JOINT SYNTHESIS OF CONTROL AND FAULT DETECTION ALGORITHMS : STUDY OF P.I. CONTROLLER INFLUENCE

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Abstract

The work reported in this paper focuses on an optimal joint synthesis of controller and fault detection modules in the presence of model uncertainties. The presented method takes the interactions between control and diagnosis into account in order to make more efficient the fault detection performances. A quadratic criterion combining both control and diagnosis aims is defined in a worst case situation according to model uncertainties and disturbances. The cases of proportional and proportional integral controllers are examined in order to study the P.I. controller influence on the joint approach. The residual evaluation is carried out using a threshold computed in an uncertain closed-loop framework.

1 Introduction

Most of methods for model-based fault detection are usually designed on an open-loop scheme of the system but only few studies consider diagnosis in the closed-loop framework. Nevertheless, the existence of interactions between control and fault detection performances is described in [1], [2] and [3]. It is straightforward to note that control and diagnosis have opposite goals. Indeed, the diagnosis module aims at detecting faults whereas controller should make the measured output insensitive to both disturbances and faults.

Some diagnosis methods have been developed in a closed-loop framework. Their main purpose is to design a fault sensitive residual robust to disturbances such as model uncertainties and noises. The authors of [4], [5], [6] and [7] present an integrated approach based on a four parameters controller, proposing a joint design of control law and fault detection algorithms. Other works in this field use modules with two or three degrees of freedom ([8], [2], [9]). In [10], the controller is synthesized from a cost function, which combines both control law and fault detection objectives. In this case, the closed loop system has not only stability robustness properties, but also ensures performance robustness for failure detection module.

When uncertain plants are involved, previous studies show that a separated synthesis of both control law and fault detection module does not lead to an efficient diagnosis. In this case, it is interesting to synthesize the control and diagnosis modules simultaneously and it is essential to trade-off control law and fault detection performances.

The work presented in this article is issued from [10]. The main technical contribution lies in the joint synthesis of both control law and fault detection algorithms by taking their interaction into account and in the analyse of P.I. controller influence on the coupled approach. This study considers the dynamic behavior of the system for the control law design while the fault detection is only based on the static mode analysis of the signals.

This paper is organized as follows. In section 2, work hypotheses are presented. Later, the controller and the residual generator are separately designed according to the minimization of a quadratic cost function established in a worst case situation. The distinction between a proportional and a proportional integral control law is carried out. The residual is evaluated in determining an optimal threshold which is based on an uncertain representation of the closed loop system. In section 4.1, an augmented cost function joining both fault detection and control law synthesis is introduced and the influence of P.I. controller on the joint approach is also examined. The concluding section 5 considers prospective developments.

2 Problem statement

The system considered in this article is assumed to be stable and controllable. Its behaviour can be characterized, around an operating point $(\underline{U}_o; \underline{Y}_o)$, by the following input-output model

$$\underline{y}(s) = G_{yu}(s)\underline{u}(s) + G_{yd}(s)\underline{d}(s) + G_{yn}(s)\underline{n}(s) + G_{yf}(s)f(s) \quad (1)$$

where $\underline{u} \in \mathbb{R}^m$ and $\underline{y} \in \mathbb{R}^p$ are respectively the input and output vectors, $\underline{d} \in \mathbb{R}^l$ represents unknown input disturbances and the p measurement noises $n_i \sim \mathcal{N}(0, \sigma_i^2)$ are uncorrelated. Each component of $\underline{d}$ is a piecewise constant signal and is such as $|d_i(t)| < d_{imax}$. The fault is denoted by unknown input $f \in \mathbb{R}$.

Note that if the model uncertainties are included in the system description, the real behavior of the process is represented as follows

$$\underline{y}_{real}(s) = \underline{y}(s) + \Delta G_{yu}(s)\underline{u}(s) \quad (2)$$

where unknown transfer matrix $\Delta G_{yu}(s)$ denotes the model uncertainties. Other parametric uncertainties in transfers $G_{yd}(s)$, $G_{yn}(s)$ and $G_{yf}(s)$ are included in the time evolution of signals $\underline{d}$, $\underline{n}$ and f.
In a closed-loop framework, the control law $\underline{u}$ is generated by means of a controller $K(s)$ such as

$$\underline{u}(s) = -K(s)\underline{y}_{real}(s). \tag{3}$$

Thus, the input-output relation (2) becomes

$$\begin{aligned}\underline{y}_{real}(s) =& S(s)\big(G_{yd}(s)\underline{d}(s) \\ &+ G_{yn}(s)\underline{n}(s) + G_{yf}(s)f(s)\big)\end{aligned} \tag{4}$$

where sensitivity function $S(s)$ is defined by

$$S(s) = \big(I + (G_{yu}(s) + \Delta G_{yu}(s))K(s)\big)^{-1}. \tag{5}$$

Efficient model-based fault detection relies on the generation of a fault-sensitive residual. Since each component of $\underline{n}$ is a zero mean noise, the output simulation error $\underline{e}(s)$ can be used as a detection residual

$$\underline{e}(s) = \underline{y}_{real}(s) - \underline{\hat{y}}(s). \tag{6}$$

Output vector $\underline{\hat{y}}(s)$ is simulated by means of expression (7) that is deduced from (1)

$$\underline{\hat{y}}(s) = G_{yu}(s)\underline{u}(s). \tag{7}$$

In the sequel, only the static behavior of the residual is considered for fault detection. This is the reason why the noise influence on the residual generation is neglected. Thus, in steady state, the relation (6) in a noiseless situation becomes

$$\underline{e} = G_{ed}\underline{d} + G_{ef}f \tag{8}$$

with

$$\begin{aligned}G_{ed} &= \lim_{s\to 0}\big(I - \Delta G_{yu}(s)K(s)S(s)\big)G_{yd}(s) \\ G_{ef} &= \lim_{s\to 0}\big(I - \Delta G_{yu}(s)K(s)S(s)\big)G_{yf}(s).\end{aligned} \tag{9}$$

In order to improve the fault detection efficiency, a secondary residual $r(t)$ is built as follows

$$r = |\underline{H}^T\underline{e}| \tag{10}$$

where $r \in \mathbb{R}$ and $\underline{H} \in \mathbb{R}^p$ is a projection vector. Diagnosis enhancement implies an optimal choice of the vector $\underline{H}$ maximizing the fault effects while minimizing the influence of disturbances.
For the sake of simplicity, the steady state transfer matrices $G_{yu}(s)$, $\Delta G_{yu}(s)$, $G_{yd}(s)$, $G_{yf}(s)$ is described by the following notations

$$P = G_{yu} \quad \Delta P_{real} = \Delta G_{yu} \quad E = G_{yd} \quad W = G_{yf}$$

Unknown uncertain matrix ΔP_{real} is assumed to belong to a known interval $\Lambda = [\Delta P_{min}; \Delta P_{max}]$. In a noiseless situation, the following expression defines a set of possible outputs $\underline{y}_\Lambda$ and inputs $\underline{u}_\Lambda$ when the model uncertainties take any value ΔP belonging to the interval Λ

$$\underline{y}_\Lambda = (P + \Delta P)\underline{u}_\Lambda + E\underline{d} + Wf \tag{11}$$

The static model useful to the simulation of the output vector is given by

$$\underline{\hat{y}} = P\underline{u}. \tag{12}$$

In the sequel, sets G_{ef_Λ} and G_{ed_Λ} denote the family of transfer functions (9) when the model uncertainties take any value $\Delta P \in \Lambda$.

Referring to (9) and (10), it appears that the design of the projection vector $\underline{H}$ depends on the controller $K(s)$. Thus, the objective of this work is to synthesize simultaneously an optimal control law and an optimal fault detection algorithm by minimizing an augmented cost function. The next section presents both control and diagnosis aims as well as the decoupled synthesis of the controller and the residual generator based on a quadratic cost function optimization.
A threshold synthesis based on a closed loop framework is also introduced.

3 Decoupled synthesis

The approach presented in this part consists in synthesizing firstly an optimal control law then an optimal residual generator.
The control law synthesis has two main objectives :

- to trade off the control signal energy and the disturbances rejection in steady state and in a worst case context according to model uncertainties ans disturbances.
- to ensure an acceptable dynamic behaviour of the closed loop system.

The aims of the fault detection synthesis are :

- to detect faults in steady state (no isolation)
- to maximize faults affects and to minimize both disturbances and model uncertainties influences on the fault detection algorithm.

Furthermore, in order to focus the consequences of the integral action on the fault detection, both proportional and proportional integral control laws are studied.

3.1 Proportional controller

The control law is issue of proportional controller $K(s) = K_p$ and since only non-zero mean disturbances are considered, the

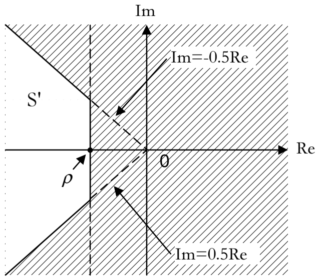

Figure 1: Set S'

problem consists in determining the optimal controller K_p^* solution of the following cost function

$$K_p^* = Arg \min_{K_p} \max_{\Delta P \in \Lambda, d} J \tag{13}$$

with

$$J = \left\| \underline{y}_\Lambda \right\|_Q^2 + \left\| \underline{u}_\Lambda \right\|_R^2. \tag{14}$$

under model constraint (11). Moreover, the controller deduced from (13) must guarantee the dynamic objective of the closed loop system. Thus, it is necessary to ensure that the controller K_p^* places the closed loop poles in set S' depicted on figure (1). The determination of K_p^* consists then in solving the static performance criterion (13) under model and pole location constrains.

Once the controller is determined, the residual generator has to be synthesized. With respect to the simulation error definition (8), the residual (10) only depends on the disturbances according to

$$r = |\underline{H}^T (G_{ed}\underline{d} + G_{ef} f)| \tag{15}$$

where matrices G_{ef} and G_{ed} (9) are described by

$$\begin{cases} G_{ed} = \left(I - \Delta P_{real} K_p^* (I + (P + \Delta P_{real}) K_p^*)^{-1}\right) E \\ G_{ef} = \left(I - \Delta P_{real} K_p^* (I + (P + \Delta P_{real}) K_p^*)^{-1}\right) W \end{cases} \tag{16}$$

In an ideal case, optimal vector $\underline{H}^*$ must be selected in order to satisfy both fault sensitivity and non-zero mean disturbances insensitivity purposes

$$\begin{cases} \underline{H^*}^T G_{ed_\Lambda} = 0 \\ \underline{H^*}^T G_{ef_\Lambda} \neq 0 \end{cases} \tag{17}$$

for all model uncertainties belonging to Λ and where

$$\begin{cases} G_{ed_\Lambda} = \left(I - \Delta P K_p^* (I + (P + \Delta P) K_p^*)^{-1}\right) E \\ G_{ef_\Lambda} = \left(I - \Delta P K_p^* (I + (P + \Delta P) K_p^*)^{-1}\right) W \end{cases} \tag{18}$$

However, these conditions of perfect decoupling are often too strong. Due to model uncertainties and disturbances, residual r is not null without fault and it is then necessary to evaluate it. In order to reduce the false alarms rate, the residual has to be compared with a detection threshold, reflecting the worst effect of disturbances and model uncertainties on the secondary residual in a non-faulty case.

According to the closed-loop framework, threshold Γ is defined by

$$\Gamma = \max_{\Delta P \in \Lambda, \underline{d}} \left(|\underline{H}^T G_{ed_\Lambda} \underline{d}| \right) \tag{19}$$

According to the definition of Γ, a fault can be detected whatever the model uncertainties and the disturbances if

$$\min_{f, \Delta P} |\underline{H}^T G_{ef_\Lambda} f| \geq 2 \max_{d, \Delta P} |\underline{H}^T G_{ed_\Lambda} \underline{d}| = 2\Gamma \tag{20}$$

The minimum detectable fault magnitude is then given by

$$|f_{min}| = \frac{2 \max_{d, \Delta P} |\underline{H}^T G_{ed_\Lambda} \underline{d}|}{\min_{\Delta P} |\underline{H}^T G_{ef_\Lambda}|} \tag{21}$$

where matrices G_{ef_Λ} and G_{ed_Λ} are defined in (18). Referring to ([11]) and ([12]), optimal projection vector $\underline{H}^*$ is then determined in solving the following optimization problem

$$\underline{H}^* = arg \min_{\underline{H}} |f_{min}| \tag{22}$$

3.2 Proportional Integral controller

If the systems considered are stable in open loop, only a proportional integral controller depicted as follows

$$\underline{u} = -K_p \underline{y}_{real} - K_i \int \underline{y}_{real}. \tag{23}$$

can efficiently rejected disturbances $\underline{d}$ in steady state . Moreover, whatever the structure of the system, the static values of the outputs and the control inputs are only influenced by the integral gain K_i. Thus, the optimal integral gain K_i^* is determined by

$$K_i^* = Arg \min_{K_i} \max_{\Delta P \in \Lambda, \underline{d}} \left(\|\underline{y}_\Lambda\|_Q^2 + \|\underline{u}_\Lambda\|_R^2 \right). \tag{24}$$

The proportional gain K_p is dedicated to the poles assignment. However, if there would not exist K_p such as the poles of the system belong to the set S' (figure(1)), an iterative search of the best couple (K_p^*, K_i^*) minimizing cost function (24) and ensuring the poles assignment must be undertaken.

The synthesis of the optimal projection vector $\underline{H}^*$ is then deduced from (22). In the case of a proportional integral controller (23), the existence of the control-diagnosis interactions is subordinated to the numbers of inputs and outputs of the system. The minimization of cost functions (24) and (22) is studied in the 3 following cases :

$p = m$**:** For systems having the same number of control inputs as outputs, the presence of integrations in the controller implies a step disturbances rejection in steady state. The static error is null by definition and the static value of the control vector is only function of the disturbance magnitudes. Parameters K_p and K_i are then selected in order to guarantee an acceptable dynamic behavior of the closed loop system. For a sake of simplicity, the poles assignment is based on the nominal model of the system. Under the assumption that the system is commandable, the choice of the dynamic performances is then arbitrary. The synthesis of the optimal projection vector $\underline{H}^*$ is then deduced from the diagnosis cost function (22) where matrices G_{ef_Λ} and G_{ed_Λ} are given by

$$\begin{cases} G_{ed_\Lambda} = P(P+\Delta P)^{-1}E \\ G_{ef_\Lambda} = P(P+\Delta P)^{-1}W \end{cases} \quad (25)$$

The main advantage of a proportional integral controller is to make a perfect step disturbances rejection possible. Furthermore, the residual generation in steady state becomes independent of the control law.

$p < m$**:** For a system where the number of outputs is lower than the number of inputs, the static error is always null. On the other hand, the steady value of each control vector component depends of the integral gain K_i. The control problem consists in solving (24) (with Q null) under the poles assignment constraint.
Then, the determination of the residual generator consists in finding the projection vector $\underline{H}^*$ solution of (22) where matrices G_{ef_Λ} and G_{ed_Λ} are defined by

$$\begin{cases} G_{ef_\Lambda} = PK_i\big((P+\Delta P)K_i\big)^{-1}W \\ G_{ed_\Lambda} = PK_i\big((P+\Delta P)K_i\big)^{-1}E \end{cases} \quad (26)$$

$p > m$**:** When the process has less inputs than outputs, the controller gains are deduced from the minimization of (24) and the projection vector is derived from (22) where the matrices G_{ef_Λ} and G_{ed_Λ} are described by

$$\begin{cases} G_{ef_\Lambda} = P\big(K_i(P+\Delta P)\big)^{-1}K_iW \\ G_{ed_\Lambda} = P\big(K_i(P+\Delta P)\big)^{-1}K_iE \end{cases} \quad (27)$$

4 joint synthesis

As indicated in (18), (26) and (27), the performance of the residual generator often depends on the controller characteristics. Thus, instead of synthesizing the control law and the residual generator sequentially, a joint synthesis of these two algorithms is suitable.

4.1 Proportional controller

Without considering the control performances, best projection vector $\underline{H}^*$ and controller K_p^* would be determined in solving the following cost function

$$\min_{\underline{H},\, K_p} |f_{min}| \quad (28)$$

where $|f_{min}|$ is defined by (21) and matrices G_{ef_Λ} and G_{ed_Λ} are given by (18). In order to take the control performances into account, the controller has to be equally solution of (13). The computation of K_p^* and $\underline{H}^*$ thus derives from the minimization of an augmented cost function

$$(K_p^*, \underline{H}^*) = \arg \min_{K_p, \underline{H}} \Big(\big(\max_{\Delta P \in \Lambda, \underline{d}} J \big) + \gamma |f_{min}| \Big) \quad (29)$$

under both model and pole assignment constraints. Indexes J and $|f_{min}|$ are respectively defined by (14) and (21). Since projection vector $\underline{H}$ appears only in the fault detection index, criterion (29) can be rewritten as

$$(K_p^*, \underline{H}^*) = \arg \min_{K_p} \big(J_c + \gamma \min_{\underline{H}} |f_{min}| \big) \quad (30)$$

where

$$J_c = \max_{\Delta P \in \Lambda, \underline{d}} J \quad (31)$$

The criterion (30) could be minimized by means of several methods as the game theory ([13]) or an algorithmic research. For a sake of simplicity, only the second approach is carried out in the sequel of this paper.
The choice of γ represents the crucial point of this problem since it allows one to balance the importance of $|f_{min}|$ in the cost function (30).

4.2 Proportional Integral controller

For a proportional integral control law, the same reasoning as previously is carried out. Whatever the system structure, the computation of optimal gains K_p^* and K_i^* and projection vector $\underline{H}^*$ consists in solving the following optimization problem

$$(K_i^*, \underline{H}^*) = \min_{K_i, \underline{H}} (J_c + \gamma \min_{K_i, \underline{H}} |f_{min}|) \quad (32)$$

where J_c and $|f_{min}|$ are respectively defined by (31) and (21) and under the constraint that the proportional gain K_p assigns the poles of the closed loop system in the location described on figure (1).
When the process have the same number of control inputs and outputs, the joint synthesis of both controller and residual generator becomes equivalent to a sequential synthesis.
In the other cases, the joint synthesis is achieved thanks to a choice of the weighting matrices and by taking the suitable matrices G_{ef_Λ} and G_{ed_Λ} into account.
The following part studies the influence of γ on the augmented cost function minimization.

4.3 Study of γ influence on the augmented cost function minimization

Since an analytic resolution of (30) is not carried out in this work, the study of γ influence on the augmented cost function minimization consists in solving (30) in the case of several randomly chosen systems. Due to the use of an algorithmic research, only S.I.M.O (1×2) systems are considered. The simulation is achieved in the case of a proportional control law and in the following context :

- the static behavior of systems is described by

$$\underline{y}_\Lambda = (P+\Delta P)(u_\Lambda + d) + Wf \quad (33)$$

with $\Delta P \in \Lambda = [-20\%P; +20\%P]$ and $|d| < 1$

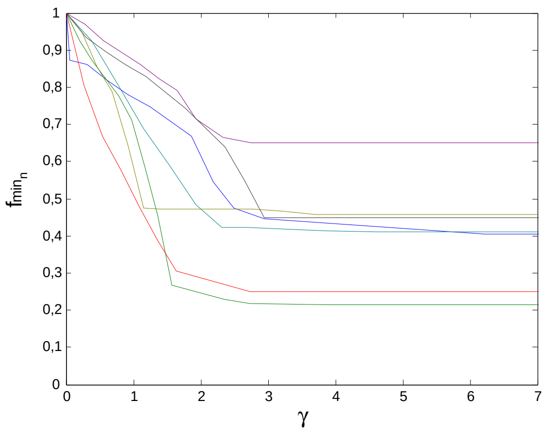

Figure 2: Evolution of $|f_{min}|_n$ with respect to γ

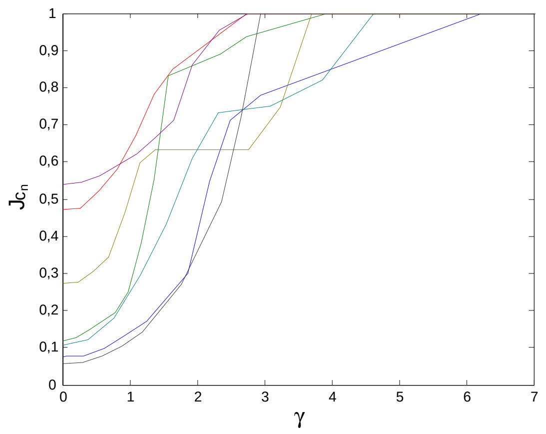

Figure 3: Evolution of J_{c_n} with respect to γ

- the fault appears on one of the two output sensors ($W = \begin{bmatrix}1 & 0\end{bmatrix}^T$ or $W = \begin{bmatrix}0 & 1\end{bmatrix}^T$)

- the algorithmic research consists in finding the optimal solutions of (30) among a set of controllers and projection vectors. The optimal projection vector $\underline{H}^*$ is defined as follows

$$\underline{H}^* = \begin{bmatrix}cos(\phi^*) & sin(\phi^*)\end{bmatrix}^T$$

  where ϕ^* is chosen in the interval $[0; 2\pi]$.

- by means of the control and fault detection indices normalization, the criterion (30) is rewritten as

$$(K_p^*, \underline{H}^*) = \arg \min_{K_p} \left(J_{c_n} + \gamma \min_{\underline{H}} |f_{min}|_n\right) \qquad (34)$$

  where

$$J_{c_n} = \frac{J_c}{J_{c_{max}}} \quad \text{and} \quad |f_{min}|_n = \frac{|f_{min}|}{|f_{min}|_{max}}. \qquad (35)$$

The evolution of indices J_{c_n} and $|f_{min}|_n$ obtained for each system and for several values of γ are depicted on figures (2) and (3).

When $\gamma \to 0$, optimal controller K_p^* minimizes only the control index J_c and the joint synthesis becomes equivalent to the decoupled synthesis exposed in section 3. In this case,

$$J_c = J_{c_{min}} \text{ and } |f_{min}| = |f_{min}|_{max}. \qquad (36)$$

If $\gamma \to \infty$, K_p^* minimizes only the fault detection index and we obtained respectively the worst and the best case for the control and the fault detection performances

$$J_c = J_{c_{max}} \quad |f_{min}| = |f_{min}|_{min}. \qquad (37)$$

It appears that whatever the value of $\gamma > 0$, $|f_{min}|$ and J_c belong to the following intervals

$$J_c \in [J_{c_{min}}; J_{c_{max}}] \qquad (38)$$

$$|f_{min}| \in [|f_{min}|_{min}; |f_{min}|_{max}] \qquad (39)$$

Furthermore, referring to figures (2) and (3), if $\gamma_j = \gamma_i + \varepsilon$ with $\varepsilon > 0$ then

$$J^*_{c_{\gamma=\gamma_j}} \geq J^*_{c_{\gamma=\gamma_i}} \qquad (40)$$

and

$$|f_{min}|^*_{\gamma=\gamma_j} \leq |f_{min}|^*_{\gamma=\gamma_i}. \qquad (41)$$

The use of normalized indexes in the optimization problem allows to obtained a satisfactory trade-off between the control and the fault detection objectives when $\gamma = 1$.

5 Conclusions

In this paper, an optimal joint synthesis of control law and fault diagnosis algorithm is proposed in the presence of model uncertainties. A cost function combining control and static fault detection objectives is defined in a worst case context. This cost function synthesis is based on the control-diagnosis interactions. By a suitable choice of the weighting matrices, the optimisation of this cost function achieves optimal trade-off between the control and the diagnosis aims. A detection threshold based on an uncertain closed-loop framework is also presented. The advantage of a proportional integral controller for both disturbances rejection and fault diagnosis is shown through the application to the 3-tank system. Future developments will concerne the extension of this work to the dynamic fault diagnosis.

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