Multivariable Control Configurations for Fluid Catalytic Cracking Units

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Abstract

The control problem of fluid catalytic cracking (FCC) units is a challenging task due to its model complexity, nonlinear dynamics, constrained variables and cross coupling interaction between inputs and outputs. This work is concerned with the design of multivariable feedback control configurations for FCC units. Sufficient conditions to achieve regulation in terms of the steady-state gain matrix are provided, allowing to obtain a systematic procedure for analyzing multivariable control configurations of complex and interacting processes. Numerical simulations on a dynamical model based on a FCC unit operating in the partial-combustion mode are used to show the effectiveness of several multivariable control configurations under disturbances and uncertainty parameters.
Keywords. Chemical industry; Conventional control; Multivariable control; Fluid catalytic cracking.

1 Introduction

Catalytic cracking has been one of the key processes in petroleum refining in the last decades. The fluid catalytic cracking (FCC) process is an important process in refiners for upgrading heavy hydrocarbons to more valuable lighter products. In a refinery that produces a substantial amount of gasoline, FCC gasoline makes up about 40 % of the overall refinery gasoline pool. The remaining gasoline is typically derived from straight-run naphta, coking, hydrocracking, and other molecular conversion units such as alkylation and reforming operations. Due to its large throughput and the high product-feed upgrade, the overall economic benefits of a refinery could be considerably increased if proper control and optimization strategies are implemented on FCC units [1,2].

There have been many studies in the literature to control FCC units. For instance, nonlinear controllers [3,4] and more complex mode predictive controllers [5-7] are used to control FCC processes. Multivariable control of FCC units has been considered for instance by Balchen *et al.*, [8] and Grosdidier *et al.*, [9]. Grosdidier *et al.*, [9] described the advanced computer controls installed on the reactor and the regenerator on a real-life FCC unit, concluding that advanced multivariable control can lead to a good dynamic performance with a margin of robustness. Balchen *et al.*, [8] used state-space predictive control to regulate temperature in FCC units. The controller is obtained by solving on-line a nonlinear programming problem, and simulations showed good closed-loop performance. However, the complex interaction among the process variables and the constraints on the manipulated and controlled variables causes the computing cost to be high and time consuming. Application of non-linear controllers to FCC based on uncertainty observer can be found in Alvarez-Ramirez *et al.*, [3] and Aguilar *et al.*, [4], where the temperature regulation problem for a regenerator-riser system is studied. These approaches produce practical controllers where the closed-loop temperature trajectories are forced to remain in a neighborhood close to the set point. In an important contribution, Hovd and Skogestad [10] addressed the problem of control structure (strategy) selection based on linear models. They concluded that a favorable selection of controlled variables is critical for good control of the FCC units. These works have shown that FCC units are nonlinear, multivariable and complex dynamic control systems. The complexity often is caused by the strong interactions existing between the control loops. On the other hand, the FCC unit is operated against one or more constraints. The most common constraint is a coke-burning limit. A controller design strategy for FCC units must therefore be capable of handling complex interactions between input-output pairs, and must be robust to modeling errors and

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nonlinearities over the operating region.

As previous works has discussed [10], [7], simple decentralized control loops are usually preferred for FCC unit control. The variable more important that should be controlled in the units of catalytic cracking is the temperature [1], [2]. Temperature is an independent variable that adjust steady-state catalyst activity and allows changing product slate and properties (such as gasoline octane). Thus, due to operation and design restrictions, temperatures in both regenerator and riser reactors must be regulated in order to have an efficient and safe process operation and to control conversion and, to a certain extent, yield distribution.

This motivated us to develop a new, general, and effective design of decentralized linear PI controllers for complex multivariable process. Decentralized control is very important in process applications because of its several advantages over a fully multivariable design. These advantages include flexibility in operation, failure tolerance, simplified design, and simplified tuning [10], [11]. On the other hand, linear control is inherently simple and has proven robust performance. In fact, simplicity of control decisions is quite important for its practical implementation and realization. The control loops are simple multivariable PI feedback controllers, which uses both riser exit and regenerator cyclone temperature measurements. For control design we have employed simple linear input/output models and we have exploited steady-state gain matrix properties in order to construct decentralized multivariable control configurations. The analysis and design are straightforward for refining industrial applications, and take full advantage of the true interactive nature of a multidimensional system to achieve good controller performance.

This work is organized as follows. In Section 2 FCC description and model identification for control purposes are presented. Section 3 presents the design of decentralized multivariable control configurations. Numerical simulations results on a FCC unit operating in partial-combustion mode are then given. Finally, the conclusion is drawn.

2 FCC description and identification

Several authors have made substantial efforts to model the behavior of FCC units. A detailed review of recent work on FCC modeling can be found in the article by Arbel *et al.,* [12]. Since FCC feedstocks consist of thousands of components, the estimation of intrinsic kinetic constants is very difficult, thus, the lumping of components according to the boiling range is generally accepted. Contributions to the modeling of FCC units vary from regenerator models over kinetic models for the reactions taking place in the reactor riser [1], [2]. The model used for this case study is one developed by Lee and Groves Jr. [13] with slight modifications according to Hovd and Skogestad [10]. It is based on the three-lump reactor model (*i.e.*, the charge material-gas oil over 370 °C, gasoline and middle distillates over C5-370 °C cut, and the remaining coke and gas), which comprises the main components in a FCC unit. The cracking is then described by the following three reactions:

$$\begin{aligned} &feed(F) \xrightarrow{1} gasoline(G) \\ &gasoline(G) \xrightarrow{2} coke + gas(L) \\ &feed(F) \xrightarrow{3} coke + gas(L) \end{aligned} \qquad (1)$$

cracking is a high conversion process in the sense that almost every molecule in the feed undergoes some change, but conversion as used here is typically $30-40$ wt %. The gasoline yield increases with conversion up to a maximum and then decreases as reaction 2 in the above equation predominates, and gasoline cracks to lighter products and to coke. The FCC model with its modifications is described in Hovd and Skogestad [10].

2.1 FCC unit description

In FCC units feed oil is contacted with hot catalyst at the bottom of the riser, causing the feed to vaporize. The cracking reactions occur while the oil vapor and catalyst flow up the riser. The residence time of the catalyst and hydrocarbon vapors in the riser is typically in the range $5-8$ seconds. The riser top temperature is typically between 750 and 820 K and is usually controlling by regulating the flow of hot regenerated catalyst to the riser. As a by-product of the cracking reactions coke is formed and is deposited on the catalyst, thereby reducing catalyst activity. The catalyst and products are separate in the reactor. Steam is supplied to the stripper in order to remove volatile hydrocarbons from the catalyst. In the regenerator which is operated in the fluidization regime, the coke is burnt off the catalyst surface by the air blown into the bed. This combustion reaction serves to reactivate the catalyst and to maintain the bed temperature ($950-980$ K for a gas oil cracker, $980-1080$ K for a resid cracker) high enough to supply the heat required for the vaporization and cracking reactions of the feed in the reactor [2].

Depending on the coke producing tendency of the feed, the FCC process can be operated in two distinct modes [2]: the partial combustion mode and the complete combustion mode. In the partial combustion mode the conversion of coke to CO_2 is not complete, which means that relatively large amounts of both CO are formed (This CO-rich regenerator flue gas can be sent to a CO boiler for further combustion to produce high pressure steam). Since it is not always possible operate an FCC unit in the complete combustion mode especially if the feed has a large coke production tendency and is also an economic incentive operating in the partial combustion mode, as the heat recovered in the CO boiler is valuable.

2.2 FCC Identification and Control Structure Selection

The research on the dynamic characteristics of FCC units reveal that FCC processes consist of a MIMO system with two inputs and two outputs. The independent variables that will be used for control in the following are the catalyst circulation rate F_s and the air flow to the regenerator F_a ([13]; [10]; [3]). We shall use the riser-regenerator cyclone (Hicks) control structure previously considered by Hovd and Skogestad [10]. The Hicks control structure uses the regenerator cyclone temperature $T_{cy}(K)$, and the riser exit temperature $T_{ri}(K)$ as controlled variables. Using measures of controllability such as right half plane (RHP) zeros and the frequency-dependent relative gain array (RGA), Hovd and Skogestad [10] suggested that both riser-regenerator (T_{ri}, T_{reg}) and Hicks (T_{ri}, T_{cy}) structures are good choices of control structures. Moreover, controlling T_{cy} avoids exceeding the metallurgical temperature limit in the regenerator cyclones and controlling T_{ri} directly affects the amount of gas products and therefore helps ensuring that the wet gas compressor operating limits are not exceeded. For this choice of pairing, there is no right half-plane (RHP) transmission zeros (which limit the achievable bandwidth) with a consequent potential for high-performance control. It must be pointed out that RHP transmission zeros are not desirable in a control structure because any controller cannot invert the plant and perfect control is not possible [10]. Moreover, RHP transmission zeros may lead to unstable dynamic compensation (nonminimum-phase systems), regardless of the design of the controller.

In the process industries, where there is a higher degree of uncertainty about process behavior, for control systems design purposes the input-output (transfer function) model approach is generally adequate. Furthermore, there is a correspondence between state-space models and their input-output counterparts. This section therefore considers only structures of input-output models of multivariable processes used in control systems design and analysis. The open-loop performance of a FCC unit is shown in Figure 1. Dynamic simulation of the FCC process was performed according to the parameters and model given in Hovd and Skogestad [10].

Each simulation run started from the steady state corresponding to the base case operating conditions and the subsequent transient response was obtained as each simulation variable went through a series of step changes

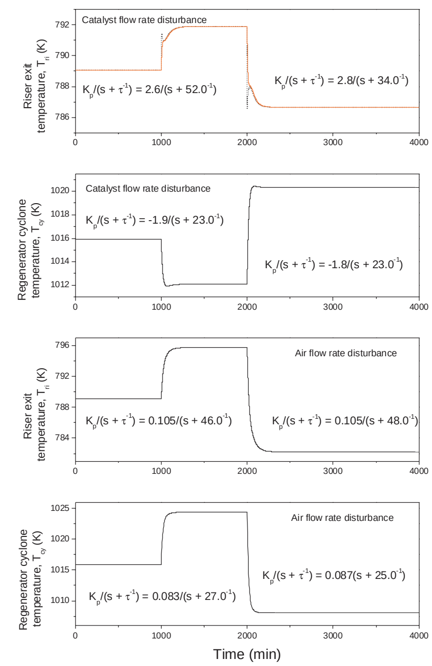

Figure 1: Step identification for a FCC unit for control design purposes.

shown in Figure 1. The transfer function of the dynamic of the FCC unit was determined from the reaction curve of the process obtained by giving step disturbances to catalyst circulation rate F_s and the air flow to the regenerator F_a. The step response in the regenerator unit is smooth, almost-monotonous, and convergent, such that it is reasonable to model the input-output response with simple first order models. On the other hand, the output response in the riser for a catalyst flow rate disturbance exhibits a large peak before it settles at the new steady-state value. However, we can use a filtered output response in order to put the transfer function in a more appropriate form. In fact, this can be considered as having a storage tank between riser and regenerator units, where the filtered time-constant corresponds to the residence time in this unit.

From the step responses, we can seen that the dynamics of both riser and regenerator units are dominated by a first order response. Moreover, regardless the type of disturbance introduced, the time constants are nearly the same. This mean that the dynamics in both units is governed by the characteristic capacitance of them. Then, we have

$$\begin{bmatrix} \Delta T_{ri} \\ \Delta T_{cy} \end{bmatrix} = \begin{bmatrix} \frac{k_{p11}}{s+\tau_a^{-1}} & \frac{k_{p12}}{s+\tau_a^{-1}} \\ \frac{k_{p21}}{s+\tau_b^{-1}} & \frac{k_{p22}}{s+\tau_b^{-1}} \end{bmatrix} \begin{bmatrix} \Delta F_s \\ \Delta F_a \end{bmatrix}$$

The elements within the blocks in the transfer function matrix defining the relationship between the respective input output pairs. The steady-state gain matrix K_p of the FCC multivariable system is then given as,

$$K_p = \begin{bmatrix} k_{p11} & k_{p12} \\ k_{p21} & k_{p22} \end{bmatrix} \quad (2)$$

In the time domain, the first-order model can be written as

$$\begin{aligned} \begin{bmatrix} \overset{\bullet}{\Delta T}_{ri} \\ \overset{\bullet}{\Delta T}_{ri} \end{bmatrix} &= -\begin{bmatrix} \tau_a^{-1} & 0 \\ 0 & \tau_b^{-1} \end{bmatrix} \begin{bmatrix} \Delta T_{ri} \\ \Delta T_{cy} \end{bmatrix} \\ &\quad + \begin{bmatrix} k_{p11} & k_{p12} \\ k_{p21} & k_{p22} \end{bmatrix} \begin{bmatrix} \Delta F_s \\ \Delta F_a \end{bmatrix} \\ \overset{\bullet}{Y} &= -TY + K_p U \end{aligned} \quad (3)$$

where $Y^T = (\Delta T_{ri}, \Delta T_{cy})$, $T = diag(\tau_a^{-1}, \tau_b^{-1})$, and $U^T = (\Delta F_s, \Delta F_a)$ are the output, open-loop time-constants, and input matrices respectively.

The optimal operating point for an FCC usually lies at one or several constraints [2]. Common constraints include: (i) Maximum regenerator cyclone temperature ($T_{cy} \leq 1000\ K$) constraint. This constraint is usually important in the complete combustion mode, and is determined by the metallurgical properties of the cyclones. (ii) Maximum and minimum air blower capacity ($0\ kg/s \leq F_a \leq 60\ kg/s$). The air blower provides the air needed for the combustion in the regenerator. (iii) Maximum and minimum catalyst circulation rate, $100\ kg_{cat}/s \leq F_s \leq 400\ kg_{cat}/s$.

Once a mathematical model of the system has been developed, and the presence of process interactions and constraints identified, the next stage of the control design procedure is to synthesize the control law.

3 Multivariable control configuration design

The objective of the control scheme is to simultaneously control the regenerator cyclone temperature and riser exit temperature. In a reactor that has the potential for multiple steady states, it is also important that the dynamic control be able to maintain unconditional asymptotic stability. In addition, the cracker has to handle a wide variety of feedstocks conditions and to change the product distribution and product specification.

Consider the nominal plant of the FCC unit given by Eq. (3), introducing the regulation error matrix $E^T = (e_1, e_2)$, where $e_1 = T_{ri} - T_{ri,ref}$ and $e_2 = T_{cy} - T_{cy,ref}$. The process is to be controlled by a classical negative feedback PI MIMO configuration given as,

$$U_c = -(\varepsilon_p HE + \varepsilon_i H \int E dt) \quad (4)$$

where $\varepsilon_p, \varepsilon_i > 0$ are closed-loop tuning parameters, H is the high-frequency closed-loop matrix, and U_c is the computed control input matrix. Moreover, the control input matrix is subjected to physical saturation. In this way, the saturated version of the above PI MIMO feedback control is given by,

$$U_r = Sat(U_c) = \begin{cases} U_c^{\min} & if \quad U_c \leq U_c^{\min} \\ U_c & if \quad U_c^{\min} < U_c < U_c^{\max} \\ U_c^{\max} & if \quad U_c \geq U_c^{\max} \end{cases} \quad (5)$$

and $U_c^{\max T} = [400\ kg_{cat}/s.,\ 60\ kg/s]$ and $U_c^{\min T} = [100\ kg_{cat}/s, 0\ kg/s]$ stand respectively for maximum and minimum limits of the catalyst circulation rate and air blower capacity.

Under this PI MIMO controller, the closed-loop equations are,

$$\begin{aligned} \overset{\bullet}{E} &= -TE + K_p U_r \\ &= -[T + \varepsilon_p K_p H] E + Z \\ \overset{\bullet}{Z} &= -\varepsilon_i K_p H E \end{aligned} \quad (6)$$

Our task here is to find high frequency closed-loop matrix H such that the closed-loop system (6) is stable with decentralized control loops. In fact, from the closed-loop equations we have,

$$\begin{aligned} \overset{\bullet}{X} &= -A_c X \\ A_c &= \begin{bmatrix} T + \varepsilon_p K_p H & 1 \\ \varepsilon_i K_p H & 0 \end{bmatrix} \end{aligned} \quad (7)$$

where $X^T = (E, Z)$. Since T is an anti-Hurwitz matrix and $\varepsilon_p, \varepsilon_i > 0$, the stability of the controlled process is completely determined by the stability of the matrix $A_H = K_p H$. The stability of matrix A_H implies that the input directionality of the closed-loop system is not lost.

From the step responses the steady-state gain matrix is given by:

$$K_p = \begin{bmatrix} 2.70 & 0.105 \\ -1.85 & 0.085 \end{bmatrix} \quad (8)$$

The closed-loop structure (7) and the matrix gain properties (condition number$\simeq$ 25 and positive eigenvalues) can be exploited to propose the following three decentralized cases for select the high-frequency closed-loop matrix H.

Diagonal matrix H (=diag(α_1, α_2)) Using a diagonal high-frequency matrix we have

$$A_H = \begin{bmatrix} 2.70\alpha_1 & 0.105\alpha_2 \\ -1.85\alpha_1 & 0.085\alpha_2 \end{bmatrix}$$

the closed-loop system is stable iff the matrix A_H is anti-Hurwitz or anti-stable (*i.e.*, all positive eigenvalues), thus $\alpha_1, \alpha_2 > 0$ are necessary and sufficient conditions for the stability of the closed-loop system of Eq (7). The PI decentralized control law is then given by,

$$\begin{bmatrix} \Delta F_s \\ \Delta F_a \end{bmatrix} = \begin{bmatrix} -\alpha_1 \left(\varepsilon_p \Delta E_1 + \varepsilon_i \int \Delta E_1 dt\right) \\ -\alpha_2 \left(\varepsilon_p \Delta E_2 + \varepsilon_i \int \Delta E_2 dt\right) \end{bmatrix} \tag{9}$$

Triangular matrix Both triangular upper and lower high-frequency closed-loop matrices selections are a special type of decentralized control since one control loop is actually acting only with information of a single temperature measurement (riser exit or cyclone regenerator temperatures).

For upper triangular matrix H selection we have,

$$A_H = \begin{bmatrix} 2.70\alpha_1 & 2.7\alpha_2 + .105\alpha_3 \\ -1.85\alpha_1 & -1.85\alpha_2 + .085\alpha_3 \end{bmatrix}$$

Thus we have two cases , (i) $\alpha_1, \alpha_3 > 0$, $\alpha_2 < 0$, and (ii) $1.4595\alpha_1 + 0.45946\alpha_3 > \alpha_2$, for achieve a stable closed loop behavior. The PI decentralized control configuration is given by

$$\begin{bmatrix} \Delta F_s \\ \\ \Delta F_a \end{bmatrix} = \begin{bmatrix} \alpha_1(\varepsilon_p \Delta E_1 + \varepsilon_i \int \Delta E_1 dt) \\ +\alpha_2(\varepsilon_p \Delta E_2 + \varepsilon_i \int \Delta E_2 dt) \\ \\ \alpha_3 \left(\varepsilon_p \Delta E_2 + \varepsilon_i \int \Delta E_2 dt\right) \end{bmatrix} \tag{10}$$

For lower triangular matrix selection of H we have

$$A_H = \begin{bmatrix} 2.7\alpha_1 + 0.105\alpha_2 & 0.105\alpha_3 \\ -1.85\alpha_1 + 0.085\alpha_2 & 0.085\alpha_3 \end{bmatrix}$$

We have two cases for achieve a stable closed-loop system: $(i) = \alpha_1, \alpha_2, \alpha_3 > 0$ and $(ii) = \alpha_1, \alpha_3 < 0; \alpha_2 > -8.0852\alpha_3 - 25.714\alpha_1$. The PI decentralized control configuration is given by

$$\begin{bmatrix} \Delta F_s \\ \\ \Delta F_a \end{bmatrix} = \begin{bmatrix} \alpha_1 \left(\varepsilon_p \Delta E_1 + \varepsilon_i \int \Delta E_1 dt\right) \\ \\ \alpha_2(\varepsilon_p \Delta E_1 + \varepsilon_i \int \Delta E_1 dt) \\ +\alpha_3(\varepsilon_p \Delta E_2 + \varepsilon_i \int \Delta E_2 dt) \end{bmatrix} \tag{11}$$

The real advantage of the proposed PI MIMO decentralized control configurations over traditional and nonlinear control MIMO designs can be summarized as follows: (1) These configurations have easy design and tuning methodologies. (2) If the high-frequency closed-loop matrix H is chosen adequately such that A_H is anti-stable, then the FCC unit can be robustly stabilized via simple decentralized multivariable PI control configurations. In fact, the control gains of the PI MIMO controller are parametrized in terms of a desired closed-loop stable matrix and two single parameters that are used for faster closed-loop response. Tuning and design

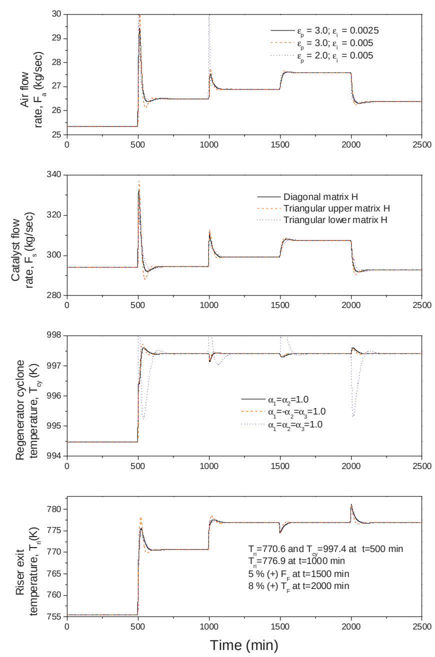

Figure 2: Closed-loop performance of three decentralized control configurations for a FCC unit.

rules can be formulated as follows. In a first step, determine high-frequency closed-loop matrix H such that A_H is an anti-stable matrix. Once that high-frequency closed-loop matrix H has been chosen, the control tuning depends only on two parameters, namely, ε_p and ε_i. We suggest select first ε_p values and then manipulate ε_i values. The underlying idea is to achieve convergence to the prescribed set point as fast as possible.

4 Numerical Simulations

Numerical simulations were carried out considering set point changes and typical disturbances to FCC reactor, namely, flow rate and temperature disturbances in the feedstock. The nominal values 25.35 and 294 kg/s are assigned to the control inputs F_a and F_s. Numerical values for other parameters are found in Hovd and Skogestad [10]. Figure 2 shows the closed-loop performance of three decentralized control configurations as previously discussed. The controller performs well in spite of a set point change and changes in feedstock conditions. Moreover, with identical tuning parameters $(\varepsilon_p, \varepsilon_i)$ for both the diagonal and upper triangular matrix selections we have found similar control performance, while for the lower matrix selection we have found the worst control performance.

For control design purposes, since the Hicks control structure is a favorable selection of controlled and manipulated variables, we proceed as follows. In a first step we select the high-frequency closed-loop matrix H such that the closed-loop system is stable and then we select the tuning controller parameters $(\varepsilon_p, \varepsilon_i)$. The tuning parameters were adjusted considering firstly a pure proportional action (ε_p), such that control actions are free of input-saturation. Then was manipulated the integral parameter, ε_i in order to eliminate steady-state off set and to achieve better control performance.

In order to prove the robustness of the proposed method, we increase the static gains in all the elements in the process transfer function matrix by 15-20 % random variations. Through numerical simulations (not shown) we have found that the closed-loop performance is not seriously affect.

5 Conclusions

In this paper, a systematic method is proposed for the design of decentralized multivariable control configurations for complex processes. The design exploits the structure of the input-direction matrix to provide decentralized PI configurations with guaranteed robust stability. Numerical simulations on a FCC unit shows that the resulting control performance with the proposed design is very satisfactory. An advantage of the proposed control design is that easy tuning procedures can be designed. Furthermore, the compensator design procedure is relatively simple and can be implemented easily. Although the control design is restricted to the type of multivariable process described here, the concepts presented in our work should find general applicability in the control of multivariable processes of similar type, such as that are commonly found in refining industrial applications.

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