An LMI Approach to Stability of Discrete Delay Systems [1]

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Keywords: time-delay systems, discrete systems, Lyapunov functionals, delay-dependent conditions, norm-bounded uncertainties

1. Introduction

During the last decade, a considerable amount of attention has been payed to stability and control of continuous-time linear systems with delays (see e.g. [?], [?], [?], [?]-[?] and the references therein). Delay-independent and, less conservative, delay-dependent sufficient stability conditions in terms of Riccati or linear matrix inequalities (LMIs) have been derived by using Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions. Delay-dependent conditions are based on different model transformations. The most recent one, a descriptor representation of the system [?]-[?], minimizes the overdesign that stems from the model transformation used. The conservatism that stems from the bounding of the cross-terms in the derivation of the derivative of the Lyapunov-Krasovskii functional has also been significantly reduced in the past few years. An important result that improves the standard bounding technique of e.g. [?] has been proposed in [?].

Less attention has been drawn to the corresponding results for discrete-time delay systems [?], [?], [?], [?], [?]. This is mainly due to the fact that such systems can be transformed into augmented systems without delay. This augmentation of the system is, however, inappropriate for systems with unknown delays or systems with time-varying delays (such systems appear e.g. in the field of communication networks).

Delay-dependent conditions for stability of discrete-time systems have been obtained in [?], for the case of time-varying delays. The LMI conditions there are convex in the upper-bound of the delay $\bar{h}$. For $\bar{h}=1$, these LMIs coincide with the well-known delay-independent conditions in the case of constant delays (see e.g. [?]) and they are therefore very conservative. Delay-dependent conditions for the case of constant delays have been obtained in [?] via a model transformation that is similar to the one in [?].

In the present paper we derive the discrete counterpart of stability criteria of [?]. We apply a descriptor model transformation to a linear discrete time-delay system. We further develop the Lyapunov-Krasovskii method for linear discrete-time systems with delay and obtain stability criteria for both, constant and time-varying delays. In the case of time-varying delays, additional delay-independent conditions are derived that are based on the Razumikhin approach. Two simple examples are given that show that our conditions are less conservative than those that have appeared in the literature.

2. Problem Statement

We consider the following unforced discrete-time state-delayed system

$$x_{k+1} = (A+H\Delta_k E)x_k + (A_1+H\Delta_k E_1)x_{k-h_k}, \quad x_k=\phi_k,\ -\bar{h}\le k\le 0 \tag{1}$$

where $x_k \in \mathcal{R}^n$ is the state vector, h_k is a positive number representing the delay, $h_k \le \bar{h}$ and A, A_1, H, E and E_1 are constant matrices of appropriate dimensions and $\Delta_k \in \mathcal{R}^{r_1\times r_2}$ is a time-varying uncertain matrix that has the form $\Delta_k = diag\{\Delta_{1,k},...,\Delta_{m,k}\}$, where

$$\Delta_k^T\Delta_k \le I, \quad i=1,...,m. \tag{2}$$

For simplicity we consider the case of a single delay. The results may be easily generalized to the case of multiple delays. It is assumed that the eigenvalues of $A+A_1$ are all of absolute value less than 1.

We address the following problems.

- **Problem 1:** For $h_k = h$ that is an unknown constant satisfying

$$0 \le h_k \le \bar{h} \tag{3}$$

find whether the system is asymptotically stable for all Δ_k satisfying (**??**).

[1]This work was supported by the C&M Maus Chair at Tel Aviv University.

- **Problem 2:** Find a stability test for all time-varying h_k that satisfy (**??**) and Δ_k satisfying (**??**).

3. Delay-Dependent and Delay-Independent Stability

We consider in this section the nominal case where $H = 0$. The case with norm-bounded uncertainty in the system dynamics is treated in Section 4.

3.1. Descriptor model transformation.

Denoting

$$y_k = x_{k+1} - x_k \tag{4}$$

the system (**??**) can be represented, in the case where $H = 0$, by the following descriptor form:

$$\begin{bmatrix} x_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} y_k + x_k \\ -y_k + Ax_k - x_k + A_1 x_{k-h_k} \end{bmatrix}.$$

Since $x_{k-h_k} = x_k - \sum_{j=k-h_k}^{k-1} y_j$ it follows that

(5a-c)
$$E\bar{x}_{k+1} = \begin{bmatrix} I_n & I_n \\ A + A_1 - I_n & -I_n \end{bmatrix} \bar{x}_k - \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \sum_{j=k-h_k}^{k-1} y_j,$$
$$E = \{I_n,\ 0\}, \quad \text{and} \quad \bar{x}_k \triangleq \begin{bmatrix} x_k \\ y_k \end{bmatrix},$$

where

(6)
$$x_0 = \phi_0, \quad y_0 = (A - I)\phi_0 - A_1\phi_{-h_0},$$
$$y_k = \phi_{k+1} - \phi_k, \quad k = -\bar{h}, ..., -1.$$

Thus, if x_k is a solution of (**??**), then $\{x_k, y_k\}$, where y_k is defined by (**??**), is a solution of (**??**), (**??**) and vise versa.

3.2. Lyapunov-Krasovskii method for discrete systems with delays.

Lemma 1. *If there exist positive numbers α, β and a continuous functional*

$$V_k = V(x_{k-\bar{h}}, ..., x_k, y_{k-\bar{h}}, ..., y_{k-1})$$

such that

(7a,b)
$$0 \le V_k \le$$
$$\beta \max\{\max_{j\in[k-\bar{h},k]} |x_j|^2, \max_{j\in[k-\bar{h},k-1]} |y_j|^2\},$$
$$V_{k+1} - V_k \le -\alpha |x_k|^2,$$

*for x_k and y_k satisfying (**??**), then (**??**) is asymptotically stable.*

Proof. From (**??**b) it follows that

$$\sum_{j=0}^{k} (V_{j+1} - V_j) = V_{k+1} - V_0 \le -\alpha \sum_{j=0}^{k} |x_j|^2.$$

Therefore, for x_k and y_k satisfying (**??**) we have

(8)
$$|x_k|^2 \le \sum_{j=0}^{k} |x_j|^2 \le \frac{1}{\alpha} V_0$$
$$\le \frac{\beta}{\alpha} \max\{\max_{j\in[-\bar{h},0]} |x_j|^2, \max_{j\in[-\bar{h},-1]} |y_j|^2\}, \ \forall k \ge 0.$$

Let x_k be a solution of (**??**) and y_k be defined by (**??**), then $\{x_k, y_k\}$ satisfies (**??**), (**??**) and thus (**??**). Eq. (**??**) implies that $|x_k|^2$ is small enough for small enough $\|\phi\|^2 \triangleq \max_{j\in[-\bar{h},0]} |\phi_{-j}|^2$. Moreover, $\sum_{j=0}^{\infty} |x_j|^2 < \infty$ and, hence, $|x_j|^2 \to 0$ for $j \to 0$. □

3.3. The case of constant delay.

Denoting:

(9a,b)
$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \quad \text{and} \quad E = diag\{I_n,\ 0\}$$

we consider the following Lyapunov-Krasovskii functional:

$$V_k = V_{1,\,k} + V_{2,\,k} + V_{3,\,k} \tag{10a}$$

where

(10b-d)
$$V_{1,\,k} = x_k^T P_1 x_k = \bar{x}_k^T EPE\bar{x}_k, \qquad 0 < P_1$$
$$V_{2,\,k} = \sum_{m=-\bar{h}}^{-1} \sum_{j=k+m}^{k-1} y_j^T R y_j, \qquad 0 < R$$
$$V_{3,\,k} = \sum_{j=k-h}^{k-1} x_k^T S x_k, \qquad 0 < S$$

Note that $V_{1,\,k}$ corresponds to necessary and sufficient conditions for the stability of discrete descriptor systems without delay [**?**], $V_{2,\,k}$ is typical for delay-dependent criteria, while $V_{3,\,k}$ corresponds to delay-independent stability conditions [**?**].

We obtain the following:

Theorem 1. *Consider the system (**??**) with the constant time delay that satisfies (**??**) and $H = 0$. This system is asymptotically stable if there exist $P = P^T$, $Z \in \mathcal{R}^{2n\times 2n}$, $S,\ R \in \mathcal{R}^{n\times n}$ and $Y \in \mathcal{R}^{2n\times n}$ that satisfy the following LMIs.*

$$\Gamma(\bar{h}) \triangleq \begin{bmatrix} \Phi & Y - \mathcal{A}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \\ * & -S + \begin{bmatrix} 0 & A_1^T \end{bmatrix} P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \end{bmatrix} < 0 \tag{11a}$$

(11b,c)
$$\begin{bmatrix} Z & Y \\ Y^T & R \end{bmatrix} \ge 0 \quad \text{and} \quad \begin{bmatrix} I & 0 \end{bmatrix} P \begin{bmatrix} I \\ 0 \end{bmatrix} > 0$$

where

$$\Phi = \mathcal{A}^T P\mathcal{A} - EPE + \begin{bmatrix} S & 0 \\ 0 & \bar{h}R \end{bmatrix} + \bar{h}Z + Y\begin{bmatrix} I & 0 \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} Y^T$$

$$\mathcal{A} = \begin{bmatrix} I & I \\ A-I & -I \end{bmatrix}.$$

Proof: We find when $V_{k+1} - V_k$ is strictly negative.

$$V_{1,\ k+1} - V_{1,\ k} = \begin{bmatrix} x_{k+1}^T & 0 \end{bmatrix} EPE \begin{bmatrix} x_{k+1} \\ 0 \end{bmatrix}$$
$$- \begin{bmatrix} x_k^T & 0 \end{bmatrix} EPE \begin{bmatrix} x_k \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} x_{k+1}^T & 0 \end{bmatrix} P \begin{bmatrix} x_{k+1} \\ 0 \end{bmatrix} - \bar{x}_k^T EPE\bar{x}_k$$
$$= \left\{ \bar{x}_k^T \bar{\mathcal{A}}^T - (\textstyle\sum_{j=k-h}^{k-1} y_j^T) \begin{bmatrix} 0 & A_1^T \end{bmatrix} \right\} P$$
$$\times \left\{ \bar{\mathcal{A}}\bar{x}_k - \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \textstyle\sum_{j=k-h}^{k-1} y_j \right\} - \bar{x}_k^T EPE\bar{x}_k$$

(12a) $$= \bar{x}_k^T \left[\bar{\mathcal{A}}^T P \bar{\mathcal{A}} - EPE \right] \bar{x}_k + \mu_k + \eta_k$$

where

$$\bar{\mathcal{A}} = \begin{bmatrix} I & I \\ A + A_1 - I & -I \end{bmatrix},$$

(12) $$\mu_k = (x_k^T - x_{k-h}^T) \begin{bmatrix} 0 & A_1^T \end{bmatrix} P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} (x_k - x_{k-h})$$
$$\eta_k = -2\textstyle\sum_{j=k-h}^{k-1} \bar{x}_k^T \bar{\mathcal{A}}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} y_j$$

(13) $$V_{2,\ k+1} - V_{2,\ k} = \bar{h} y_k^T R y_k - \textstyle\sum_{j=k-\bar{h}}^{k-1} y_j^T R y_j$$
$$= \bar{x}_k^T \begin{bmatrix} 0 & 0 \\ 0 & \bar{h}R \end{bmatrix} \bar{x}_k - \textstyle\sum_{j=k-\bar{h}}^{k-1} y_j^T R y_j$$

(14) $$V_{3,\ k+1} - V_{3,\ k} = x_k^T S x_k - x_{k-h}^T S x_{k-h}$$
$$= \bar{x}_k^T \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} \bar{x}_k - x_{k-h}^T S x_{k-h}$$

and thus

(15)
$$V_{k+1} - V_k = \bar{x}_k^T \Gamma_1 \bar{x}_k - x_{k-h}^T S x_{k-h} - \sum_{j=k-\bar{h}}^{k-1} y_j^T R y_j + \mu_k + \eta_k$$

where

$$\Gamma_1 = \bar{\mathcal{A}}^T P \bar{\mathcal{A}} - EPE + \begin{bmatrix} S & 0 \\ 0 & \bar{h}R \end{bmatrix}.$$

By [**?**], for any $a \in R^n, b \in \mathcal{R}^{2n},\ \mathcal{N} \in \mathcal{R}^{2n\times n},\ R \in \mathcal{R}^{n\times n},\ Y \in \mathcal{R}^{n\times 2n},\ Z \in \mathcal{R}^{2n\times 2n}$, the following holds

(16)
$$-2b^T\mathcal{N}a \leq \begin{bmatrix} b \\ a \end{bmatrix}^T \begin{bmatrix} Z & Y - \mathcal{N} \\ Y^T - \mathcal{N}^T & R \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix},$$
$$\begin{bmatrix} Z & Y \\ Y^T & R \end{bmatrix} \geq 0.$$

Applying the latter to η_k, where $\mathcal{N} = \bar{\mathcal{A}}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix}$, $a = y_j$ and $b = \bar{x}_k$, we obtain the following:

$$\eta_k \leq \textstyle\sum_{j=k-h}^{k-1} \begin{bmatrix} \bar{x}_k^T & y_j^T \end{bmatrix} \begin{bmatrix} Z & Y \\ -\bar{\mathcal{A}}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} & \\ * & R \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ y_j \end{bmatrix}.$$

Hence,

(17)
$$\eta_k \leq \textstyle\sum_{j=k-\bar{h}}^{k-1} y_j^T R y_j + \bar{h}\bar{x}_k^T Z \bar{x}_k + 2\bar{x}_k^T \left[Y - \bar{\mathcal{A}}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \right] x_k$$
$$-2\bar{x}_k^T \left[Y - \bar{\mathcal{A}}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \right] x_{k-h}$$

and one obtains

$$V_{k+1} - V_k \leq \xi_k^T \Gamma(\bar{h}) \xi_k$$

where $\xi_k = col\{\bar{x}_k,\ x_{k-h}\}$.

The LMI $\Gamma(\bar{h}) < 0$ together with (**??**b,c), guarantees that $V_k \geq 0$ and $V_{k+1} - V_k < 0,\ \forall\ 0 \leq k$. The asymptotic stability of the system (**??**) is thus guaranteed by Lemma 1. □

The result of Theorem 1 depends on the delay bound $\bar{h}$. The corresponding criterion for asymptotic stability which is delay-independent can be readily derived as a special case of Theorem 1. Choosing $Z = \rho I_{2n},\ R = \rho I_n$ and $Y = \rho \begin{bmatrix} 0 & I_n \end{bmatrix}$, where ρ is a positive scalar and letting ρ tend to zero we obtain the following.

Corollary 1. *The system (**??**) with $H = 0$ and with constant delay is asymptotically stable independently of the delay if there exist $P = P^T \in \mathcal{R}^{2n\times 2n}$ and $S \in \mathcal{R}^{n\times n}$*

that satisfy the following LMIs.

(18)

$$\begin{bmatrix} \mathcal{A}^T P\mathcal{A} - EPE + \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} & \mathcal{A}^T P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \\ * & -S + \begin{bmatrix} 0 & A_1^T \end{bmatrix} P \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \end{bmatrix} < 0$$

$$\begin{bmatrix} I & 0 \end{bmatrix} P \begin{bmatrix} I \\ 0 \end{bmatrix} > 0.$$

The results of this section have been derived for the case where h is a constant that satisfies (**??**) (Problem 1). The case where the delay is time-varying (Problem 2) is treated next.

3.4. **Delay-dependent stability in the case of time-varying delay.** We treat in this section the case where the delay h is bounded by (**??**) but is time-varying. Similarly to the derivation of Theorem 1 we apply the Lyapunov-Krasovskii of (**??**a) with $V_{3,k} = 0$. Unlike the continuous case, the conditions for time-varying delays are also obtained by applying the bounding of (**??**) (in the continuous case the conservative version of (**??**) with $Y = \mathcal{N}$ is used). In the case of nonsingular A_1 we readily obtain the following.

Corollary 2. *Consider the system (**??**) with $H = 0$ and with time-varying delay that satisfies (**??**). Assume that A_1 is nonsingular. This system is asymptotically stable if there exist $P = P^T \in \mathcal{R}^{2n\times 2n}$ and $R \in \mathcal{R}^{n\times n}$ that satisfy the LMIs (**??**), where $S = 0$.*

Note that in the case of time-varying delay the conditions of Corollary 2 imply that $P_3 < 0$.

In the case of general (probably singular) A_1, modifying derivations of Corollary 2 by choosing the Lyapunov-Krasovskii of (**??**a) with $V_{3,k} = 0$ and with

$$V_{2,k} = \sum_{m=-\bar{h}}^{-1} \sum_{j=k+m}^{k-1} y_j^T A_1^T R A_1 y_j, \quad 0 < R,$$

we obtain the following

Corollary 3. *Consider the system (**??**) with $H = 0$ and with time-varying delay that satisfies (**??**). This system is asymptotically stable if there exist $P = P^T \in \mathcal{R}^{2n\times 2n}$ and $R \in \mathcal{R}^{n\times n}$ that satisfy (**??**b,c) and the following LMI*

$$(19) \quad \begin{bmatrix} \Psi & Y - \mathcal{A}^T P \begin{bmatrix} 0 \\ I_n \end{bmatrix} \\ * & \begin{bmatrix} 0 & I_n \end{bmatrix} P \begin{bmatrix} 0 \\ I_n \end{bmatrix} \end{bmatrix} < 0.$$

where

$$\Psi = \bar{\mathcal{A}}^T P \bar{\mathcal{A}} - EPE + \begin{bmatrix} 0 & 0 \\ 0 & \bar{h} A_1^T R A_1 \end{bmatrix} + \bar{h} Z$$
$$+ Y \begin{bmatrix} A_1 & 0 \end{bmatrix} + \begin{bmatrix} A_1^T \\ 0 \end{bmatrix} Y^T$$

3.5. **Delay-independent conditions in the case of time-varying delays.** As in the continuous-time situation, this case is treated adopting the Lyapunov-Razumikhin approach (see [**?**]).

Theorem 2. *Consider the system (**??**), where $H = 0$, with time-varying delay that satisfies (**??**). This system is asymptotically stable if there exist $0 < P \in \mathcal{R}^{n\times n}$ and scalars $\alpha \in (0, 1)$ and $q > 1$ that satisfy the following LMI:*

$$(20) \quad \bar{\Gamma}_{ind} \triangleq \begin{bmatrix} A^T P A - \alpha P & A^T P A_1 \\ * & A_1^T P A_1 - \frac{1-\alpha}{q} P \end{bmatrix} < 0.$$

Proof: Choosing the Lyapunov-Razumikhin function $V_k = x_k^T P x_k$ and assuming that for some $q > 1$

$$V_{k-i} \le q V_k, \quad -\bar{h} \le i \le -1, \; k \ge 0,$$

we find:

$$\begin{aligned} &V_{k+1} - V_k = (x_k^T A^T + x_{k-h_k} A_1^T) P (A x_k + A_1 x_{k-h_k}) \\ &-x_k^T P x_k = x_k^T (A^T P A - \alpha P) x_k + 2 x_{k-h_k}^T A_1^T P A x_k \\ &+ x_{k-h_k}^T A_1^T P A_1 x_{k-h_k} \\ &-(1-\alpha) x_k^T P x_k \le [x_k^T \; x_{k-h_k}^T] \bar{\Gamma}_{ind} \begin{bmatrix} x_k \\ x_{k-h_k} \end{bmatrix} \end{aligned}$$

and thus due to (**??**) $V_{k+1} - V_k < 0$, which implies the asymptotic stability of (**??**) (see [**?**]). □

4. Robust stability

We treat the uncertain case, where in (**??**) $H \neq 0$ and $[E \; E_1]$ is not zero. Since $\Gamma(\bar{h})$ in (**??**a) can be written as

$$\Gamma(\bar{h}) = \Gamma_0(\bar{h}) + \mathcal{M}^T P \mathcal{M},$$

where Γ_0 is the part of Γ that does not depend on $\mathcal{A}$ or A_1 and

$$\mathcal{M} = \begin{bmatrix} \mathcal{A} & \begin{bmatrix} 0 \\ -A_1 \end{bmatrix} \end{bmatrix},$$

we replace A in $\mathcal{A}$ with $A + H\Delta_k E$ and A_1 with $A_1 + H\Delta_k E_1$ and obtain, applying Theorem 1 to the uncertain system (??)-(??), that the stability of the system is guaranteed if there exist $P = P^T$, $Z \in \mathcal{R}^{2n\times 2n}$, S, $R \in \mathcal{R}^{n\times n}$ and $Y \in \mathcal{R}^{2n\times n}$ that satisfy (??b,c) and the following inequality for all $0 \geq k$:

$$\text{(21a)} \qquad \Gamma_0 + (\mathcal{M}^T + \tilde{E}^T \Delta_k^T \tilde{H}^T) P (\mathcal{M} + \tilde{H}\Delta_k \tilde{E}) < 0$$

where

$$\text{(21b,c)} \qquad \tilde{E} = \begin{bmatrix} E & 0 & E_1 \end{bmatrix} \quad \text{and} \quad \tilde{H} = \begin{bmatrix} 0 \\ H \end{bmatrix}.$$

It is well known that the following holds true for any two real matrices α and β of the appropriate dimensions and for Δ_k that satisfies (??)(see e.g. [?]).

$$\text{(22a)} \qquad \alpha\Delta_k\beta + \beta^T\Delta_k^T\alpha^T \leq \alpha D^{-1}\alpha^T + \beta^T D\beta$$

where

$$\text{(22b)} \qquad D = diag\{d_1 I, ..., d_m I\} > 0.$$

Choosing $\alpha = \mathcal{M}^T P\tilde{H}$ and $\beta = \tilde{E}$ and requiring P_3 to be negative-definite, we apply (??a,b) to (??a) and obtain the following.

Theorem 3. *Consider the system (??) with the constant delay that satisfies (??) and with Δ_k that satisfies (??). This system is asymptotically stable if there exist $P = P^T$, $Z \in \mathcal{R}^{2n\times 2n}$, S, $R \in \mathcal{R}^{n\times n}$, $Y \in \mathcal{R}^{2n\times n}$ and D of the structure (??b) that satisfy (??b,c) and the following LMIs.*

$$\text{(23a,b)} \qquad \begin{bmatrix} \Gamma(\bar{h}) & \mathcal{M}^T P\tilde{H} & \tilde{E}^T D \\ * & -D & 0 \\ * & * & -D \end{bmatrix} < 0, \quad P_3 < 0.$$

The corresponding criteria for robust stability in the delay-independent (constant and time-varying delay) and delay-dependent (time-varying delay) cases may be derived similarly.

5. Examples

Example 1: We consider the system (??) where:

$$\text{(24)} \quad A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix} \quad \text{and} \quad H = 0.$$

Assuming that h is constant, we seek the maximum value of $\bar{h}$ for which the asymptotic stability of the system is guaranteed. We compare three methods: The criterion of [?], Theorem 1 in [?] and Theorem 1 above. It is found that the method of [?] does not provide a solution even for $\bar{h} = 1$.The maximum value of $\bar{h}$, achievable by the method of [?], is 12, whereas a value of $\bar{h} = 16$ was obtained by applying Theorem 1 of the present paper.Using augmentation it is found that the system considered is asymptotically stable for all $h \leq 18$. The criterion of Corollary 1 did not provide a solution, so that no delay-independent solution has been found. Allowing h to be time-varying we apply Corollary 2. We obtain that asymptotic stability is guaranteed for all $h \leq 8$.

Treating next the case where the system parameters are uncertain with A and A_1 given in (??) and with $H = diag\{0.1, 0.2\}$, $E = I_2$ and $E_1 = 0.5I_2$, where $m = 1$ and $r_1 = r_2 = 2$, we apply Theorem 3 and obtain that the system (??) with constant delays is stable for all Δ_k that satisfy (??) if $h \leq 5$. This is achieved by taking $D = 159.3I_2$.

Example 2 [?]: We consider the system (??) where

$$A = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.4 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad H = 0.$$

In the case of constant delay, this system is delay-independently stable by the conditions of [?] and by Corollary 1 of the present paper. In the case of time-varying delay, by conditions of [?] the system is asymptotically stable for $0 < h_k \leq 2$. By Theorem 2, it is verified that also in the case of time-varying delay the system is delay-independently stable. This is achieved by taking $\alpha = 0.5$ and $q = 1.01$.

6. Conclusions

Delay-dependent criteria have been derived for determining the asymptotic stability of discrete-time systems with uncertain delay and norm-bounded uncertainties. It is the first time that the descriptor model transformation is applied in the discrete-time case and, similar to the corresponding continuous-time case, the resulting criteria are most efficient. The approach that is adopted in this paper allows for considering the case of time-varying delays that cannot be treated by using augmentation techniques. A delay-independent condition for the case where the delay is time-varying is also obtained which is based on the Lyapunov-Razumikhin approach.

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