

# ROBUST VARIABLE STRUCTURE MODEL FOLLOWING LOAD FREQUENCY CONTROLLER

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## Abstract

A systematic procedure for design of variable structure model following load frequency controller for a single-area power system has been presented. The variable structure model following control (VSMFC) technique presented in this paper not only guarantees invariance to a class of parameter variations and disturbances but also possess other attractive features of variable structure systems (VSS) like robustness to unknown disturbances, simplicity in design, and reduced order dynamics when in sliding mode. The simulation result of the variable structure model following control strategy to load frequency control is presented by changing all parameters by 30% to 50% from their nominal values. These results show that systems performance is robust to parameter variations and disturbances.

## 1 Introduction

Load Frequency control is one of the important problems in electric power system design and operation. There has been an increasing interest in load frequency control (LFC) problem for better performance during past 30 years. Many strategies for load frequency control have been proposed in past including the concept of variable structure system for both single-area and multi-area systems [1]-[4], [6]. When a load frequency controller is designed, one of the problems encountered is the parametric uncertainty in the power systems. Therefore, in the design of controllers the uncertainties have to be considered. The usual design approach for load frequency controller employs the linear control theory to develop control law on the basis of the linearized model with fixed system parameters. However as the system param-

eters cannot be completely known the controller designed based on fixed parameter model may not work properly for the actual plants. To take these parametric uncertainties into account several authors have applied variable structure systems to the design of load frequency controllers [4, 1, 12]. However their approach in selecting the parameters of switching vector is based on trial and error procedure and no systematic method is recommended for the proper choice of switching vector and specifying performance expected from the system. Several adaptive control techniques have also been proposed for dealing with parameter variations [10, 9]. In [15] a robust controller, based on Riccati-equation approach is presented but they could not find a robust controller when governor time constant and the speed regulation constant were changed by 50% from their nominal values. This limitation was overcome in [16] by combining robust control design technique, the Riccati equation approach and adaptive control design to design a new robust adaptive load frequency controller for power system. The system parametric uncertainties are obtained by changing parameters by 30% to 50% simultaneously from their typical values. As in [16], one has to find two control laws using robust control and adaptive control technique, the controller design becomes complex requiring much computations. The VSMFC technique presented here not only guarantees invariance to a class of parameter variations and disturbances [7] but also possess other attractive features of variable structure systems (VSS) [14]. Young [17, 19] made use of the theory of VSS in designing AMFC system. Young's method not only produces asymptotically stable systems, but is also capable of prescribing error transients. In [19] Young has designed an adaptive model following controller for a multivariable system where he employs the hierarchy of controls method to achieve sliding mode on the intersection of the hyperplanes. In [20] VSMFC techniques has been applied to a third order single input single output numerical example where the plant considered is in phase variable canon-

ical form. In this paper we apply this technique to design a load frequency controller for power system under parameter variations. This adaptive strategy is much simpler than approach in [16]. The controller possess attractive features like robustness to parameter variations, modeling errors and unknown disturbances, simplicity in design, reduced order dynamics when in sliding mode. The simulation results of the variable structure model following control strategy to load frequency control is presented by changing all parameters by 30% to 50% from their nominal values to show efficacy of this control scheme.

The paper is organized is as follows: Section 2 describes dynamic model for load frequency control. Section 3 gives a brief review of the concept of variable structure model following control. The modified variable model following control law for load frequency controller is proposed in Section 4. Simulation results and discussions are presented in Section 5 and finally paper is concluded in Section 6.

## 2 Dynamic model for load frequency control

In general the power system model are complex, nonlinear, dynamic systems. The usual practice is to linearize the model around the operating point and then develop the control laws. Since the system is exposed to small changes in loads during its normal operation, the linearized model will be sufficient to represent the power system dynamics. The block diagram of the linearized model is shown in Fig. 1. The state equations can be written as,

$$\Delta \dot{f} = \frac{-1}{T_p} \Delta f + \frac{K_p}{T_p} \Delta P_g - \frac{K_p}{T_p} \Delta P_d \quad (1)$$

$$\Delta \dot{P}_g = \frac{-1}{T_t} \Delta P_g + \frac{1}{T_t} \Delta X_g \quad (2)$$

$$\Delta \dot{X}_g = \frac{-1}{RT_g} \Delta f - \frac{1}{T_g} \Delta X_g - \frac{1}{T_g} \Delta P_c + \frac{1}{T_g} \int \Delta f dt \quad (3)$$

Integral control of  $\Delta f$  can be given as,

$$\Delta E = K \int \Delta f dt$$

$$\Delta \dot{E} = K \Delta f \quad (4)$$

where,  $X_1 = \Delta f$  Incremental frequency deviation in Hz  
 $X_2 = \Delta P_g$  Incr. change in generator output power in p.u. MW  
 $X_3 = \Delta X_g$  Incr. change in governor valve position in p.u. MW  
 $X_4 = \Delta E$  Incremental change in voltage angle in radians  
 $\Delta P_g$  Load disturbance in p.u. MW  
 $\Delta P_c$  Incr. change in speed changer position in p.u. MW  
 $T_g$  Governor time constant in seconds  
 $T_t$  Turbine time constant in seconds  
 $T_p$  Plant time constant in seconds  
 $K_p$  Plant gain  
 $R$  Speed regulation due to governor action in Hz p.u. MW<sup>-1</sup>

The dynamic model in state variable form can be written as:

$$\dot{X} = AX + Bu + F\Delta P_d$$

where,

$$A = \begin{bmatrix} \frac{-1}{T_p} & \frac{K_p}{T_p} & 0 & 0 \\ 0 & \frac{-1}{T_t} & \frac{1}{T_t} & 0 \\ \frac{-1}{RT_g} & 0 & \frac{-1}{T_g} & \frac{-1}{T_g} \\ K & 0 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \\ 0 \end{bmatrix}; F = \begin{bmatrix} \frac{-K_p}{T_p} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

## 3 Review of variable structure model following control

The theory of variable structure control can be applied for the designing of the adaptive model following control systems. This method is referred to as variable structure model following control (VSMFC) [20], which not only guarantees the asymptotic stability of the system but also succeeds in prescribing the transient response of the error. In VSMFC, the plant is controlled in such a way that its dynamic behavior approximates that of a specified model. The model is part of the system and it specifies the design objectives. The actual plant is required to follow the model. The adaptive controller should force the error between the model and the plant states to zero as time tends to infinity.

Consider the plant and model described by,

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x, u) \quad (6)$$

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \quad (7)$$

where  $x$  is plant state  $n$  vector,  $x_m$  is model state  $n$  vector,  $u$  is the plant control  $m$  vector,  $f$  represents nonlinear additive terms, and  $r$  is the model input.

The tracking problem is stated as follows:

Given a plant in equation (6), a reference model in equation (7) and reference input  $r$ . Find a control  $u(t)$  which guarantees that the state  $x(t)$  perfectly tracks the reference state  $x_m(t)$ . The tracking error vector is  $e(t) = x_m(t) - x(t)$ . We assume that the pair  $(A, B)$  and  $(A_m, B_m)$  are stabilizable. The model matrix  $A_m$  is assumed to be stable. The plant matrices  $A, B$ , and the vector  $f$  may be uncertain and time varying. The upper and lower bounds of the elements of these matrices and  $f$  are assumed to be known. It is seen that:

$$\dot{e} = A_m e + (A_m - A)x + B_m r - Bu - f \quad (8)$$

Perfect model matching occurs if, for zero initial conditions, the error vector  $e$  is null for any input belonging to the class of piecewise-continuous vector functions. The necessary conditions [8, 5] are:

$$\text{rank} B = \text{rank}[B, A - A_m] = \text{rank}[B, B_m] \quad (9)$$

The above perfect model following conditions are always satisfied if the reference model and plant have

similar Lunberger type controllable canonical structure. In VSMFC design, the switching surfaces are chosen as:

$$S = Ge = 0, \quad (10)$$

where  $S^T = [S_1, S_2, \dots, S_n]$ . The controller which ensures sliding mode on the intersection of the switching surfaces is of the form,

$$u = -K_i^T V \quad (11)$$

where

$$V^T = [e^T x^T r^T]. \quad (12)$$

The design objective is to find switching vector  $G$  and the vectors  $K_i^T$  which will ensure that sliding mode occurs on  $S = Ge = 0$ .  $G$  can be obtained by pole placement problem [19] and  $K_i^T$  can be obtained by applying existence condition for sliding mode [14], i.e.,

$$S\dot{S} \leq 0 \quad (13)$$

#### 4 Synthesis of sliding surface

Basically the switching vector  $G$  is obtained by solving a pole placement problem, i.e.,  $G$  has to be selected such that the error system (8) in sliding mode is asymptotically stable.

Consider the error system of (8). In sliding mode this reduces to [19]:

$$\dot{e} = [I - B(GB)^{-1}G]A_m e. \quad (14)$$

By applying the sliding mode condition  $S = Ge = 0$  in equation (14),  $m$  error state variables can be eliminated and the resulting error system can be transformed into [18]:

$$\dot{e}'_1 = (A_m^{11} - A_m^{12}G_2^{-1}G_1)e'_1, \quad e'_1 \in R^{n-m} \quad (15)$$

by the transformation,

$$\dot{e}' = We = \begin{bmatrix} \dot{e}'_1 \\ \dot{e}'_2 \end{bmatrix} \quad (16)$$

where

$$W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad W_1 B = 0, \quad \text{and}$$

$$WA_m W^{-1} = \begin{bmatrix} A_m^{11} & A_m^{12} \\ A_m^{21} & A_m^{22} \end{bmatrix} \quad (17)$$

$$GW^{-1} = [G_1 \quad G_2]. \quad (18)$$

Now  $G$  has to be chosen such that the eigenvalues of the equivalent error system (15) be placed in the desired locations. Using the above design strategy, we first obtain  $W$  such that:

$$WB = \begin{bmatrix} 0 \\ b_2 \end{bmatrix} \quad (19)$$

where  $b_2$  is a nonzero scalar. Then  $G$  is selected such that  $G_1$  and  $G_2$  as in equation (18) make the system (15) asymptotically stable with desired error transients.

#### 5 Variable structure model following load frequency controller design

The plant considered is in equation (5) with the parameter values as [1, 12],

$$T_p = 20 \text{ s}, \quad T_t = 0.3 \text{ s}, \quad T_g = 0.08 \text{ s}, \quad K_p = 120 \text{ Hz p.u. MW}^{-1}$$

$$K = 0.6 \text{ p.u. rad}^{-1}, \quad R = 2.4 \text{ Hz p.u. MW}^{-1}$$

The state space model is given by,

$$A = \begin{bmatrix} -0.05 & 6 & 0 & 0 \\ 0 & -3.333 & 3.333 & 0 \\ -5.208 & 0 & -12.5 & -12.5 \\ 0.6 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 12.5 \\ 0 \end{bmatrix}; \quad F = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

In order to satisfy the model following conditions we will convert above system into phase variable form by using transformation,

$$Z = TX$$

in phase variable form. Then the plant in equation (5) becomes,

$$\dot{Z} = TAT^{-1}Z + TBu + TF\Delta P_d \quad (21)$$

where,

$$TAT^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -149.985 & -106.2327 & -42.4545 & -15.833 \end{bmatrix}$$

$$TB = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad TF = \begin{bmatrix} 0 \\ -0.024 \\ 0.0012 \\ -0.0001 \end{bmatrix}.$$

The model selected was a critically damped model such that,

$$\dot{x}_m = A_m x_m + B_m r \quad (22)$$

where

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -24 & -50 & -35 & -10 \end{bmatrix}; \quad B_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 24 \end{bmatrix}.$$

The switching surface  $S = Ge$  is chosen as,

$$S = G_1 e_1 + G_2 e_2 + G_3 e_3 + G_4 e_4$$

$$\dot{S} = G_1 \dot{e}_1 + G_2 \dot{e}_2 + G_3 \dot{e}_3 + G_4 \dot{e}_4 \quad (23)$$

where,

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 + 0.0024\Delta P_d \\ \dot{e}_3 &= e_4 - 0.012\Delta P_d \\ \dot{e}_4 &= -24x_{m1} - 50x_{m2} - 35x_{m3} - 10x_{m4} + 24r + 149.985x_1 \\ &\quad + 106.2327x_2 + 42.4541x_3 + 15.833x_4 - u + 0.001\Delta P_d \end{aligned} \quad (24)$$

The Switching vector was designed by using pole assignment technique as explained in Section 4. The poles of the matrix  $[A_m^{11} - A_m^{12}G_{11}]$  were chosen as -4, -6, -8. The transformation matrix considered is a unit matrix. The switching vector was obtained as,

$$G^T = [ 192 \quad 104 \quad 18 \quad 1 ] \quad (25)$$

The error is defined as,

$$e_i = x_{mi} - x_i \quad ; \quad i = 1, 2, 3, 4 \quad (26)$$

A modified VSMFC law suited to this system is chosen as,

$$u = -K^T V - k_{10} \quad (27)$$

where,

$$K^T = [ k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6 \quad k_7 \quad k_8 \quad k_9 ]$$

and

$$V^T = [ e_1 \quad e_2 \quad e_3 \quad e_4 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad r ] .$$

The term  $k_{10}$  in control law (27) compensates for  $f(x, u)$  in (6).

Using (24), (26) and (27) equation (13) becomes,

$$\begin{aligned} S\dot{S} = & (k_2 + 142)Se_2 + (k_3 + 69)Se_3 \\ & + (k_1 - 24)Se_1 + (k_4 - 10)Se_4 \\ & + (k_5 + 125.985)Sx_1 + (k_6 + 56.2327)Sx_2 \\ & + (k_7 + 7.4541)Sx_3 + (k_8 + 5.833)Sx_4 \\ & + (k_9 + 24)Sr + (k_{10} + 0.0346\Delta P_d)S \leq 0 \quad (28) \end{aligned}$$

The right hand side of the above equation will have to be less than zero in order to satisfy existence condition for the sliding mode.

## 6 Simulation Results and Discussions

Letting each term in (28) separately less than zero we can solve it to obtain the controller gains. The controller gains were found as,

$$\begin{aligned} k_1 = & \begin{cases} 0 & \text{if } Se_1 > 0 \\ 25 & \text{if } Se_1 < 0 \end{cases}, \quad k_2 = \begin{cases} -142 & \text{if } Se_2 > 0 \\ 0 & \text{if } Se_2 < 0 \end{cases} \\ k_3 = & \begin{cases} -69 & \text{if } Se_3 > 0 \\ 0 & \text{if } Se_3 < 0 \end{cases}, \quad k_4 = \begin{cases} 0 & \text{if } Se_4 > 0 \\ 10 & \text{if } Se_4 < 0 \end{cases} \\ k_5 = & \begin{cases} -126 & \text{if } Sx_1 > 0 \\ 0 & \text{if } Sx_1 < 0 \end{cases}, \quad k_6 = \begin{cases} -57 & \text{if } Sx_2 > 0 \\ 0 & \text{if } Sx_2 < 0 \end{cases} \\ k_7 = & \begin{cases} -8 & \text{if } Sx_3 > 0 \\ 0 & \text{if } Sx_3 < 0 \end{cases}, \quad k_8 = \begin{cases} -6 & \text{if } Sx_4 > 0 \\ 0 & \text{if } Sx_4 < 0 \end{cases} \\ k_9 = & \begin{cases} -24 & \text{if } Sr > 0 \\ 0 & \text{if } Sr < 0 \end{cases}, \quad k_{10} = \begin{cases} -0.035\Delta P_d & \text{if } S > 0 \\ 0.035\Delta P_d & \text{if } S < 0 \end{cases} \end{aligned}$$

The initial conditions for the plant were chosen as,

$$x_1 = x_2 = x_3 = x_4 = 0.1.$$

The initial positions of the reference model were placed at

$$x_{m1} = x_{m2} = x_{m3} = x_{m4} = 0.8.$$

Simulation studies of the controller were made using the software package MATLAB. Fig. 2 and 3 shows the system responses and control torque with nominal parameters of the system when the system is subjected to a step load change of 0.03 p.u.. The figures reveals the ability of the controller to drive the system to follow the reference model. The robustness of the controller was tested by changing all the system parameters by 30 % to 50% from their nominal values. However the results presented here are only for 50%. Fig. 4, 5 shows the system responses and control torque for 50% parameter variation. It is observed that system remains invariant to the imposed parameter variation. In spite of 30% to 50% parameter variation from their nominal value, the controller has the ability to follow the reference model which specifies the performance expected from the system.

There is one major drawback associated with variable structure control, namely chattering. The control signal emerging from the control law (27) is seen to comprise high-frequency components which lead to control chattering. Chattering, in general, highly undesirable because it involves extremely high control activity and furthermore, it may excite high-frequency unmodelled dynamics [13]. However, this drawback can be overcome by maintaining sliding motion inside a small boundary layer neighbouring the switching line [13] or, as pointed out by Qian and Ma [11], by the insertion of a low-pass filter ahead of the plant to yield a smooth control signal. Figures 6 and 7 shows the system responses and control torque obtained with the insertion of low-pass filter for 50% parameter variations. The Figures shows the ability of the low-pass filter to reduce control chattering. The transfer function of the low-pass filter used is  $G(s) = \frac{10}{s+10}$ .

## 7 Conclusion

In this paper, a new design technique based on the concept of variable structure model following control is presented to design load frequency controller. It provides robustness to parameter variations and disturbances. Compared with the schemes of [15] and [16] this design method is simple and straightforward. In addition to this, the variable structure model following control (VSMFC) techniques presented in this paper not only guarantees invariance to a class of parameter variations and disturbances but also possess other attractive features of variable structure systems (VSS) like robustness to unknown disturbances, simplicity in design, and reduced order dynamics when in sliding mode. These advantages are not found in [16], as one has to find two control laws using robust control and adaptive control technique and hence the controller design becomes complex requiring much computations. The simulation results of the variable structure model following control strategy to load fre-

quency control is presented by changing all parameters by 30% to 50% from their nominal values to show efficacy of this control. These results show that systems performance is robust to parameter variations and disturbances.

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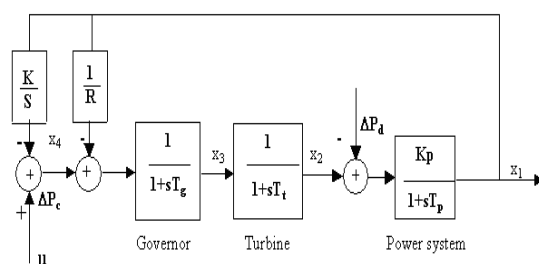


Figure 1: Block diagram of single area control

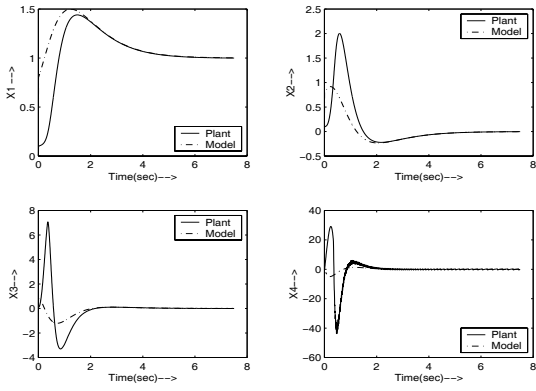


Figure 2: System responses for nominal parameters

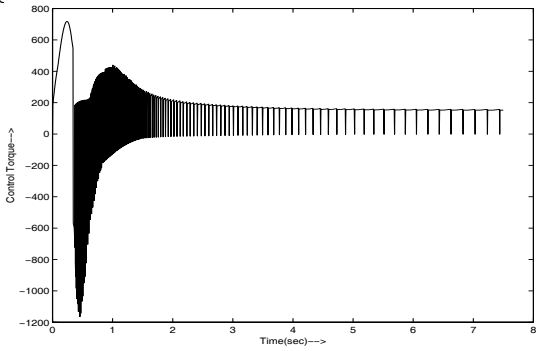


Figure 3: Control torque for nominal parameters

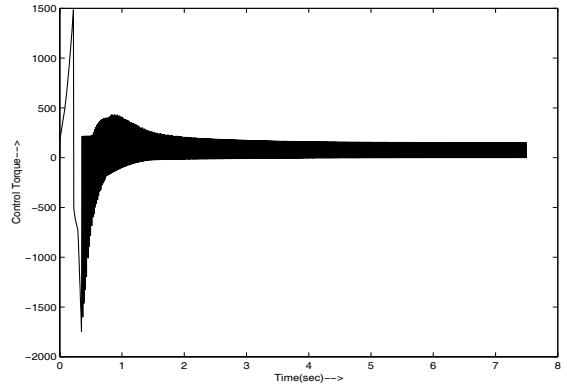


Figure 5: Control torque for 50% parameter variations

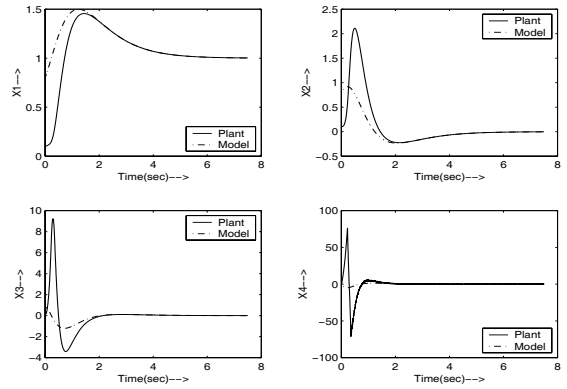


Figure 6: System responses for 50% parameter variations with insertion of low pass filter

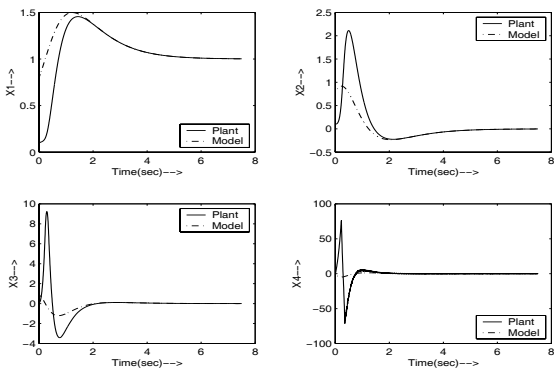


Figure 4: System responses for 50% parameter variations

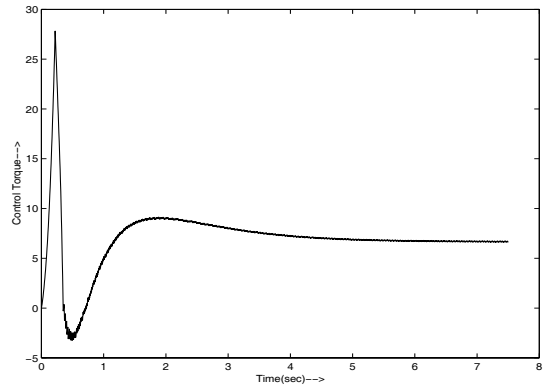


Figure 7: Control torque for 50% parameter variations with insertion of low pass filter