CONTROLLER TRANSFER UNDER SAMPLING RATE DYNAMIC CHANGES

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Abstract: In this paper, the effects due to changes in the sampling period of digital controllers are analyzed. The swapping between controllers could produce instability problems and, in any case, the control performances could be degraded. In order to minimize this effect, the new control actions are computed based on the new controller parameters and estimations of the required past values of the state or appropriated sequences of its input and output. The proposed algorithm uses interpolation and numerical optimization to evaluate the value of these variables at the new sampling periods for the next controller. The improvements obtained by this approach are shown by simulation examples.

Keywords: Computer control systems, real-time systems, scheduling, controller transfer.

1. INTRODUCTION

This paper is concerned with the effect that variations on the sampling rate in the digital control may produce on the controlled system performances. Such situation can arise due to different reasons. One of them is the change in the scheduling policy to achieve optimal usage of central processing unit (CPU) resources. Several proposals for optimising the CPU usage, consisting in adjusting dynamically the period of the tasks in order to improve what is commonly named as Quality of Service (QoS), can be found at (Buttazzo, *et al*., 1998), (Kosugi and Moriai, 1997), (Shin and Meissner, 1999), (Seto *et al*., 1996), (Stankovic *et al*., 1999).

It is assumed that several tasks are executed at the same CPU and one or some of them will implement digital controllers. It is also well known that the control performances under digital control are degraded if the sampling time is too large. If enough computing resources are available, the tasks could be executed more frequently. Otherwise, the CPU would be overloaded and the real time constraint will be violated.

But this change in the resources availability will determine a change in the activation rate of the tasks. This, for the control tasks, will imply a variation on the sampling period that can modify the dynamic behaviour of the controlled system, becoming unstable or, at least, having degraded performances. This is the typical situation of transfer between controllers, although in this case both controllers are designed for the same purpose but with different sampling rates.

Under these conditions, the new controller should be launched and, even if the controller parameters can be pre-computed and available, the appropriated input-output data are not known because the stored system signals were taken at different instants of time. In this paper, an approach to reduce the impact of the sampling rate change is presented. The solution involves the computation of the controller parameters and the estimation of the sequence of input-output signals at the new sampling rate.

The easiest solution requires the output interpolation as well as the estimation of the previous control actions. The spline technique is used for carrying out the interpolation and a numeric method (multivariable optimisation) is used for tuning the values of the control actions. The applied optimisation method is the Simplex algorithm based on the evaluation of a cost function in order to find the zone where the optimal solution is located. The achieved improvements are shown in simulation examples and some draft conclusions are summarized in the last section.

2. PROBLEM STATEMENT

Consider a linear time-invariant SISO continuous time (CT) system described by:

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + b_c u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in R^n$, is the state of the system, $y \in R$ is the output, and $u \in R$ is the control input being updated by a computer at a sampling period T through a zero order hold (ZOH). So, for a given sampling period, there exist a discrete time function and a state space representation defined by:

$$\left(1+\sum_{i=1}^{n} a_i q^{-i}\right) y_k = \sum_{i=1}^{m} b_i q^{-i} \cdot u_k \tag{2a}$$

$$x((k+1)T) = Ax_k + bu_k \quad y_k = cx_k \tag{2b}$$

where $A = e^{A_c T}$ and $b = \int_0^T e^{A_c \tau} d\tau \cdot b_c$, and the subindex k stands for time kT, that is, i.e., $u_k = u(kT)$. Eq: (2a) can be written as:

$$y_k = -\sum_{i=1}^{n} a_i y_{k-i} + \sum_{i=1}^{n} b_i u_{k-i} = \Psi_{k-1}^T \cdot \theta \tag{3}$$

where,

$\Psi_{k-1}^T = \begin{bmatrix} -y_{k-1} & \dots & -y_{k-n} & u_{k-1} & \dots & u_{k-n} \end{bmatrix}$

is the regressor vector and,

$\theta^T = \begin{bmatrix} a_1 & \dots & a_n & b_1 & \dots & b_n \end{bmatrix}$ is the vector of process model parameters, being a function of the sampling period. If the output of this system is being controlled with a discrete-time (DT) controller, this will be designed either from the discrete expression of the process or by discretizing a CT controller, and its coefficients will depend on the used sampling period.

Similarly, a DT controller can be modelled by the general expression:

$$\begin{aligned} u_{k,T_j} &= -\sum_{i=1}^{m} p_i(T)_j \cdot u_{k-i,T_j} + \sum_{i=0}^{m} q_i(T_j) \cdot e_{k-i,T_j} \\ &= \Phi_k^T \nu(T_j) \end{aligned} \tag{4}$$

where the parameters of these controllers also depend on the used sampling period, T_j. The possible sampling periods that could be used will belong to an interval between a minimum and a maximum (T_j, $j \in [T_{min}, T_{max}]$). The resources of the computing system will limit the minimum sampling period that can be used, while the characteristics of the controlled system will limit its maximum value.

So, from the expression (4), the problem of changing the sampling period could be interpreted as a problem of changing from one controller to another, and can be viewed as a switching system. The controller structure could consist on several controllers (one for each possible sampling period) being executed in parallel, as is proposed in (Morse, 1995). The supervisor should decide if a sampling period change is required and, in that case, will choose the latest control action computed by the selected controller. Note that each controller must compute the control at different rates, using sampled variables also at different time. The computation is rather complex.

In our case, the changing of the sampling period of the tasks was proposed as a solution to improve the CPU utilization when a set of tasks has to be scheduled. So, it is not efficient to have so many controllers working in parallel and, in order to allow more flexibility in the selection of the new sampling period, other approaches should be investigated.

Anyway, even if there is not a pre-established set of controllers, the system operation will require to shift from one controller to another one. One of the most important issues for any control systems is the stability. Many stability results have appeared in the literature during the last decade for hybrid and switched systems. (DeCarlo *et al.*, 2000), (Liberzon and Morse, 1999), (Michel, 1999) are excellent survey papers that summarize some significant progress in the stability and stabilization of these systems.

In our case, we will try to implement the new controller in order to get a behaviour as close as possible to that of the controlled system under normal operation (without the switching). To this end, the new controller parameters and the appropriated past values of the required variables should be computed before the switching.

3. CHANGE ON THE SAMPLING RATE

Let us consider two activities: to compute the parameters of the new controller and to initialise the control algorithm.

3.1 Controller design

As previously mentioned, a common approach to designing a digital controller is to discretise a CT one. Of course, the coefficients of the DT controller are functions of both the sampling period and the discretisation approach. In order to illustrate the options, consider two typical control strategies, the classical PID controller and the state feedback control.

PID controller. The CT control action is given by:

$$u(t) = K_p e(t) + K_d \frac{d}{dt} e(t) + K_i \int_0^t e(\tau) d\tau \qquad (5)$$

A DT counterpart, assuming a sampling period *T* and a zero-order hold device, is:

$$u_k - u_{k-1} = q_0 e_k + q_1 e_{k-1} + q_2 e_{k-2} \qquad (6)$$

The coefficients q_i, by applying an Euler approximation such as:

$$u_k = K_p e_k + \frac{K_d}{T}(e_k - e_{k-1}) + K_i T \sum_{j=0}^{k-1} e_j$$

are given by:

$$\begin{aligned} q_0 &= K_p + \frac{K_d}{T}; q_2 = \frac{K_d}{T}; \\ q_1 &= -K_p - 2\frac{K_d}{T} + K_i T; \end{aligned} \qquad (7)$$

showing their dependence on the sampling period. Similar expressions are obtained if any other discretisation approach is used to get eq. (6) from eq. (5). Simple formulas, such as (7), allow the controller coefficients to be directly updated if the sampling period changes. Similar expressions can be derived for any DT transfer function (Salt and Albertos, 1990).

State feedback control. Assume a conventional state-space process model (1) and a control law given by

$$u(t) = K \cdot x(t) + r(t)$$

where *r(t)* is the external input. If a regular sampling period T is applied, to keep equivalent closed-loop poles (only valid for small time intervals) the DT feedback law should be (Aström and Wittenmark, 1984)

$$u_k = K' \cdot x_k + r_k = (K + K^* T) x_k + r_k \qquad (8)$$

where $K^* \cong K(A_c + B_c K)/2$.

3.2 Information required

The problem that intuitively can be realised is that when a change in the sampling period occurs all the past stored information is not valid any more because some past values of the output and the control action, or the current state, available at the switching point, have been obtained with a temporal pattern that differs from the one that will be used from the switching point ahead.

Depending on which situation the system is, this could be a problem or not. If the system is at the steady state, a change in the sampling period does not affect its response. But, if the change occurs during the transient state this change could affect the performance of the system.

The main problem in implementing a multi-controller structure is the computational efficiency. Keeping in mind that at least one task will be needed for each controller, this will result in having so many tasks on execution as possible sampling periods, and each one working at a different sampling rate. A better solution could be to activate the new controller only when a change is decided but in this case the past information is not available. So, the proposed solution is to estimate these values using for that purpose the past outputs obtained with the previous controller and its sampling period, the inputs actually applied to the process and any result of state estimation, if this is the case.

Two scenarios have been considered. First, it is assumed that the new sampling period is multiple (divisor) of the previous one. Secondly, any arbitrary value of the new sampling period (but not too far from the previous one) is assumed.

4. MULTIPLE (DIVISOR) PERIOD

Let us assume that the current sampling period is T', and the new one, T, is a fraction of it (T'=λT). In this case, for an input/output DT controller, the main problem is that the regressor components, leading to the current control, (4), are unknown.

A solution for estimating these components is by interpolation. This idea is also used in (Albertos, *et al.*, 2003), applied to scarce measurements.

So, when a change in the sampling period is needed, the interpolated error values are computed by an interpolator, for instance, a cubic spline.

In addition to the error samples, the appropriated past values of the control action leading to the current control are also needed. In this case, as the current sampling rate is a multiple of the new one, it is possible to consider that the control action has been kept constant during several samples (particularly, λ samples).

The error and control signals are illustrated in fig. 1.

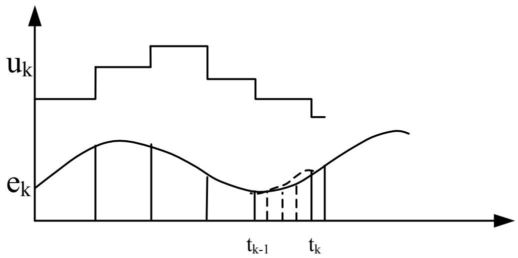

Fig. 1: Transfer to a faster controller with multiple sampling rate.

At the instant of time t_k a change of the sampling period is decided. So, the values of the error at the past instants are interpolated. These samples are represented by dotted lines. The control action has been constant.

Now, consider an increment in the sampling period, the new sampling period being multiple of the previous one ($T = \lambda T'$).

Under this assumption, the past error samples corresponding to the new sampling period are known. The only warning is to have a data structure large enough in order to ensure that, when a change at the sampling period is decided, the required (λxm)-past samples are stored.

The problem now is to compute the equivalent past control actions leading to the current behaviour, as illustrated in figure 2.

So, if the sampling period is duplicated, as shown in fig. 2, the control action values are already stored and the equivalent “average” value should be computed. The solution to the problem could be to calculate the past controller inputs that, using the new sample period, would be able to generate the same measured outputs, based on the process model.

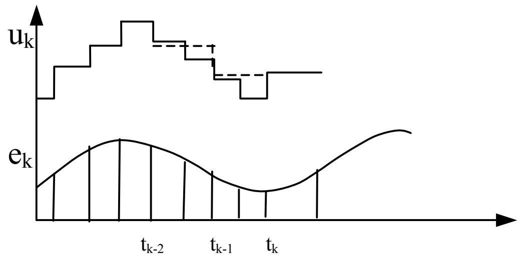

Fig. 2: Transfer to a slower controller with a multiple sampling period.

So, let us assume that the input-output DT model of the process using the T_j sampling period is as given by (3). We must estimate the values of the *n* past control actions leading to the same (or as much similar as possible) y_k, at the change instant, although we only need *m* past values to run the controller.

This problem can be seen as an optimisation one where the index to be minimised is the difference between the real output at the switching time and the estimated value obtained by (3). The values for the past control actions can be estimated using, for instance, the Simplex algorithm.

For a state feedback controller, the feedback gain can be approximately computed using (8), if the sampling periods are small enough. Otherwise, the approximate relationship is not valid.

5. ARBITRARY PERIOD CHANGE

The above methods can also be applied if the relation between the sampling periods is not an integer, but some considerations are needed.

To transfer to a faster control rate, the assumption of considering the past control actions as constants, as it was assumed before, is not valid anymore. The reason is that, if the relationship between the two sampling periods is not an integer value, a change could happen for the control action between two new samples.

So, at this situation the past values for the outputs as well as the past values for the inputs are unknown. For the case of the outputs, the previous approach of using interpolation with a spline is also proposed. In order to obtain the values of the past control actions, the Simplex algorithm can be also used but using the estimated past outputs.

In the case of a slower control rate, the problem for the past control actions is as before. The difference is that the past values for the outputs are unknown. The interpolation between the known samples is also used in this case, and the obtained values will be used in the Simplex algorithm. This situation is shown in fig. 3.

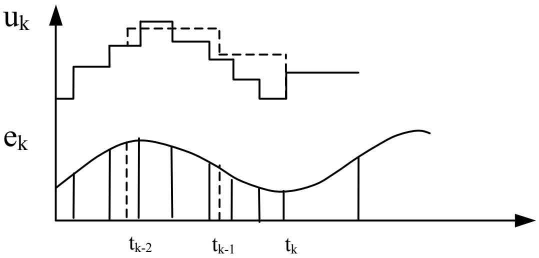

Fig. 3: Arbitrary change in the sampling period

6. EXAMPLES

In order to illustrate the results of using this approach some simulations of changing from one sampling period to another have been run. At these simulations, the input-output and the state-space representation for the systems have been considered. The tool used for doing this simulation is the toolbox based on Matlab TRUETIME. For more information about how it works see (Henriksson *et al.*, 2002).

Let us control the open-loop unstable system

$$G(s) = \frac{100}{(s+10)(s-1)}$$

and assume that the required dynamical specifications are defined by the closed-loop poles:

$$s = -3.5 \pm 7j$$

and steady-state error equal to zero.

Let us assume that, when a change at the sample period is decided, the possible variations are between a sampling period T=0.01 sec. and another T=0.08 sec. The algebraic controllers obtained for this sampling periods ensuring the matching of the specifications are:

$$G_R(z) = \frac{(3.12z^2 - 3.903z + 1.349)}{(z+0.5119)(z-1)}$$

for T=0.08 sec.

$$G_R(z) = \frac{(105.6z^2 - 202.7z + 97.7)}{(z+0.498)(z-1)}$$

for T=0.01 sec.

As a reference, a step of amplitude 10 is applied. The response of the controlled system, using each controller, is shown in fig. 4.

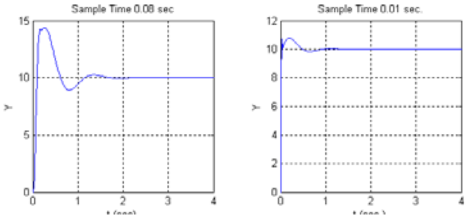

Fig. 4: Step responses (sampling period 0.08, and 0.01 sec.)

Now, let us assume that the process is being controlled with T=0.01 sec., and the supervisor decides to change the sampling period to 0.08 sec. in order to preserve the scheduling of the CPU. Let us assume that the change is decided at t=0.03 sec. after a step in the reference, when the system is still in its transient.

In the case that no estimation is used, the assumption is that the parameters of the controller are changed at the switching point but as past samples, the controller will use the ones obtained with the previous sampling period. The response of the system is shown in fig. 5 (Y_1).

If the proposed algorithm is used, the multiple period case solution applies. So, the past outputs are known in order to evaluate the control action at the switching point but the past control actions will be estimated using the Simplex algorithm. The comparative response appears in the fig. 5, (Y_2). As can be realised, the improvement is evident.

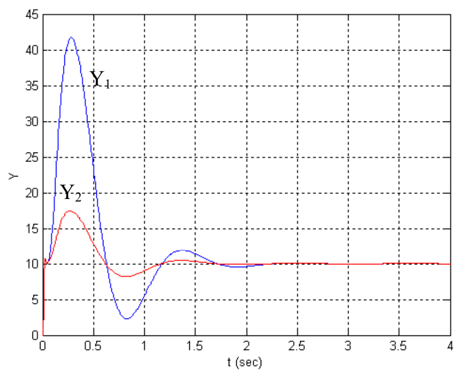

Fig.5: Comparative graphic of the performance when the proposed algorithm is used.

Now, the change is assumed in the other sense, going from a sampling period of 0.08 sec. to 0.01 sec. because the CPU is not too much busy and this allows a better use of the resources.

As previously, let us compare the responses with and without updating the stored information.

The change in the sampling period is carried out at t =0.2 sec., as shown in fig. 6. The controlled system response (Y_1) presents a big oscillation.

If the algorithm is applied, using in this case the interpolation of the outputs, the response is much better (Y_2).

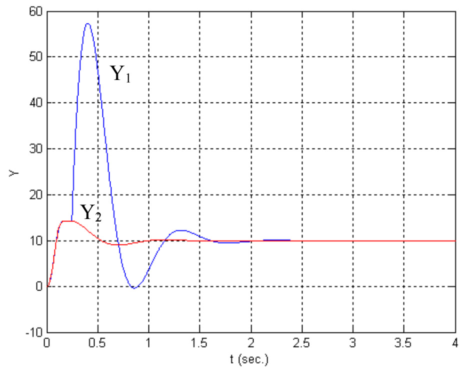

Fig.6: Reducing the sampling period.

CONCLUSIONS

In this paper, a control solution to reduce the performance degrading under changes in the sampling period in order to optimize the CPU usage has been presented. The main problem was the switching from one controller to another.

In hybrid control systems, the problem of determining the stability under a sequence of changes of controllers is a relevant one. But, even in the case that the system remains stable under whatever switching sequence, a loss of performance may appear at the switching time. This loss of performance is due to the fact that the past information available from the system at the switching time has not been obtained with the pattern of time that the next controller will use.

A solution based on an algorithm that uses the interpolation for the outputs and the estimation of the control actions based on a numerical optimization problem, has been presented.

The improvement of the performance in an open-loop unstable simulated plant has been shown.

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