PREDICTIVE STORAGE CONTROL FOR A CLASS OF POWER CONVERSION SYSTEMS

Bram de Jager

Department of Mechanical Engineering, WH 0.132
Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
A.G.de.Jager@wfw.wtb.tue.nl, Fax: +31 40 2461418, Phone: +31 40 2472784

Keywords: Predictive control, storage control, power conversion system, power-based model, QP problem, piecewise linear

Abstract

This paper discusses the synthesis of a predictive controller for storage problems in power conversion systems. The control algorithm is based on solving an optimization problem, to optimally schedule the power stored in a storage device so the total efficiency of the system, in which the storage device is embedded, is improved. The model employed in the controller is power-based, including losses for the main components of the system. The losses are modeled by quadratic, linear, and piecewise linear relations.

In general the systems for which this approach is applicable will consist of a primary power converter that converts primary power (chemical) to secondary power (mechanical), *e.g.*, for propulsion, but also has a power take-off for a secondary power converter that converts secondary power to tertiary power (electrical) to fulfill the needs of tertiary power users. When using a device to store tertiary power, the actuation of this device can be scheduled to minimize the consumption of primary power. To compute this schedule we formulate and solve a standard QP problem. Piecewise linear relations are handled by embedding in a larger design space.

We show that this approach can be effective, because the efficiencies of the converters depend on their workloads. Taking advantage of sweet spots in the efficiency characteristics may improve the total efficiency, depending on the characteristics of the storage device. The storage device achieves these savings by decoupling the consumption of and conversion to tertiary power. It appears that a reduction of the primary power that is used to generate tertiary power is achievable in the application presented. This reduction is determined by the efficiency characteristics of converters and storage device, and by the workloads foreseen. The horizon of the predictive controller has to be large enough to detect possible sweet spots, and therefore will depend largely on the characteristic of the signals that determine the workloads. It is possible to pose the problem so the required horizon is very small.

1 Introduction

We describe a power-based model approach to optimize the schedule of stored (and thus generated) power, so as to get minimal primary power consumption in systems equipped with a primary power converter, *e.g.*, a fossil fuel converter, converting from primary to secondary power, and a secondary power converter, *e.g.*, an electrical generator, converting from secondary to tertiary power due to a power take-off at the primary power converter. These systems are typically of a hybrid nature.

To be able to generate tertiary power mainly at times that do not cause much increase in primary power consumption, while still being able to meet the required tertiary power loads, the system needs to be equipped with storage devices, *e.g.*, a battery. Systems where this type of device configurations occur are, for instance, ships and aircraft. Here, the primary power converter is a gasturbine and the secondary power converter is an electrical generator mechanically coupled to the turbine. The power generated by the primary power converter is mainly used for propulsion. The remaining power is used to drive the secondary power converter, that supplies power to electrical devices. Other power converters, like DC-DC converters, could also be included. An objective in these systems is to get low consumption of primary power (fossil fuel) while still meeting propulsion requirements and the needs of tertiary power users. Our goal is thus to schedule the tertiary power storage system, to minimize the primary power consumption.

The areas in the working space where savings in primary power can be achieved, the sweet spots of the converters, are those where the primary power consumed does not increase that much when more secondary power is converted. In Fig. 1 a typical relation between power consumed, P_c, and power produced, P_m, in the primary converter is given. A comparable relation holds for the secondary converter. The area where the

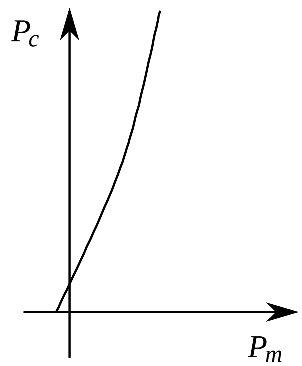

Figure 1: Relation between power consumed, P_c, and power produced, P_m, by a power converter

slope of this characteristic is smallest, so for low or negative P_m in Fig. 1, gives the best conditions to convert to secondary power, because an increase in P_m only causes a relatively small increase in P_c, so it is an area with low incremental cost. Therefore, it may sometimes be worthwhile to postpone conversion to tertiary power to a later and more profitable time interval, and satisfy the requirements for tertiary power by taking power from a storage device, or vice versa.

This type of control problem can be solved by scheduling [1], but to do this the workloads need to be known in advance, at least when using static schedulers. We target the case where the conditions under which the converters are working vary quite a lot in a short time, mainly due to hard to foresee external influences. The scheduling problem needs therefore to be solved on-line, placing restrictions on the type of techniques that can be employed, and on the intricacy of the models used, to cope with real-time issues. This type of problems has also been studied in the area of computer networks with variable service requests, see [2] for approaches using control theory and dynamic scheduling.

In this paper we will use a predictive type of control algorithm to produce the schedule. The controller computes the complete schedule within the control horizon using predictions of the workloads and a model, but only the first computed control action is implemented at each sampling instant. After this instant the control actions over the complete horizon, shifted 1 time instant, are recomputed using new information that has become available: this is the receding horizon principle. See [3] for an introduction to this principle and for predictive control in general.

We solve this problem within a QP (Quadratic Programming) formalism, for efficiency and because we have to take account of all kinds of constraints and losses, *e.g.*, quadratic and piecewise linear ones. The embedding of piecewise linear storage losses in a QP setup seems to be a novel feature for this type of applications, which should be useful for other applications as well, even in a more general setting.

The other key element in our approach is a power-based model, that does contain hardly any dynamics, and is therefore efficient to implement. Devices that can be power controlled are discussed, *e.g.*, in [4, 5].

In the following sections we present the power-based model, the control objective and constraints, and give a worked example of the approach using synthetic but realistic data, while we will start with some assumptions and restrictions of the proposed approach.

2 Assumptions and restrictions

It is assumed that the workloads, characterized by the speed, ω, at which the converters are running, by the secondary power consumed directly for propulsion, P_p, and by the tertiary power consumed directly by certain loads, P_l, is known a certain time interval in advance. This information is available for the complete horizon of a predictive controller, but may change completely at each time instant. To get this data a suitable prediction facility is assumed. Also, the controller is assumed to be geared to solve optimization problems, but only with an objective that is quadratic and constraints that are linear in the design variables. This restriction facilitates the on-line implementation, because fast solution techniques can be employed.

The model is formulated in terms of power, and it is assumed that at least the storage device can be power controlled. Whatever is needed to achieve this is not characterized by the model. Also variables internal to the devices that are physically restricted cannot be bounded directly by this approach. One has to supply suitable models for the devices to make that possible. More involved models, however, make the control strategy less insightful, complicate the design, and hamper the on-line implementation.

The simple power-based model uses only a single dynamic equation, namely for the storage device, but the simplicity of the model facilitates the on-line implementation. The time scale of interest, above about 1 [s], and the assumption that workload data is available, makes this simplification possible. To get reliable results we assume to have accurate static data for the efficiencies of the devices, which is a challenge to acquire in itself, but not the subject of this paper.

To be able to efficiently handle piecewise linear loss terms within a QP setup with linear constraints, we assume $P_p \geq 0$ and $P_l \geq 0$, so monotonicity of the objective function can be implied, as will become clear later. This assumption makes the approach inappropriate for some applications, *e.g.*, automotive propulsion systems, without modifications.

3 Power-based model

The relations between the powers for the basic system in Fig. 2 are as follows:

$$\begin{aligned} P_c &= \phi_c(\omega, P_m), \\ P_m &= P_p + P_g, \\ P_g &= \phi_g(\omega, P_e), \\ P_e &= P_l + P_b, \\ P_b &= \phi_b(E_s, P_s), \end{aligned}$$

with P_c the power consumed by the primary converter, P_m the power delivered by this converter, P_g the power consumed by the secondary converter, P_e the power delivered by this converter, P_b the power consumed by the storage device, P_s the power effectively stored, and E_s the energy in the storage device.

The static characteristics that define the efficiency of the three main components of the system, primary converter, secondary converter, and storage process, are:

- $\phi_c(\omega, P_m)$: inverse efficiency of the primary converter times outlet power, as function of speed ω and outlet power P_m, it also represents the losses when no net power is generated, *e.g.*, friction and aerodynamic losses in a fuel converter,
- $\phi_g(\omega, P_e)$: inverse efficiency of the secondary converter times outlet power, as function of speed ω and outlet power P_e, it includes the losses when no net power is generated, *e.g.*, friction losses,
- $\phi_b(E_s, P_s)$: inverse efficiency of the storage process times stored power, as function of stored energy E_s and stored power P_s, including leakage effects, *etc.*

The relation between energy stored, E_s, and power stored, P_s, is given by a simple integrator model,

$$E_s(t) = E_s(0) + \int_0^t P_s(\tau)\mathrm{d}\tau.$$

4 Objective

To be able to get an objective that is at most quadratic in the design variables – still to be determined –, we have to approximate the different characteristics of the devices. To start with, we approximate the primary converter with

$$P_c = \phi_c(\omega, P_m) \approx a_c(\omega)P_m^2 + b_c(\omega)P_m + c_c(\omega),$$

by neglecting cubic and higher order terms in P_m. Figure 1 indicates that this is a reasonable approximation. If this relation is not accurate enough and a tighter fit is needed we can use a more “localized” relation, *i.e.*, around the current or expected workload,

$$P_c = \phi_c(\omega, P_m) \approx a_c(\omega, P_p)P_m^2 + b_c(\omega, P_p)P_m + c_c(\omega, P_p).$$

This relation needs to provide a fit for the interval $[P_p, \min(P_m^{\max}(\omega), P_p + P_g^{\max}(\omega))]$. Here, P_p is used as an indicator of the workload for the primary converter because it determines the relevant interval.

For the secondary converter we can use the same type of function

$$P_g = \phi_g(\omega, P_e) \approx a_g(\omega)P_e^2 + b_g(\omega)P_e + c_g(\omega),$$

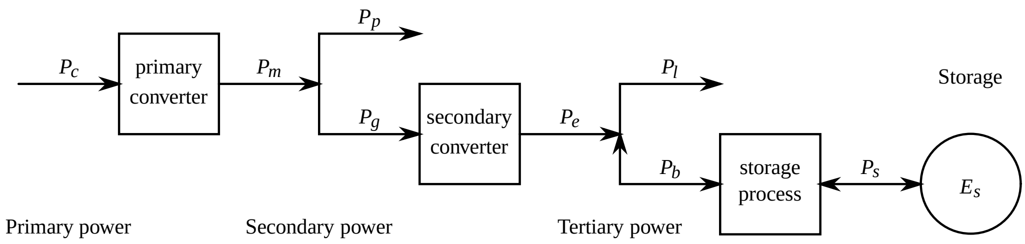

Figure 2: Outline of the basic power conversion system with storage

which is a common approximation for electrical systems with copper and iron losses.

For the storage process, in general, we cannot use this model structure, because the stored power P_s can be both positive and negative, which is no problem for the quadratic term in the approximation, but it is for the linear one, which should change slope depending on the sign of P_s, so it is piecewise linear. This can be solved with a max-function objective. Furthermore, the function ϕ_b depends on two variables that are not known in advance, so that are functions of the potential design variables.

This last problem can be solved in different ways. One is to use an approximation in both variables. Another one is to use predicted values for E_s, because this variable changes relatively slowly. For the storage device we can therefore use

$$P_b = \phi_b(E_s, P_s) \approx a_b(\hat{E}_s)P_s^2 + \max(b_b^-(\hat{E}_s)P_s, b_b^+(\hat{E}_s)P_s) + c_b(\hat{E}_s),$$

based on predicted values $\hat{E}_s$ for E_s, or

$$P_b = \phi_b(E_s, P_s) \approx a_b P_s^2 + \max(b_b^- P_s, b_b^+ P_s) + c_b E_s + d_b E_s P_s,$$

or another functional form for the approximation that better matches the device data. It holds that $0 < b_b^- < 1$ and $b_b^+ > 1$ for storage devices with losses. The main difference between these two relations is that the first one may involve complicated relations for $\hat{E}_s$, while the second one may not for E_s, because we need to end up with a quadratic relation in the end. The relation between E_s and P_s should be used in both cases, in the first case to predict the future E_s, *e.g.*, based on the previously computed optimal sequence of P_s, in the last to eliminate E_s and write the relation solely in terms of P_s.

The solution for the first problem we noted, a max-function in the objective instead of piecewise linear ones, poses a problem itself. This is because it cannot be included directly in a quadratic criterion, a problem we circumvent by introducing an auxiliary variable and two constraints, in such a way that the auxiliary variable will be equal to the outcome of this max function. Recall from [6, p. 18] that this can be achieved by solving the LP (Linear Programming) problem

$$\min_{P_a} P_a \quad \text{sub} \;\; b_b^-(E_s)P_s \le P_a, \; b_b^+(E_s)P_s \le P_a,$$

as long as this problem is well defined, *e.g.*, the solution is not unbounded. See Fig. 3 for an illustration how this looks.

This figure expresses that for positive P_s we need to supply more power than is stored, $P_a > P_s$, while for negative P_s the power becoming available is less than taken from the storage, $|P_a| < |P_s|$. By adding more constraints, the relation between P_a and P_s can also approximate the quadratic term, so $a_b P_s^2$ could be skipped from the relation for P_b, but then the final problem will have a Hessian that is not strictly positive definite, which restricts the class of solvers that can be used, and the number of constraints increases rapidly. The set of constraints should also be convex.

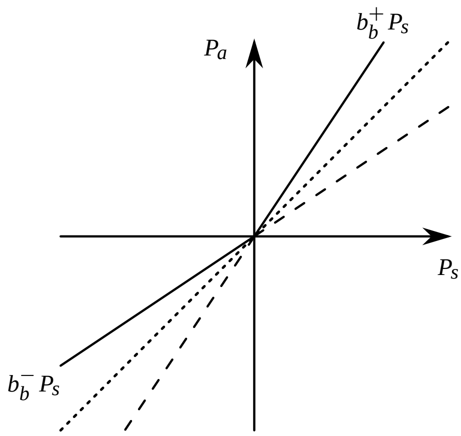

Figure 3: Relation between auxiliary power P_a and stored power P_s

The LP problem for P_a needs to be embedded in the original optimization problem of finding minimal primary power P_c, which is possible if the objective is monotonous, but for now we are just going to write

$$P_b = \phi_b(E_s, P_s) \approx a_b(\hat{E}_s)P_s^2 + P_a,$$

neglecting leakage also, because this is a slow phenomenon, and will only give a small discrepancy between computed and measured E_s. Other effects will probably have a larger influence on the accuracy of the model.

Using the relation for P_b in the relation for P_g and this again in the relation for P_c we can write

$$P_c \approx aP_a^2 + c(P_a + a_b P_s^2),$$

with

$$a = a_c b_g^2 + 2a_c a_g c_g + b_c a_g + 2a_c a_g P_p + 6a_c a_g b_g P_l + 6a_c a_g^2 P_l^2,$$
$$c = 2a_c b_g c_g + b_c b_g + 2a_c b_g P_p + 4a_c a_g P_p P_l + (2a_c b_g^2 + 4a_c a_g c_g + 2b_c a_g)P_l + 6a_c a_g b_g P_l^2 + 4a_c a_g^2 P_l^3,$$

by dropping all terms of order 3 and higher in P_a and P_s and also dropping terms that do not depend on P_a or P_s, because these do not influence the solution. Because higher order terms are dropped, the relations for a and c can better be fitted directly, as functions of ω, P_p, and P_l, using the available device data. The coefficients a and c are always positive, as are their constituent parts, in the physically relevant domain with $P_p \ge 0$ and $P_l \ge 0$.

The design variables are now apparent, namely P_a and P_s. Thus, we now have to minimize the expression for P_c over these design variables. The relation between P_c and P_a is normally strictly monotonic in the physically relevant domain, with the sign assumptions on P_p and P_l, so the minimal value for P_c is achieved at the constraints for P_a, which should therefore better consist of only those that enforce the relation between P_a and P_s. The piecewise linear loss relation between P_a and P_s is thus taken care of automatically when we minimize P_c subject to the two constraint relations between P_s and P_a.

5 Constraints

Besides the constraints for the relation between P_a and P_s we have several other constraints and simple bounds. The design variable P_s can be simply bounded by its allowed values, $P_s^{\min} \le P_s \le P_s^{\max}$. Other common min/max physical constraints, *e.g.*, on secondary power generated, $P_m^{\min}(\omega) \le P_m \le P_m^{\max}(\omega)$, tertiary power generated, $P_e^{\min}(\omega) \le P_e \le P_e^{\max}(\omega)$, and stored energy, $E_s^{\min} \le E_s \le E_s^{\max}$, are to be guaranteed also, where the lower bounds are normally equal to 0. We cannot involve P_a in these constraints, nor bound it directly, otherwise we are not sure the piecewise linear relation between P_a and P_s is effected, so we formulate the physical constraints simply as

$$dP_s \le e,$$

where d and e can be functions of ω, P_p, and P_l. Some approximations may be necessary to get this form, because only E_s is a linear function of P_s, while the variables P_m and P_e are not. To put the bounds on E_s in the stated form is therefore easy, and will not be detailed.

To get the bounds on P_m and P_e in the required form can be done in several ways

- by solving the relations between P_m, P_e, and P_s, as equations at the bounds, for P_s,
- by approximating these relations conservatively, by ones linear in P_s,
- by using an (embedded) optimization problem formulation.

The bounds on P_m and P_e are independent at each time instant, unlike the bounds on E_s, so they can be handled more efficiently. A possibility is to compute these bounds off-line and to tabulate the resulting data, expressed as lower and upper bounds on P_s, as functions of ω, P_p, and P_l.

We omit further details of the constraints handling.

6 Predictive control problem

For predictive control, the criterion to be minimized is normally the sum of the criteria for each time instant, while the constraints should be satisfied at all time instants inside the complete horizon N. The constrained variables P_m and P_e change fast and their bounds depend on ω, so they need to be bounded at all time instants within the horizon, while for E_s bounds at a restricted number of points within the horizon are sufficient.

To summarize, the predictive controller should solve the following QP optimization problem

$$\min_x \sum_{i=1}^{N} P_c(i) = \min_x \frac{1}{2} x' H x + g' x, \quad \text{sub } Ax \le b,$$

where x contains the variables $P_a(i)$, $P_s(i)$, $i = 1, \ldots, N$, H is the sparse (diagonal) Hessian, and $Ax \le b$ is the collection of constraints for $i = 1, \ldots, N$.

The objective normally causes the storage to be drained. We can add an end-point constraint on E_s, so $E_s(N) \ge E_s(0)$, to avoid depleting the storage. This could also be handled by discounting the stored energy in the criterion to be minimized, but the rate at which to discount is not known in advance very accurately. In the next section we give results for both cases and discuss their pros and cons.

From the solution x the first element of P_s is implemented, which will act as setpoint for the power controller of the storage device.[1] To continue for the next sampling time, the measurements, normally only E_s, are received, together with new predictions for ω, P_p, and P_l, that will have to be generated based on measurements and other information. With this information the next schedule can be computed 1 time instant further into the future.

7 Application

The problem summarized in the previous section is setup, using synthetic, but realistic, data for the efficiency characteristics, physical constraints, and workloads, and solved for different lengths of the horizon. The application is a model aircraft equipped with a micro-turbine, for propulsion, and with a power take-off to a micro-generator. The tertiary power is used to drive the control surfaces and to feed the communication equipment.

This application should give information about the physical characteristics of the control actions, to gain insight in the problem, about the potential benefits, about the influence on the results of the length of the prediction/control horizon, and about the time required to solve the problem at each step. A compromise between available on-line computing time and quality of the controller should give a lead to a useful horizon length.

We start with the results using an end constraint on the stored energy and then give those for an objective where the stored power is discounted.

7.1 End constraint $E_s(N) \ge E_s(0)$

Figure 4 shows the contours of the criterion P_c, for a single workload, while the optimal solutions for P_s and P_a are given for all times, which also outlines the constraint relation between P_s and P_a. The contours demonstrate the monotonic relation between P_a and P_c. The reason the solution is not always at the lower left is the endpoint constraint on E_s, the lower bounds on P_s due to constraints on P_s itself and on P_e, and the dependency of P_c on the workload. The optimal solutions are for a pattern of ω, P_p, and P_l of 1800 [s] length, with a sampling period of 1 [s], so we process sequences of at most 1800 data points in length, and the largest size of x is 3600 when we use a horizon equal to the total data length, for which the results are presented in this figure.

Figure 5 shows the values of the criterion for several situations. Baseline situations are

- a situation where no tertiary power is consumed, $P_l = 0$, this will give the lowest primary energy consumption,

[1] Here we assume that the actions of the power controller for the storage device do not influence the behavior of the controllers for the power converters, due to local feedback loops for these converters. Sending the storage power setpoint signal to these control systems may facilitate this.

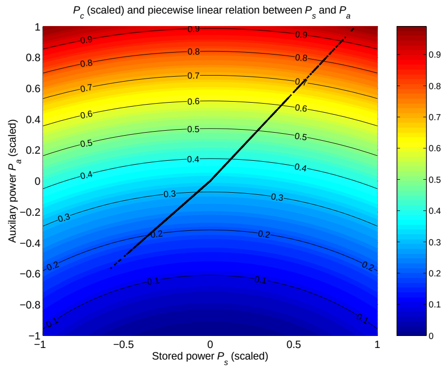

Figure 4: Objective contours and piecewise linear relation

- a situation where no scheduling is taking place, so the tertiary power demand is met by generating this power instantly, $P_e = P_l$, and no power is stored, this will give the highest primary energy consumption.

The two baseline situations are given in Fig. 5 at $N = 0$. The other data points in Fig. 5 reflect results for different horizons of the predictive controller. The results show how much primary energy is saved with the predictive controller compared to the situation without scheduling, and give an indication of the required horizon to realize the benefits.

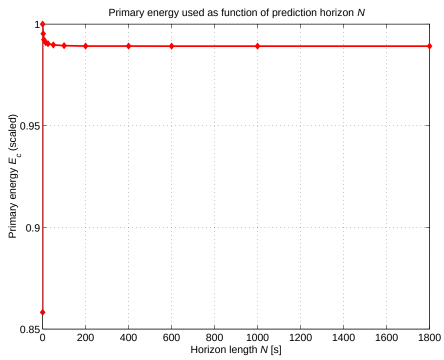

Figure 5: Normalized primary energy needed for a workload of 1800 [s]

Figure 6 shows the average computing time needed for several control horizons. These results show, as expected, a more than linear increase in time. Note that Fig. 6 shows the average time for a solution, while for real time implementation the worst case is relevant, which is about 10 times as costly as the average.

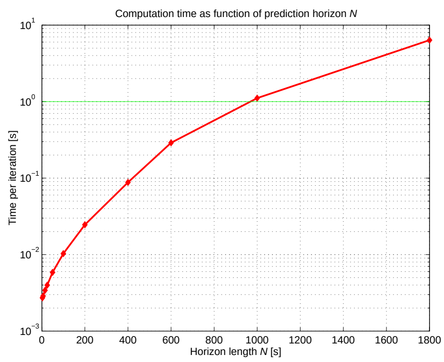

Figure 6: Computing time needed for different control horizons

From these results it appears that a control horizon of about $N = 50$ is sufficient to achieve savings in primary energy close to those achieved with scheduling over the full data set, but this is a function of the frequency of workload changes. The savings are about 8% of the primary energy needed to feed the secondary converter when no scheduling takes place, but this number is a function of the efficiency data, physical limitations, and workload characteristics. A horizon of $N = 50$ can be easily achieved, due to the simple model employed, but even horizons up to $N = 400$ are no problem with commodity hardware, because also in this case setting up and solving a single problem needs less than the assumed sampling time of 1 [s], even in the worst case. This indicates that more involved problems can be solved on-line without difficulty.

7.2 Discount stored power

When eliminating the endpoint constraint, and replacing it by discounting the stored power P_s, to guarantee $E_s(N) \approx E_s(0)$, and assuming the influence of $\hat{E}_s$ can be neglected, the QP problem is completely independent at each time instant. The optimal solution then does not require a prediction horizon larger than $N = 1$, a substantial simplification. A reasonable way to discount P_s is to include the term $-\bar{c}P_s$ in the criterion, with $\bar{c}$ the mean of c over a certain time interval. This will not guarantee E_s to be exactly equal at the start and end of this interval, so small corrections to $-\bar{c}P_s$ are needed, like $-\bar{c}(1-\epsilon(E_s - E_s^d))P_s$, or a quadratic term in $E_s - E_s^d$ could be added to the objective $\frac{1}{2}x^T H x + g^T x$, with E_s^d a desirable value for E_s.

To show all this, Figs. 7–9 give the contours of the criterion for three different cases. The cases are selected for a low, zero, and high value of P_s, specifically marked in the figures, which also give all other values for the optimal solutions for P_s and P_a, which are virtually identical, after tuning $\bar{c}$, to the previous results, but obtained with $N = 1$, so the computing time and memory requirements are negligible. A much higher frequency than 1 [s] is then possible.

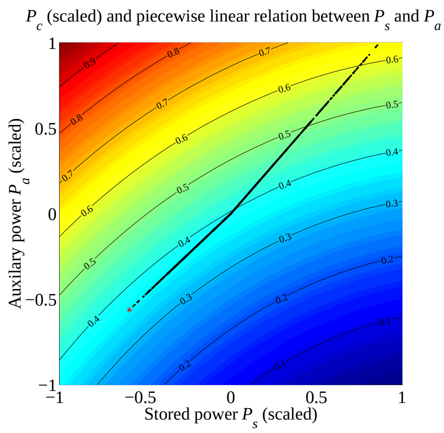

Figure 7: Objective contours and piecewise linear relation

From Figs. 7–9 it is easy to see how the control for $N = 1$ works. If the gradient of the objective is in the sector bounded by two lines perpendicular to the two linear constraints, there is no advantage in storing energy, so $P_s = 0$ will be optimal.

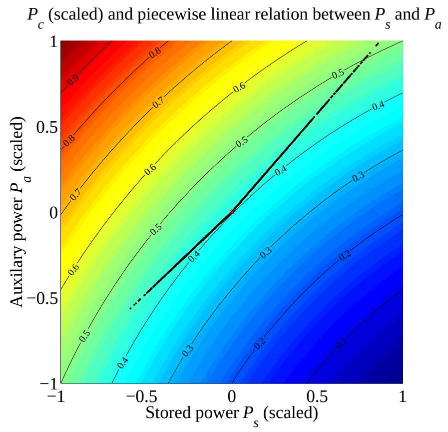

Figure 8: Objective contours and piecewise linear relation

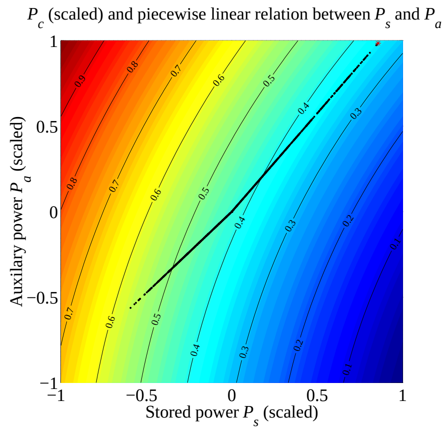

Figure 9: Objective contours and piecewise linear relation

When the value of c changes, the gradient of the objective rotates and can leave this sector, leading to an optimal value of $P_s \neq 0$. Referring to Fig. 1, $P_s < 0$ for large P_m, $P_s = 0$ for intermediate P_m, and $P_s > 0$ for small P_m will be optimal.[2] The boundaries between small, intermediate, and large are determined by the storage losses (the sector's angle) and the changes in incremental cost (variations in, mainly, c) compared to the average $\bar{c}$ expected. One has to set $\bar{c}$ so $\sum P_s \approx 0$ over a suitable time interval. Suitable normally means as large as possible, without leaving the sweet spot of the storage efficiency characteristic. When that is happening one has to reset $\bar{c}$. It would be more transparent to incorporate this in the predictive controller, *e.g.*, following one of the previous suggestions.

A result of using $N = 1$ is that control actions do not depend on the far future, the only influence of the far future is in the factor $\bar{c}$, because only this value needs to be predicted, which completely eliminates prediction of the workloads. Not the workloads itself, amounting to $3N$ data points at each time instant, are needed, but only $\bar{c}$, a single number that does not even need computation at each point in time, but could be set in advance, not based on the actual workloads foreseen, but on some characteristics, *e.g.*, based on expected environmental conditions or restrictions.

[2] When the power conversion system without storage control meets the bounds on P_m, then also with this type of control the bounds on P_m will be met, because the control "tends to the middle." This means the bounds on P_m need not be taken into account by the controller in this case.

Another advantage of $N = 1$ is that non-monotonous objective functions can easily be handled. A way to do this is to solve the optimization problem several times, once for each part of the piece-wise linear objective terms, eventually by formulating several problems with a single equality constraint instead of a single problem with several inequalities, and to choose the solution with the smallest objective. This method is not practical for larger N, because in this case we are in fact solving a mixed integer optimization problem.

Still another advantage of a small N lies in the relation for the storage process losses, which has no need to use a predicted $\hat{E}_s$, but only the latest measurement of E_s, which is more accurate.

8 Conclusions

A power-based model approach to control a storage device in a power conversion system with frequently changing workloads is outlined. The efficiencies of power converters and storage device are taken into account when computing the control actions. The controller employs a receding horizon to cope easily with frequent workload variations in changing environments.

The results show that nice savings in primary energy can be achieved by scheduling the storage device. These results, that could be indicative of practical applications, suggest that the primary energy needed to feed the secondary power converter during a certain sequence of workloads can be reduced by 8% using storage control. This number depends quite a lot on the efficiency characteristics of the converters and storage device, on the physical limitations of the devices, as well as on the foreseen and realized changes in workload.

The approach is practical, because the demands on the horizon of the predictive controller are moderate and the model used in the controller is simple, so the scheduling can be done on-line to cope with changing circumstances.

When avoiding the use of an end constraint on E_s, which seems quite simple to achieve, the computation of the predictive controller is almost trivial, and therefore implementation seems not to be a problem at all.

References

[1] K. Kogan and E. Khmelnitsky, *Scheduling: Control-Based Theory and Polynomial-Time Algorithms*. Dordrecht: Kluwer Academic Publishers, 2000.

[2] C. Lu, J. A. Stankovic, G. Tao, and S. H. Son, "Feedback control real-time scheduling: Framework, modeling, and algorithms," *J. of Real-Time Systems*, vol. 23, pp. 85–126, July 2002.

[3] J. M. Maciejowski, *Predictive Control with Constraints*. Prentice Hall, June 2001.

[4] T. G. Habetler and R. G. Harley, "Power electronic converter and system control," *Proceedings of the IEEE*, vol. 89, pp. 913–925, June 2001.

[5] A. S. Iskhakov, S. G. Obukhov, A. V. Ushakov, and N. V. Dedkov, "The new control methods of converters systems," in *Fifth European Conference on Power Electronics and Applications*, vol. 4, (Brighton, UK), pp. 278–281, 1993.

[6] K. G. Murty, *Linear Programming*. New York: John Wiley & Sons, 1983.