact set Ω_θ of admissible values. Here we can drop this assumption, it is easy to see, that such task can be reformulated to solved in this paper one if we introduce a new parameterisation $\omega(\mathbf{x},t)\boldsymbol{\theta} \le m\,r(\mathbf{x})$, which always exists. In classical adaptive control theory [8, 9, 15] a compensation control law is usually used:

$$\mathbf{u} = -\omega(\mathbf{x},t)\hat{\boldsymbol{\theta}} + \mathbf{k}(\mathbf{x}),$$

where $\mathbf{k}(\mathbf{x})$ provides input-to-state stabilisation of nominal system without unknown parameters $\boldsymbol{\theta}$ for any disturbances $\mathbf{w}$; vector $\hat{\boldsymbol{\theta}}$ is an estimate of vector $\boldsymbol{\theta}$. If it happens, that in asymptotic $\hat{\boldsymbol{\theta}}(t)\to\boldsymbol{\theta}$, then such control annihilates any influence of unknown parameters $\boldsymbol{\theta}$. Unfortunately, presence of disturbances $\mathbf{w}$ leads to additional parametric feedbacks in adaptation algorithm for $\hat{\boldsymbol{\theta}}$, which ensure additional static error for adaptive system without disturbances, as mentioned above in Remark 3.

In this paper another approach is used, where adjusted parameters $\hat{m}$ or $\hat{\mu}$ determine robust stabilisation property of control algorithm (5). Consequently, increasing of values of these parameters during their adjusting leads to improving robust properties of control (5). In this way this control compensate not only exact uncertain function $\omega(\mathbf{x},t)\boldsymbol{\theta}$ like in classical approach, but some class of uncertainties, which can be upper bounded by $m\,r(\mathbf{x})$. This advance help us to obtain some new properties for adaptive system, like it was stated in Theorem 1, and unify solving of several different tasks in one framework.

4.2. Nonlinear parameterisation

In previous paragraph a task of adaptive stabilisation of linearly parameterised by vector of unknown parameters $\boldsymbol{\theta}$ system was considered. In fact, nonlinear fashion of system (1) equation dependence on function $\boldsymbol{\omega}$ allows to solve an adaptive stabilisation task for systems with nonlinear parameterisation. The main idea, that helps to handle nonlinear appearance of vector of unknown parameters $\boldsymbol{\theta}$ in plant equations, is linear reparameterisation of initial equation, or more precisely, linear reparameterisation of time derivative of storage function V, as was done above in this paper. The same approach was used, for example, in paper [17] for more particular class of plant equation (1) and without disturbance $\mathbf{w}$ presence.

Let system equation takes form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\boldsymbol{\theta}) + \mathbf{G}(\mathbf{x})[\mathbf{u} + \mathbf{w}(t)],$$

where $\boldsymbol{\theta}\in R^j$ is some vector of unknown parameters. There is some differentiable storage function $V: R^n \to R_{\ge 0}$, such, that $\alpha_1(|\mathbf{x}|) \le V(\mathbf{x}) \le \alpha_2(|\mathbf{x}|)$, $\alpha_1,\alpha_2\in\mathcal{K}_\infty$ and

$$L_{\mathbf{f}(\mathbf{x},\boldsymbol{\theta})}V(\mathbf{x}) \le a(\mathbf{x}) + \mu\rho(\mathbf{x}),$$

where constant μ is a function of $\boldsymbol{\theta}$, and all conditions of Assumption 2 is also valid. Thus, all conditions are satisfied to use proposed in this paper robust adaptive technique to stabilise nonlinear parameterised system. It is worth to note, that there is no assumption about compactness property of set of admissible values of $\boldsymbol{\theta}$.

5. Conclusion

In the paper adaptive, robust adaptive and switching adaptive controls are synthesised, which ensure asymptotic stability of equilibrium $\mathbf{x}=0$ for the indicated class of nonlinear plants in the presence of functional uncertainties and provide asymptotic boundedness of state $\mathbf{x}$ variable for additional disturbance signal. The proposed adaptive control law (5) allows to adaptive stabilise systems, which are satisfied Assumptions 1 and 2 without any additional restrictions. Opposite to classical adaptive stabilisation technique, in this paper adjusting of parameters during adaptation increases robust ability of control (5), while in classical adaptive theory such controls are simply tuned on some special class of uncertainties exact compensation.

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