Clearance of a Small Scale Remotely Piloted Aircraft by Means of a Polynomial Based Analysis Method

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Abstract

In this paper an overview on polynomial based analysis methods for application to robust stability of linear systems subject to uncertain parameters is presented. A comparison among the most important Kharitonov type approaches proposed in literature and their applicability to the flight control law clearance problem of highly augmented aircraft is also discussed. A novel algorithm is then proposed, which can deal with high order uncertain dynamic aircraft models within reasonable computation time by introducing some degree of approximation in determining the clearance region's shape. Application to robustness analysis of an augmented small scale unmanned aircraft is finally presented, whose uncertain high order open loop dynamic model has been tuned with in-flight experimental data.

1 Introduction

The proposed clearance analysis technique is mainly based on some theoretical results that allow verification of whether the eigenvalues of an uncertain (linear) dynamic system belong to a predetermined region *D* of the complex plane (*Robust D-stability problem*). This allows a direct application of the proposed method for clearance of the unstable eigenvalue criterion [11]. The proposed method might also be used for any linear clearance criteria that can, in some way, be mapped in a test on system eigenvalue locations in the complex plane.

In the past, a large effort has been spent to deal with the robust stability problem of linear systems subject to uncertain parameters. A strong impulse to the research has been given by the paper of Kharitonov [12], where a necessary and sufficient condition for robust stability of a family of polynomials with uncertain coefficients has been provided. Although an elegant mathematical result, Kharitonov's theorem is not suited to engineering applications since it assumes uncorrelated polynomial coefficients. Indeed, in practical situations, the coefficients of the characteristic polynomial of a given system depend on the same physical parameters which implies that the coefficients themselves are related to each other. Kharitonov's result has been introduced in the western literature by Barmish [5]. Since then many papers have been published on this topic, trying to extend the original result to cope with more general parameter dependencies and/or to take into account also performance as well as stability. We recall the work by Petersen [14], which extends Kharitonov's theorem to deal with the so-called robust *D*-stability problem (see definition 1), with *D* a given domain in the complex plane (see for example Figure 1); the fundamental result by Bartlett et al [6], which states that to check stability of an uncertain polynomial with coefficients ranging into a given polytope it is necessary and sufficient to check the edges of the polytope; the works by Sariderely and Kern [16], Tesi and Vicino [17], Cavallo et al [7], all dealing with robust stability analysis of uncertain polynomials with coefficients depending affinely on parameters ranging in a given box. These results together with further insights on the topic can be found in [11]. Unfortunately the above results are not useful when: a) the characteristic polynomial depends on parameters in a nonlinear way (this is the case of many flight control applications as shown in [8]), and/or b) we are interested in the more general problem of determining the region shape in the parameter space where the system is robustly *D*-Stable.

In this context an algorithm will be described which allows to deal with the two issues mentioned above. This algorithm is based on the results provided in [7] and on a method for adaptive grid generation. More precisely, in [7] a necessary and sufficient condition for the D-stability of an uncertain polynomial depending affinely on parameters is given, while here a procedure to approximate a nonlinear vector function with a minimal set of affine ones is proposed.

Roughly speaking, this algorithm uses these results to:

1) determine a set of boxes whose union includes the initial uncertain parameter region and such that, in each box, the uncertain polynomial coefficients can be considered to be affinely dependant on parameters,
2) compute the actual D-stability region in the parameter space by applying the algorithm proposed in [4,9] to each box of the above step.

In this way the D-stability region is approximated up to the desired resolution by the union of the final resulting boxes which satisfy the condition given in [7].

The key point to guarantee that the stability region found via this way converges to the true stability region, is that the errors due to the use of a set of affine functions instead of the actual non linear vector function (which gives the characteristic polynomial coefficients of the uncertain system) can be neglected if the boxes are sufficiently small. This is always true when the mapping of the parameter space into the polynomial coefficient space is continuous.

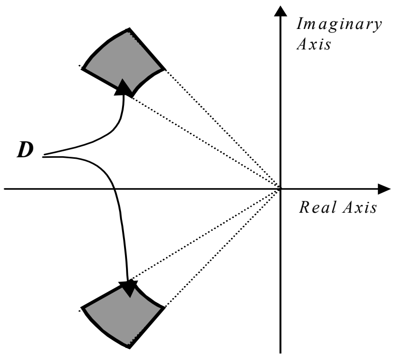

Fig. 1 - Typical D-Stability domain in the complex plane

2 Theoretical Background

Let us consider an uncertain linear system described by the differential equations:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{\pi})\boldsymbol{x}(t) \qquad (1)$$

where $\boldsymbol{x}(t) \in R^n$ and $\boldsymbol{A}: R^k \to R^{n \times n}, \boldsymbol{\pi} \to \boldsymbol{A}(\boldsymbol{\pi})$, is a continuous matrix-valued function of the parameter vector π, R^n is the model state space (of dimension n) and R^k is the uncertain parameter space (of dimension k). In this context, we need the following definition.

Robust *D*-Stability:

Given the compact set $\Gamma \subset R^k$ (i.e. a region in the parameter space) and the open domain D in the complex plane, system (5.1) is said to be robustly D-stable in Γ if $\lambda_i(\boldsymbol{A}(\boldsymbol{\pi})) \in D, i = 1,..,n$, for all $\boldsymbol{\pi} \in \Gamma$, where $\lambda_i(\boldsymbol{A})$ denotes the i-th eigenvalue of the matrix $\boldsymbol{A}$.

In other words, system (1) is defined to be robustly *D*-stable if its poles all belong to a given region *D* of the complex plane (see Figure 1 for an example), for each point π in the uncertain parameter region Γ. Note that, when D coincides with the left half of complex plane, we simply talk to about robust stability.

Now let us refer to the system described by Eq.(1) and let $\boldsymbol{a}(.): R^k \to R^n, \boldsymbol{\pi} \to \boldsymbol{a}(\boldsymbol{\pi})$, the vector-valued function containing the coefficients of the characteristic polynomial of the matrix $\boldsymbol{A}(\boldsymbol{\pi})$. We denote by:

$$L: R^n \to P^n, \boldsymbol{a} = (a_1,...,a_n)^T \to p(s,\boldsymbol{a})$$

where

$$p(s,\mathbf{a}) = s^n + a_1 s^{n-1} + \cdots + a_n,$$

the linear operator mapping a vector of R^n into P^n, the set of monic polynomials of degree n. Finally, define the compound operator $L_a := L \circ \boldsymbol{a}$. From a robust *D*-stability point of view, the complete behaviour of system (5.1) is described by the following family of monic polynomials:

$$L_a(\Gamma) = \{p(\cdot, \boldsymbol{a}(\boldsymbol{\pi})) \mid \boldsymbol{\pi} \in \Gamma\} \qquad (2)$$

Indeed it is clear that the system described by eq. (1) is robustly D-stable within the given set Γ if and only if the roots of all polynomials belonging to the family $L_a(\Gamma)$ are in the domain *D*. Now consider the following problem.

Problem 1:

Determination of the Robust *D*-Stability Region in the parameter space R^k. Determine the region $\Pi^*_D \subset R^k$ such that system (1) is robustly *D*-stable in Π^*_D. As we shall see, the idea behind the polynomial coefficient based approach proposed here is that of approximating the *D*-Stability Region Π^*_D by the union of boxes in the space R^k. To check robustness in the given box, it is necessary to have a procedure to solve the following basic problem.

Problem 2: *Basic Problem*

Given a box $\Pi^*_D \subset V$, determine if system (1) is robustly *D*-stable in *V*. With the methods currently available in the literature, the above stated Basic Problem can be solved without conservatism when the dependence of the characteristic polynomial on parameters is affine (see [6,7,16,17]).

The nonlinear dependence has been considered in [15] and [10] (multiaffine dependence), while in [13] a multivariate dependence has been assumed. In these last papers the stability analysis is performed by introducing fictitious parameters which allow the multivariate dependence to be transformed into a multiaffine one. Then the test is performed (at the price of some conservatism) on the fictitious multiaffine characteristic polynomial by using one of the approaches proposed in the literature. Another algorithm dealing with nonlinear dependency on parameters, implements the idea proposed in [3]. In this approach, known as the Polytopic Covering approach, the image of the given nonlinear function is "immersed" into that of an affine function. In [2] it is shown that the polytopic covering approach leads to less conservative results than those obtainable with other methods.

The main drawback of these "polytopic set covering" based methods is that the dimension of the parameter space in which the *D*-stability analysis algorithm needs to be applied, can dramatically increase.

In [4] it has been shown that good results can be achieved when the augmented parameter space dimension is at least the same as the polynomial order. For the aeronautical application under investigation and, specifically, in flight control law clearance problems, the order of the closed loop polynomial is too high to allow these methods to work well and to obtain results in reasonable time.

3 The Proposed Algorithm

Let us come back to the solution of Problem 1; here we consider a slight variation of the problem, taking into account the fact that in flight control problems the range of parameter variations or parameter uncertainties can be estimated. Thus, let us consider that:

$$\underline{\pi_i} \le \pi_i \le \overline{\pi_i} \qquad \forall\ i=1..K \tag{3}$$

where the underline is used to indicate the minimum value of a parameter while the overlain stands for the maximum value.

Hence we have that $\pi \in \Pi$ (i.e. a box in the parameter space), where

$$\Pi = \left[\underline{\pi_1}, \overline{\pi_1}\right] \times \left[\underline{\pi_2}, \overline{\pi_2}\right] \times \cdots \times \left[\underline{\pi_k}, \overline{\pi_k}\right] \tag{4}$$

Therefore, our goal is to determine the set $\Pi_D := \Pi^*_D \cap \Pi$, where Π^*_D is the robust D-stability region defined in Problem1.

The nonlinear mapping $\boldsymbol{a}(\Pi)$ can be approximated by a set of affine mappings, each of them suitably defined on a partition of Π. In other words, let us consider instead of $\boldsymbol{a}(\Pi)$, the following mapping:

$$\bigcup_{i=1}^{N} \mathbf{a}_i^*(\Pi_i), \quad \bigcup_{i=1}^{N} \Pi_i = \Pi \wedge \ \ \Pi_i \cap \Pi_j = \varnothing, \forall i \ne j \in 1..N \tag{5}$$

where $\boldsymbol{a}^*_i(\Pi)$ is an affine approximation of $\boldsymbol{a}(\Pi_i)$, calculated by linear regression methods, and N is the number of subsets in which the initial box Π has been divided. It is expected that the D-stability region $\hat{\Pi}_D$ corresponding to the polynomial coefficient mapping defined in (2) will approach the true stability region Π_D provided that the linear regression approximation error tends to zero as the volumes of boxes tend to zero. In this respect, the following procedure gives an approximate solution to Problem 1. It computes the boundary of the stability region $\partial\Pi_D$ up to a desired resolution (actually an estimation $\partial\hat{\Pi}_D$ will be evaluated instead of $\partial\Pi_D$). Any dependence of the system matrix on uncertain parameters can be covered.

As said, the procedure is made up of two main steps:

a. Compute an optimal partition $\{\Pi_i\}$ of Π (trying to minimise N, the number of subsets Π_i) where the generic nonlinear map $\boldsymbol{a}(.)$ can be approximated by an affine map $\boldsymbol{a}^*_i(.)$ in each Π_i with a maximum estimated error of d_{eps}. The algorithm also stops when subsets Π_i become smaller than a pre-defined grid size eps_1.

b. Compute (up to a desired resolution eps_2) the D-stable region in the uncertain parameter space for each partition Π_i by using the approximated affine vector function $\boldsymbol{a}^*_i(.)$.

Specifically, we can schematically describe the first procedure as follows:

Procedure 1 – Adaptive Grid Generation

Put the box Π in the *List*

For each box of the *List*

  Evaluate coefficients in the box vertices and in the centre;
  Compute an affine function approximation in the box (linear regression fitting);
  Compute error d_{err} (from linear regression algorithm);
  If $d_{err} < d_{eps}$ **or** $\|box\| < eps_1$ **then** add box to the final list;
   Else divide box in sub-boxes and update *List;*
  End

End

End of Procedure 1 – Adaptive Grid Generation

In this procedure and in the second one, given a generic box V, the operation $\|V\|$ is defined as follows:

$$\|V\| = \max_{i=1,\ldots,2^k} l_i \tag{6}$$

where l_i is the i-th side of the box V. In other words the size of the box is given by the length of the longest side of the box. A more sophisticated algorithm for adaptive grid generation (i.e. a grid where the number of partitions is not a priori defined) could be investigated and implemented to increase the reliability of the error fitting, but this work is beyond the scope of this chapter.

The above procedure can treat points where the nonlinear vector function $\boldsymbol{a}(.)$ is not defined during the Evaluate statement. It should also be noted that by using mathematical manipulations, only dot products between matrixes and vectors (no matrix pseudo-inversion) are required, thus leading to a very fast algorithm. The only time consuming task is actually the evaluation of the nonlinear vector function in 2^k+1 points for each examined box. More precisely, because 2^k smaller boxes are generated each time we divide a box and the algorithm used for the Evaluate statement does not allow multiple evaluations of the same point in the uncertainty space, the maximum total number of polynomial coefficient evaluations after j steps (i.e. the number of evaluations required in case no boxes are below the maximum error d_{eps}) is: $(2^{(j-1)}+1)^k+2^{(j-1)k}$. In other words, the maximum number of trimming and linearisations is equivalent to the number of evaluations performed with a grid of $2^{(j-1)}+1$ points for each uncertainty, plus all centre points of boxes generated at step j. Thus, the effectiveness of the proposed technique can be also assessed by comparing it with a grid of the same size. Furthermore, by putting j=1 in the above relation, the minimum number of characteristic polynomial coefficient evaluations is obtained, which is actually equivalent to only evaluate the polynomial coefficients in the vertices of Π (i.e. min/max combinations of the uncertainties), plus its centre point.

The output of this first procedure is a list of boxes $\{\Pi_i\}$ where the initial nonlinear vector function can be considered affinely dependent on the uncertain parameters.

The main steps of the second procedure are schematically listed below (see [4,9] for details).

Procedure 2 – Computation of D-Stable Region

Put the box set $\{\Pi_i\}$ in the *List*

For each *box* of the *List*

If *cond(box)* **then**

Compute eigenvalues of the system in the centre point of *box;*

If all the eigenvalues belong to domain *D* **then** *box* is *D*-stable;

Else *box* is *D*-unstable

Elseif $\|box\|$ <eps_2 then box is not *D*-stable

Else divide box in sub-boxes and update *List*

End

End

End of Procedure 2 - Computation of D-Stable Region

Given a generic box *V*, the logical operation *cond(V)* gives a necessary and sufficient condition that guarantees the box *V* is entirely included in the *D*-stable or *D*-unstable regions of parameter space. For the sake of brevity, we do not detail the procedure here, but only highlight that it is based on the simple knowledge of the polynomials coefficients in the vertices of the considered parameter space box (see [7] for details). Because in this procedure we use the affine vector functions $\{\boldsymbol{a}_i^*(.)\}$ computed in procedure 1, evaluation of such vertex polynomial coefficients can be implemented with simple matrix and vector dot products, thus leading to a very fast execution time. Finally, it should be noted that eigenvalues of the system are only computed in the centre point of each box when *cond(V)* is true, so dramatically reducing the number of eigenvalue evaluations.

4 System Description and Numerical Set-up

The methods discussed so far have been used for checking stability of the control augmentation laws developed for a remotely piloted small scale aircraft. The latter is a commercial, one-third replica of a Piper PA-18 Super Cub, whose main features are given in Table 1.

Aircraft's main characteristics	
Wingspan	3.85 m
Length	2.53 m
Wing area	2.21 m^2
Take-off weight	30 kg
Engine	Two-stroke, 10 kW
Cruise speed	25 m/s

Table 1

The aircraft is equipped with a full set of navigation sensors, including an AHRS and a GPS system operating in differential mode. The stability and control laws are implemented onto the on-board, real-time flight control computer. The radio control commands can be switched during the flight between normal 'RC modeler' direct link mode and the augmented one.

A 6DOF, rigid-body, non-linear simulation model of the aircraft, sensors and control laws has been developed under Matlab/Simulink to be used during the control laws design process. The aerodynamic database fitted into the model has been built either by numerical (CFD) and standard literature methods (ie. DATCOM), and it has been subsequently refined by in-flight testing. Both numerical and experimental data have been gathered to model the engine/propeller performances, and mass and inertia properties.

The analysis tool implementing the two procedures described above has been developed under *Matlab/Simulink*. It allows a user to easily specify algorithm parameters. The main steps of the analysis cycle based on the use of this tool are: trimming and linearisation routine of the aircraft model, stability domain and uncertainty range specification, algorithm parameter setting and procedure running and result visualization. These steps have been performed to check system stability under variation of several couple of uncertain parameters. The dimension of the model state space for the linearized closed-loop system is 34. The minimum grid size (*eps1*) and the maximum estimated error (d_{eps}) set in the adaptive grid generation procedure (AGP) are 0.0313 and 0.04 while the region shape resolution (*eps2*) in the D-stability region shape computation procedure (RSC) is 0.0156. Because all these parameters refer to a unity uncertainty interval, to obtain the right values in the treated cases it is necessary to multiply *eps1* and *eps2* per the range of the uncertain parameters.

5 Results

The analyses performed has been oriented mainly to clear the closed-loop controller robustness with respect to the uncertainties of the inertial and aerodynamic databases used for the simulation model. In the following, selected results of several analysis performed are presented. The left-half region of the imaginary plane has been selected as the D-stability domain. Since the on-board mass distribution can vary with changes of the installed equipments, some of the 'inertial' analysis' results can also be interpreted as 'allowed loading limits'. The aircraft is always trimmed in straight-and-level flight. Among the inertial parameters, the centre of gravity position and the moments of inertia have been considered. The longitudinal stability of an aircraft is strongly influenced by the longitudinal position of the aircraft's centre of gravity, X_{CG}. Figure 2 shows the cleared area for the closed-loop airplane, where *Airspeed* and X_{CG} are considered as uncertain parameters, ranging respectively in [10, 40] m/s and [26, 56] percent of the mean aerodynamic chord. Since the controller's design point is at an airspeed of 25 m/s, and no speed scheduling has been implemented here, the cleared region is reduced at higher flight speed. The small not-cleared area on the left side is where the aircraft stalls.

Results from an open-loop analysis with the same two parameters as above are shown in Figure 3, in order to evaluate the overall effect of the controller's action, especially on the allowable c.g. range.

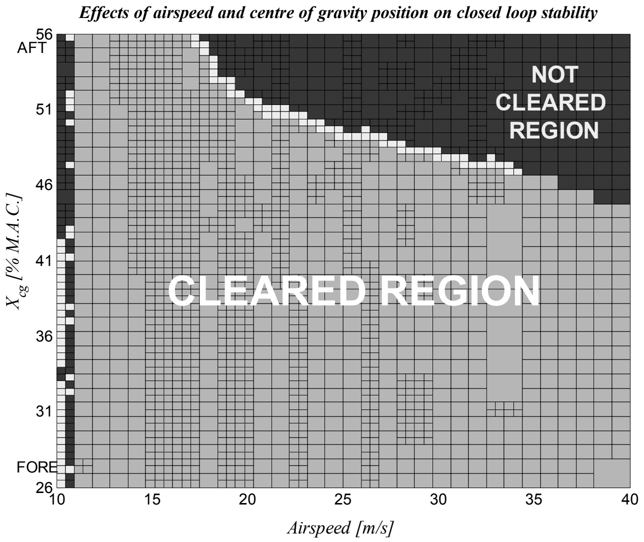

Fig. 2

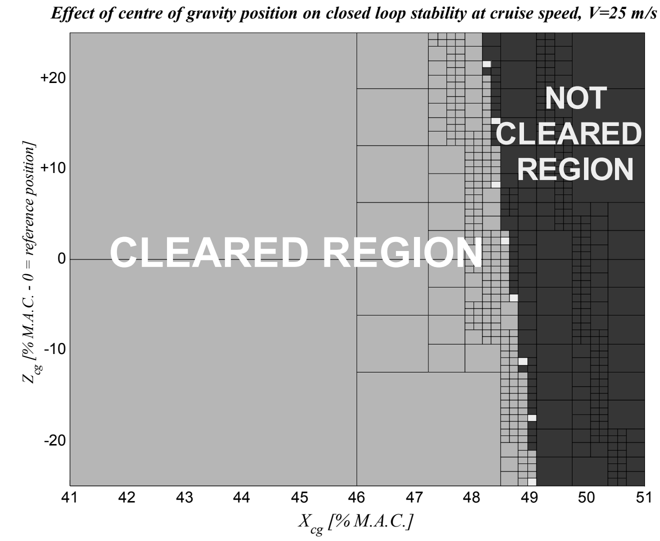

Fig. 4

Two not cleared regions are present, which an additional analysis has shown to be caused by an unstable lateral-directional spiral mode at low speed (trapezoidal dark area on the left side) and by the longitudinal instability which occurs when the c.g. moves near and aft of the aircraft's neutral point (upper dark area). The bound between the cleared region and the upper dark region marks the neutral point position.

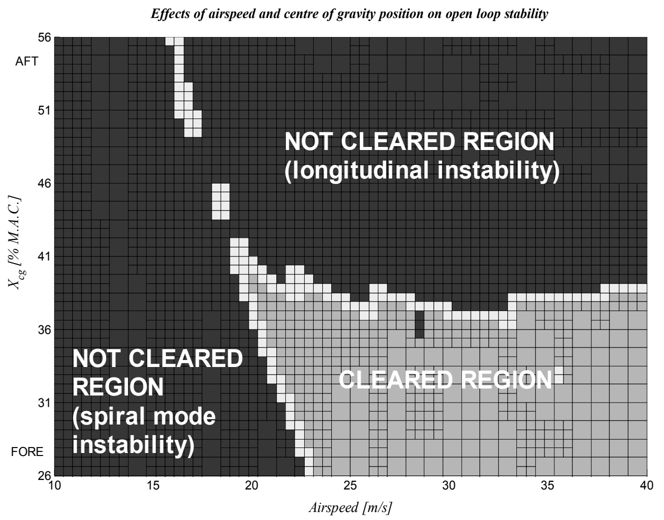

Fig. 3

The effect on the closed-loop stability of the c.g. position in the aircraft plane of simmetry at cruise airspeed is shown in Figure 4. Once again, the stability is strongly dependent by the longitudinal position, while the vertical position has only a negligible effect. The closed-loop controller's robustness has been checked against the aerodynamic uncertainties which can affect the simulation model, especially if aerodynamic data are built from computational fluid dynamics codes.

A selected result from this analysis is shown in Figure 5, where variations in the ±50% range have been considered for the wing-body pitching moment vs. angle of attack curve slope, and for the slope of the curve which gives the elevator contribution to the horizontal tail lift coefficient (the controller acts on the elevator to stabilize the aircraft). Since the former contribution is destabilizing, the "worst case" is given in the right lower corner of the parameters' plane, for percentual uncertainties of +12% and –15% respectively.

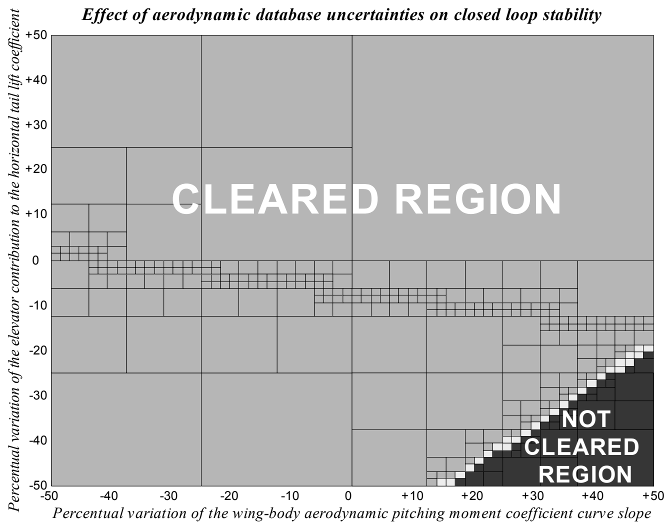

Fig. 5

Other than the type and quality of results, computational effort is another important aspect for evaluation of this method compared to conventional gridding methods. The most time consuming task is trimming and linearization of the aircraft model in a given point of the uncertain parameter space (this took typically more than 80% of total computational effort). For example, the completed analysis in the case of Figure 5 has required 483 evaluated points while a conventional gridding method would obtain results of the same accuracy by evaluating 4225 points. These figures clearly demonstrate the capability of the proposed clearance methods to reduce computational effort up to 10 times less than conventional gridding ones, provided that the same resolution is used in determining region boundaries. This is obviously obtained at the expense of a much more complicated algorithm and of some degree of inaccuracy.

6 Conclusions

In this paper we illustrated the clearance results for the eigenvalue criterion obtained with the polynomial based approach proposed in [5]. This method basically determines the region(s) in the uncertainty space where all eigenvalues of a (linear) uncertain dynamic system belong to a pre-defined domain D in the complex plane. This technique can be used for analysing all clearance criteria which can be mapped in the locations of a system's eigenvalue. Also, any kind of dependence between the characteristic polynomial coefficient and the uncertain parameters can be taken into account.

A key advantage with respect to classical methods, where analysis is conducted on simple discrete points, is that this technique allows the shape of cleared, not-cleared and trimmable regions in the uncertain parameter space to be determined. Because no assumption has been undertaken when choosing the kind of uncertain parameter to be investigated, this technique also allows the parameter space to be investigated continuously to determine the cleared regions in order to discover hidden weaknesses in this space and/or to gather information about further analysis to be conducted.

Some further developments could also improve the results and applicability of this technique. For example, the adaptive grid generation algorithm can be improved to identify the parameters that are mostly not linear (this will give the possibility to reduce complexity to $\sim 2^l$ where l is less than the number of uncertain parameters). Also, some techniques can be investigated for extending applicability to other linear clearance criteria (like, for example, stability margins) which do not map directly in a condition on the location of the eigenvalues. For example, it is possible to map the stability margin criterion to a problem of robust stability which can be analysed with the proposed method. Extending the applicability of this method to other criteria gives, in principle, the opportunity to perform the analysis by using the same trim and linearization points evaluated by the adaptive grid generation algorithm (i.e. the adaptive grid is generated only once), thus leading to a computational effort only slightly higher than required for analysis of only one criterion.

Finally, the results can be very easily interpreted: one can directly look at the regions in the uncertain parameter space where a clearance criteria is fulfilled. However, this image can be very difficult to plot and interpret when the number of uncertain parameters under consideration is more than three.

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