

# MULTIPLE FAULT ISOLATION IN DIAGNOSTICS OF INDUSTRIAL PROCESSES

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## Abstract

The paper concerns the problems of on-line diagnosis of industrial processes with multiple faults. The authors discussed some hypotheses of multiple faults isolation conditions. Particularly, the multiple fault isolation analysis under assumption of single faults hypothesis is discussed. The simple, practical algorithm of multiple fault isolation was given.

## 1. Introduction

The on-line diagnostics of industrial processes is mostly performed by assumption of single faults. This significantly simplifies fault isolation algorithms. However the problem of admissibility of such assumption seems to be disputable. In large-scale technological installations, due to the plant scale and reliability factors the faults have to be considered as unavoidable. The faults may appear with differentiated frequency as a single or multiple faults. The most dangerous in practice and the most difficult for isolation are particularly those of the faults that appear simultaneously.

Staroswiecki *et al.* have been developed the structural approach for the design of FDI in large-scale industrial plants [7]. The applicable idea of FDI algorithm based on concept of hierarchical decomposition was described in [8]. Boolean inference scheme and least Hamming distance as a practical fault separability index in case of single and multiple faults was proposed by Staroswiecki [7].

The main aim of the paper is to discuss the following problems associated with multiple fault occurrences:

- In what cases the diagnostics based on assumption of single faults is allowed?
- Are the system state signatures with multiple faults based on single faults state signatures sufficiently certain?
- Is it necessary to scan along the state space of the system with multiple faults to formulate multiple fault diagnosis?
- What influence does multiple fault isolation have on the diagnostics of decentralized structures?

## 2. Fault isolation in case of single and multiple faults

### 2.1. Faults and diagnosed system states

Let us define the set of possible faults –  $F$ . Faults are considered as destruction events lowering the quality of overall or part of system functionality.

$$F = \{f_k : k = 1, 2, \dots, K\} \quad (1)$$

Faults should be detected in diagnostics process. System state  $z(f_k)$  related to fault  $f_k$  is defined as follows:

$$z(f_k) = \begin{cases} 0 - \text{fault free state} & f_k \\ 1 - \text{faulty state} & f_k \end{cases} \quad (2)$$

Let the set of existing faults will be defined as follows:

$$F(1) = \{f_k \in F : z(f_k) = 1\} \quad (3)$$

Let us assume that the diagnosed state of the system is determined by the states of all the faults from the set  $F$ .

$$z = \{z(f_1), z(f_2), \dots, z(f_K)\} \quad (4)$$

Set  $Z$  of all states  $z_i$  of diagnosed system:

$$Z = \{z_i : i = 0, 1, \dots, I\}, \quad |I| = 2^K \quad (5)$$

may be defined as a sum of states' subsets with the number of faults  $m$  varying from 0 do  $K$ :

$$Z = \bigcup_{m=0}^K Z_{(m)} \quad (6)$$

where:

$$Z_{(m)} = \left\{ z_i \in Z : \sum_{k=1}^K z(f_k) = m \right\} \quad (7)$$

is the system state subset with  $m$  multiple faults. The state  $z_i$  of diagnosed system may be unequivocally described by the set  $F(1)_i$  of faults occurred in this state.

## 2.2. System state and fault signatures

The set of diagnostics tests is used for fault isolation purposes. Every diagnostic test generates diagnostic signals  $s_j$  based on process variables (Fig. 1).

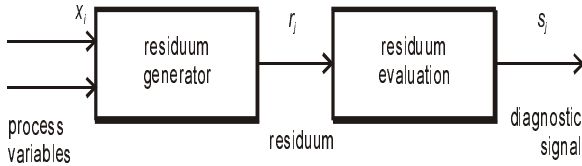


Fig. 1. Block diagram of a diagnostic test based on model of the system

The results of all the diagnostic tests are collected in the set of diagnostic signals  $S$ :

$$S = \{s_j : j = 1, 2, \dots, J\} \quad (8)$$

Inverse inference necessary to isolate faults by means of diagnostic signals is based typically on the *a priori* knowledge of relation faults-symptoms.

$$R_{FS} \subset F \times S \quad (9)$$

Relation faults-symptoms the most frequently has a form of binary diagnostic matrix (Fig. 2.). Every matrix element is defined as follows:

$$r(f_k, s_j) = \begin{cases} 0 & \Leftrightarrow \langle f_k, s_j \rangle \notin R_{FS} \\ 1 & \Leftrightarrow \langle f_k, s_j \rangle \in R_{FS} \end{cases} \quad (10)$$

The element of diagnostic matrix is equal to 1 if diagnostic signal  $s_j$  points out fault  $f_k$ . The columns of diagnostic matrix are called signatures of faults. Signatures may be interpreted as a reference patterns of particular faults in case of single faults.

$$V(f_k) = \{r(f_k, s_j) : s_j \in S\} \quad (11)$$

S/F	$f_1$	$f_2$	...	$f_k$	..	$f_K$
$s_1$						
$s_2$						
...						
$s_j$				$r(f_k, s_j)$		
...						
$s_J$						
	$V(f_1)$			$V(f_k)$		$V(f_K)$

Fig.2. Binary diagnostic matrix

The references of diagnostic signals of all system states may be defined similarly (see Fig. 3). The full matrix of system states consists of the diagnostic signals states in the healthy system state, in the states with single faults, in the states with double, triple etc. faults. The matrix of system states in case of single faults is identical to the binary diagnostic matrix.

	$Z_{(0)}$	$Z_{(1)}$				$Z_{(2)}$	...	$Z_{(K)}$
S/Z	$z_0$	$z_1$	$z_2$	...	$z_K$		$z_i$	$z_l$
$s_1$	0							1
$s_2$	0							1
...	0							1
$s_j$	0						$v_{ij}$	1
...	0							1
$s_J$	0							1

Fig. 3. The matrix of diagnosed system states.

The column vector of reference diagnostic signals values is defined as the particular state of the system signature.

$$V(z_i) = \{v(z_i, s_j) : s_j \in S\} \quad (12)$$

The states are undistinguishable if the state signatures are identical. The state signatures may be created on the basis of binary diagnostic matrix. Typically [1, 3, 6], the union operator of all fault signatures in particular system state is used to create elements of system state signature.

$$v(z_i, s_j) \equiv v_{ij} = \bigcup_{k: f_k \in F(1)_i} r_{jk} \quad (13)$$

The question is to what extent such assumption is admissible? Is it possible to define the system state signatures with multiple faults on the basis of the state signatures with single faults? The example below shows that sometimes the results of multiple faults may compensate each other.

**Example 1.** Let us analyse the influence of two chosen faults on the residuum value gained from balance equation for the tank system with inflow  $F$  and free outflow. Let  $L$  denotes the medium level in the tank.

$$r = F - \alpha_{12} D \sqrt{2gL} - A \frac{dL_1}{dt} - F_l \quad (14)$$

Simultaneously appearing of two faults: partly blocking the outflow (decrease of area  $D$ ) and increase of tank leakage (increase of  $F_l$ ) may compensate each other what in particular case may not change residual value. Residual is not sensitive to double fault, however it is sensitive to the single faults (blocking the outflow and leakage).

The above mentioned example shows that one can infer that operator (13) are not valid in all cases. Application of this operator does not guarantee that determination of reference state signatures will be correct. However, the probability of fault results compensation is in fact very low. So the operation (13) may be assumed as practicable.

### 2.3. Diagnosing based on assumption of single and multiple faults

In case of diagnostics with the assumption of single faults, the diagnosis points out these faults that's signatures conform with the achieved diagnostic signals.

$$DGN(F) = \{f_k \in F : \forall_{j: s_j \in S} s_j = r(f_k, s_j)\} \quad (15)$$

The faults are unisolable if the fault signatures are equal. In case of diagnostics assuming multiple faults, the diagnosis points out those system states that conforms to the achieved diagnostic signals.

$$DGN(Z) = \{z_i \in Z : \forall_{j: s_j \in S} s_j = v(f_k, s_j)\} \quad (16)$$

Those states are not distinguishable by the given set of diagnostic signals.

## 3. Multiple faults contra single faults diagnostics

### 3.1. Sequence of faults

Very common practice in diagnostics of industrial processes is assumption of the occurrence of exclusively single faults. This substantially reduces the complexity of fault isolation procedures. The following problem arises: is this assumption admissible?

In case of huge technological installations consisting of thousands of apparatus, the states with multiple faults are not the exceptional. But taking into account that the diagnostics

is performed in on-line mode the diagnosis  $DGN$  in moment  $n$ , should isolate the subset of faults which has appeared since previous diagnosis was generated:

$$\begin{aligned} DGN_n &= F(1)_n - F(1)_{n-1} = \\ &= \{f_k : [z(f_k)_n = 1] \wedge [z(f_k)_{n-1} = 0]\}. \end{aligned} \quad (17)$$

It is unlikely that more than one fault will appear within short time interval. Therefore if faults appear in sequences with time intervals longer than the time necessary to formulate subsequent diagnoses, the diagnoses generated by assumption of single faults may be classified as allowable. It can be stressed that after each subsequent diagnosis, the set of allowable diagnostic signals should be reduced by the signals that are sensitive to the fault detected [3, 6]. Enabling of the disabled diagnostic signals should be done immediately after recovering of fault free state of the system. Set  $S$  is dynamically modified according to principles given in [3, 6].

### 3.2. Simultaneous faults

The diagnostic inference may fail when two or more faults occur in the time interval shorter than the time of diagnosis formulation. In this case the set of diagnostic signals is inconsistent with the set of signatures of possible faults and diagnosis in the form of equation (15) can not be formulated. In particular cases the signature of the state with double or triple faults may conform to the signature of the single fault. In this case the false diagnosis will be generated.

The above mentioned inference scheme is correct if the diagnostic inference is based on the availability of full set of diagnostic signals. In fact, the only subset of diagnostic signals is sufficient to undertake the fault decision even in complex diagnosed systems. The principles of diagnostic inference based on dynamic decomposition of diagnosed process are given in [3, 6, 5].

The decomposition of the process starts after detection of first symptom. The symptom associated fault subset is determined after symptom occurrence. This subset consists of all possible faults and diagnosed signals necessary to identify these faults. Further diagnosis is limited to the given subset.

Let us assume that for the fault detection purposes the diagnostic signals are sampled in the predefined periods. First symptom  $s_j^1 = 1$  will start fault isolation procedure. The subset of possible faults is easy to create taking into account this symptom. This subset consists of all faults detected by the signal  $s_j^1$ :

$$(s_j^1 = 1) \Rightarrow F^1 = \{f_k \in F : f_k R_{FS} s_j\} \quad (18)$$

The following subset of diagnostic signals is created for the recognition of faulty state:

$$S^1 = \{s_j \in S_n : F^1 \cap F(s_j) \neq \emptyset\} \quad (19)$$

where:

$$F(s_j) = \{f_k \in F : f_k R_{FS} s_j\} \quad (20)$$

Let us see that the subsets  $F^1$  and  $S^1$  are determining the subset of diagnostic relation:

$$R_{FS}^1 = \{ \langle f_k, s_j \rangle \in R_{FS} : f_k \in F^1, s_j \in S^1 \} \quad (21)$$

The diagnosis formulated on this relation has the following form:

$$DGN(F)/S^1 = \{f_k \in F^1 : \forall_{j: s_j \in S^1} s_j = r(f_k, s_j)\} \quad (22)$$

If another fault appears in the system in the same time and if this fault does not belong to  $F^1$  then it will be detected by one of the diagnostic signals sensitive to this fault. Similarly to (18) the new subset of possible faults will be created:

$$(s_j^2 = 1) \Rightarrow F^2 = \{f_k \in F : f_k R_{FS} s_j\} \quad (23)$$

and a subset of diagnostic signals suitable for fault isolation:

$$S^2 = \{s_j \in S : F^2 \cap F(s_j) \neq \emptyset\} \quad (24)$$

what determine the subset of diagnostic relation :

$$R_{FS}^2 = \{ \langle f_k, s_j \rangle \in R_{FS} : f_k \in F^2, s_j \in S^2 \} \quad (25)$$

If the diagnostic sets  $S^1$  and  $S^2$  are disjunctive:

$$S^1 \cap S^2 = \emptyset \quad (26)$$

then both faults are isolated correctly in separate fault isolation processes, assuming single faults occurrence. In this case the relation subsets  $R_{FS}^1$  and  $R_{FS}^2$  are also disjunctive.

The above given above conclusion may be generalized: multiple faults appearing simultaneously or within short time intervals will be correctly localised under assumption of single faults if the subsets of faults suitable for localisation are disjunctive.

If condition (26) is not fulfilled, then the diagnostic signal is sensitive to both appeared faults. In this case the diagnostic reasoning should be performed under assumption of double faults or faults with appropriate higher multiplicity. The set of possible faults must fulfil the following condition:

$$F^* = F^1 \cup F^2 \cup \dots \quad (27)$$

Analogue the set of suitable diagnostic signals may be determined from:

$$S^* = S^1 \cup S^2 \cup \dots \quad (28)$$

**Example 2.** Let us consider the system with binary diagnostic matrix given on Fig.4. Let us assume that fault  $f_3$  and  $f_9$  appeared simultaneously and the following set of diagnostic signals is achieved:

$(s_1=0), (s_2=1), (s_3=0), (s_4=0), (s_5=1), (s_6=0), (s_7=0), (s_8=1), (s_9=1)$ . If the diagnostics will be based on the full binary matrix under assumption of single faults, then diagnosis may not be possible. Values of diagnostic signals are not conforming to any fault signature.

Let us take into further consideration the diagnostic case based on the dynamically determined subsets of diagnostic relations.

S/F	f <sub>1</sub>	f <sub>2</sub>	f <sub>3</sub>	f <sub>4</sub>	f <sub>5</sub>	f <sub>6</sub>	f <sub>7</sub>	f <sub>8</sub>	f <sub>9</sub>	f <sub>10</sub>	f <sub>11</sub>
s <sub>1</sub>		1		1							
s <sub>2</sub>			1			1					
s <sub>3</sub>				1		1					
s <sub>4</sub>	1				1	1					
s <sub>5</sub>					1		1		1		
s <sub>6</sub>							1				
s <sub>7</sub>										1	
s <sub>8</sub>								1	1		
s <sub>9</sub>								1	1		1

Fig.4. Binary diagnostic matrix of the system

Let us consider that fault  $f_4$  and  $f_9$  are detected by diagnostic signals respectively:  $s_2$  and  $s_9$ . The following subsets of faults are created:

$$(s_2^1 = 1) \Rightarrow F^1 = \{f_3, f_6\}, (s_9^1 = 1) \Rightarrow F^2 = \{f_8, f_9, f_{11}\}.$$

The corresponding subsets of diagnostic signals  $S^1 = \{s_2, s_3, s_4\}$  and  $S^2 = \{s_5, s_8, s_9\}$  are disjunctive. Diagnosis formulated on the grounds of formula (22) will point out both faults.

#### 4. Simple algorithm of multiple fault isolation

Multiple fault isolation based on formula (17) demands scanning of enormous set of system states. The system state signatures are created on the basis of formula (13). The described algorithm is not practical because of low efficiency. The subsets of possible faults and diagnostic signals are created to allow reduction of the number of considered system states, according to formula (27) and (28). As a result, the subsets of possible system states and subsets of system state signatures are also created.

Fault isolation algorithms based on this idea were presented by Kościelny in [3, 6]. From practical point of view it is important to answer whether searching of the system states space (part of the system) is necessary to achieve the diagnosis isolating multiple faults.

The simple algorithm that points out all possible fault subsets without examining system states will be presented. The following inference scheme is applied:

- values of the achieved diagnostic signals with the signatures of existing faults does not conform due to multiple faults occurrence.
- if multiple faults occur, all the symptoms of each of the fault are known
- seeking the possibility of occurrence of particular fault is sufficient to examine occurrence of its all symptoms.

The assumption of occurrence of multiple faults is done if the diagnoses can not be formulated due to inconsistency of the achieved values of diagnostic signals with the signatures of particular faults. If diagnosis is performed in accordance to the rules given in section 3.2, the subset of possible faults will be created according to formula (28) and the subset of diagnostic signals for isolation of these faults according to equation (29).

If diagnosis is performed on the basis of full diagnostic matrix, the set of possible faults should include all the faults that may cause the symptoms obtained during diagnosing when assuming single faults.

$$F^* = \bigcup_{j: s_j \in S \cap s_j = 1} F(s_j) \quad (29)$$

The set of signals suitable for fault isolation is created in accordance to following formula:

$$S^* = \bigcup_{k: f_k \in F^*} S(f_k) \quad (30)$$

where:

$$S(f_k) = \{s_j : \langle f_k, s_j \rangle \in R_{FS}\} \quad (31)$$

denotes the set of diagnostic signals which in case of occurrence of  $k$ -th fault should be equal to value "1".

The set  $S(f_k)$  is created for every fault from the set  $F^*$ . Then the set  $S(f_k)$  is checked whether all "1" values will occur. If this condition is fulfilled then this fault is treated as one of possible faults. The diagnosis is formulated on the basis of all the faults complying with this condition.

$$DGN(F) = \{f_k \in F^* : \forall_{j: s_j \in S(f_k)} s_j = 1\} \quad (32)$$

The set of faults defined in the diagnosis comprise not only the occurred faults but also:

- unisolable faults with existing faults (by using all currently available diagnostic signals)
- faults  $f_m$ , for which the subsets  $S(f_m)$  are subsets of the sum of the existing faults sets  $S(f_k)$ .

**Example 3.** Let us suppose that fault isolation procedure is performed on the basis of full binary diagnostic matrix shown on Fig. 2 Let us assume that faults  $f_5$  and  $f_6$  occur simultaneously. The following set of diagnostic signals is available:  $(s_1=0), (s_2=1), (s_3=1), (s_4=1), (s_5=1), (s_6=0), (s_7=0), (s_8=0), (s_9=0)$ .

Diagnosis performed on the basis of full binary matrix, assuming single faults leads to the conclusion of inconsistency of achieved diagnostic signals with all the single fault signatures. One can conclude that this is a case of multiple faults. Let us apply presented above algorithm to multiple fault isolation.

The following set of possible faults  $F^* = \{f_1, f_3, f_4, f_5, f_6, f_7, f_9\}$  is build up on the basis of (29). According to (29) and (30) the set of diagnostic signals suitable for reasoning is created  $S^* = \{s_1, s_2, s_3, s_4, s_5, s_6, s_8, s_9\}$ . Diagnosis  $DGN(F) = \{f_1, f_3, f_5, f_6\}$  points out, besides existing faults, two additional for those occurred all characteristic symptoms.

#### 5. Multiple fault diagnosis in decentralised systems

Diagnosing method described in section 3 is based on the dynamic assigning the subsets of possible faults and diagnostic signals suitable for further fault isolation. Certain subsystem is therefore considered and determined by the appropriate subsets of binary diagnostic matrix. Similar effects bring the decomposition of the system and diagnosing in decentralised structures. In many cases it is possible to isolate the multiple faults even if assuming single faults occurrence performs the inference. This problem is discussed in the paper of Kościelny [4, 6] and in chapter 18 in the monograph [2].

## 6. Summary

The paper refers to the problems of diagnostics of multiple faults. The following conclusions may be formulated:

- a) Presently, there is no absolutely certain method of determining system state signatures on the basis of signatures of single faults.
- b) Diagnosis generated by assumption of single faults are correct if the faults occur sequentially with the occurrence intervals longer than the time necessary to formulate subsequent diagnosis.
- c) Multiple faults appearing simultaneously or within short time intervals will be identified correctly under assumption of single faults if the dynamically determined subsets of diagnostic signals suitable for its localisation are disjunctive.
- d) It is possible to determine the set of faults including existing fault without considering system states with multiple faults. Appropriate inference algorithm was given.
- e) System decomposition and decentralised diagnosing gives the opportunity to isolate of multiple faults even if the inference is performed under assumption of single faults.

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## References

- [1] J. Gertler. "Fault Detection and Diagnosis in Engineering Systems", Marcel Dekker, Inc. New York - Basel - Hong Kong, (1998).
- [2] J. Korbicz, J.M. Kościelny, Z. Kowalczyk, W. Cholewa. "Process diagnostics" (in polish). WNT, Warsaw, p. 828, (2002).
- [3] J.M. Kościelny. "Fault Isolation in Industrial Processes by Dynamic Table of States Method", *Automatica*, Vol.31, No.5, pp. 747-753, (1995).
- [4] J.M. Kościelny. "Diagnostics of Processes in Decentralized Structures", *Archives of Control Sciences*, Vol. 7, No3/4, pp.181-202, (1998).
- [5] J.M. Kościelny, D. Sędziak, K. Zakroczymski. "Fuzzy Logic Fault Isolation in Large Scale Systems", *International Journal of Applied Mathematics and Computer Science*, Vol.9, No.3, 637-652, (1999).
- [6] J.M. Kościelny. "Diagnostics of Automated Industrial Processes", (in polish), Akademicka Oficyna Wydawnicza Exit, Warsaw (2001).
- [7] R.J. Patton, P.M. Frank, R.N. Clark (eds.). "Issues of Fault Diagnosis for Dynamic Systems", Springer-Verlag London, p.597, (2000).
- [8] M. Staroswiecki, M., J.P. Cassar and M. Bayart. "Hierarchical data validation in control system using smart actuators and sensors", IFAC Symposium: SICICI'92, Malaga, Spain, (1992).