ADAPTIVE THRESHOLD GENERATION USING INTERVAL MODELS: TIME VERSUS FREQUENCY DOMAIN APPROACHES

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Abstract

In this paper, adaptive threshold generation using interval models is analysed in time and frequency domains. In time domain, the optimal threshold is generated through determining the worst-case time evolution of the residual's energy, while in the frequency domain this worst-case evolution is determined by means of the evaluation of the worst-case evolution of the residual's spectral contents. Both results will be related through the Paserval's Theorem. Finally, a common application will be used to assess the validity of both approaches and check the validity of the derived results.

1 Introduction

Adaptive threshold generation has been a very active area of research in robust fault diagnosis. Since the seminal works of Horak [19] in case of structured uncertainty and Emami-Naeini [14] in case of unstructured uncertainty, many researchers have analysed how the effect of model uncertainty should be taken into account when determining the optimal threshold to be used in residual evaluation. Two approaches have been followed: the first based on determining the optimal threshold in the time domain [19] [28] and the second in the frequency domain [11] [14] [15] [30]. In the time domain approach, the uncertainty mainly has been modelled using structured uncertainty ("***interval models***"), while in the frequency domain approach first unstructured uncertainty was used and recently structured uncertainty has been taken into account. However, no connection between time and frequency domain approaches has been established yet. In Hamelin [18], it is suggested as a future research. One of the difficulties in connecting both approaches is that in the time-domain approaches the optimal threshold is determined directly by determining an interval for thre residual by propagating parameter uncertainty on the residual. On the other hand, in the frequency domain approaches, optimal threshold is determined by worst-case evaluating the residual's energy on a time window. This difference is due to that, either in frequency or in time domain, the optimal threshold is computed using the most direct form.

The structure of this paper is the following: in Section 2 adaptive thresholds using interval models are presented in time-domain and in Section 3 in frequency-domain. In Section 4 a relation between time-domain and frequency-domain adaptive thresholding methods is presented using Parseval's Theorem. In Section 5, an application example based on the DAMADICS benchmark is presented and finally in Section 6 conclusions close the paper.

2 Adaptive thresholds in time-domain

Adaptive thresholding in time-domain has its roots in Horak's seminal work published in 1988 [19]. In this paper, it is proposed to use an interval model to represent the model uncertainty. And, then, this uncertainty is propagated to the residual using a dynamic optimisation in continuous time-domain. Since then, adaptive thresholding using interval models has been studied by several researchers [1] [3] [27] [28].

2.1 Basic adaptive thresholding

The problem of adaptive threshold generation in time-domain using an interval model to describe the behaviour of the monitored system, can be formulated mathematically as follows: at every time-instant an interval for the residual

$$r(t) = y(t) - \hat{y}(t) \in \left[\underline{r}(t), \overline{r}(t)\right] \quad (1)$$

should be computed subject to the interval for the predicted behaviour $\hat{y}(t) \in \left[\underline{y}(t), \overline{y}(t)\right]$ will be obtained from the system interval model

$$\dot{\hat{x}}(t) = A(\theta)\hat{x}(t) + B(\theta)u(t)$$
$$\hat{y}(t) = C(\theta)\hat{x}(t) \quad (2)$$

where: θ is the vector of uncertain parameters that belongs to a convex set Θ representing a bounded region of the parameter space:

$$\Theta = \{\theta \in \Re^p \mid \underline{\theta} \le \theta \le \overline{\theta}\} \quad (3)$$

and $y(t)$ is real measurement. Then, in case that equation (1) holds, no fault can be indicated. Otherwise, a fault should be indicated. Of course, this simple test is very naive and should be complemented with some other strategy that takes into account the effect of the noise, as the ones proposed by Basseville [6]. Alternatively, it can be used the residual evaluation function proposed in the adaptive threshold approaches coming from the frequency domain based on the evaluation of the residual's energy. In this paper, this will be the approach proposed. At least three possible strategies for generating the residual (1) are possible depending in which scheme the model (2) is used, namely, ***simulation***, ***observation*** or ***prediction*** [29]. The most general scheme is the observer strategy, being possible to consider the two others as a particular cases. In case of using an observer, equation (2) must be modified according to:

$$\dot{\hat{x}}(t) = A(\theta)\hat{x}(t) + B(\theta)u(t) + L(y(t) - C(\theta)\hat{x}(t))$$
$$\hat{y}(t) = C(\theta)\hat{x}(t) \quad (4)$$

where L is the observer gain. Depending on which strategy is followed a ***worst-case*** simulation, observation or prediction algorithm must be used [29].

2.2 Worst-case temporal response

In order to determine (1) or, alternatively:

$$y(t) \in \left[\underline{\hat{y}}(t), \overline{\hat{y}}(t)\right] \quad (5)$$

observer equation (4) can be expressed as:

$$\dot{\hat{x}}(t) = A_o(\theta)\hat{x}(t) + B_o(\theta)u_o(t)$$
$$\hat{y}(t) = C(\theta)\hat{x}(t) \quad (6)$$

where: $A_o(\theta) = A(\theta) - LC(\theta)$, $B_o(\theta) = [B(\theta) \quad L]$ and $u_o(t) = [u(t) \quad y(t)]^t$.

Then, in order to compute the interval (5) enclosing the temporal response two optimisation problems must be solved for each output:

$$\overline{\hat{y}}_i(t) = max\ \hat{y}_i(t) \quad (7)$$

$$\underline{\hat{y}}_i(t) = min\ \hat{y}_i(t) \quad (8)$$

subject to (6) and (3). This problem was first formulated by Horak [19] in the context of adaptive threshold generation. He proposed an algorithm called ***RMI*** ("***reachable minimum interval***") based on dynamic optimisation to solve it[1]. The difficulty of this problem is due to that in general is not true that considering only the responses at vertices of parameter uncertain intervals would produce the tightest interval containing all possible temporal responses. Only a small number of systems with uncertain parameters satisfy this desirable property [17]. In case that this property is not satisfied the vertex solution only provides an inner solution, according to Kolev [20]. The existent algorithms to determine (5) follow mainly two approaches after applying some kind of time-discretisation: the first based on trying to solve the associated optimisations problems (7) and (8), as for example Puig [28], or trying to find one step recursion that provides interval (5) at present instant from previous intervals determined in previous time instants, as for example, El Ghaoui (1999). However, this is not in general an easy task appearing some problems that must be handled, namely: the wrapping effect, the uncertain parameter time-invariance and the determination of global solution of optimisation problems (7) and (8).

2.3 Advanced adaptive thresholding

In this paper, as proposed in Section 2.1, an advanced adaptive thresholding technique will be used based not in bounding directly the residual but instead a residual evaluation function. The residual evaluation function is the residual's energy in a temporal window T:

$$\|r\|_e = \|r\|_{2,t,T} = \sqrt{\int_{\tau-T}^{\tau} r'(t)r(t)dt} \quad (9)$$

where T is a time window. Evaluation function (9) is the most accepted by the frequency domain approaches. In this case, the adaptive threshold J_{th} should be set to be:

$$J_{th} = \sup_{\theta} \|r\|_e \quad (10)$$

This threshold can be computed evaluating in the time interval $t \in [\tau - T, \tau]$, the interval for residual $r(t)$ given by (1). The evaluation of this interval will follow the same methodologies than in the case of the basic adaptive thresholding strategy proposed in Section 2.1.

3 Adaptive thresholding in the frequency domain

Adaptive thresholding in the frequency domain has started with the seminal work of Emami-Naeini [14]. Then, it was followed by Ding [11] and Frank [15]. In these works, the

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[1] In the literature this problem is found in many places: ***Qualitative Reasoning*** [8] [22], ***Constraint Satisfaction*** [10], ***Validated Initial Value Problems*** [7] [21] [23] [24] [25] [26], ***Automatic Control*** [4] [13] [31], ***Fault Detection*** [3] [19] [28] and ***Circuit Tolerance Analysis*** [16] [20].

uncertainty was considered unstructured. But, more recently, Rambeaux [30] and Hamelin [18] have considered the case of structured uncertainty. All these works try to bound the residual's energy taking into account the uncertainty in the model. The first approaches considering unstructured uncertainty use H_∞ techniques to bound the effect of this uncertainty and the second approaches considering structured uncertainty use instead Kharitonov polynomials. In the frequency domain approach, although the considered residual evaluation function is again the residual's energy, it is evaluated in the frequency domain according to:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{R}\|_{2,\omega,W} = \sqrt{\frac{1}{2\pi}\int_{\omega_l - W}^{\omega_l} \boldsymbol{R}'(\omega)\overline{\boldsymbol{R}}(\omega)d\omega} \quad (11)$$

where W is a frequency window.

In order to evaluate (11), the expression for the residual $\boldsymbol{R(s)}$ will be derived. Observer equation (6) can be expressed in input-output form as:

$$\boldsymbol{y}(s) = \boldsymbol{G}_{yu}(s,\boldsymbol{\theta})\boldsymbol{u}(s) \quad (12)$$

taking into account only the effect of parametric uncertainty and ignoring noise and disturbance effects. Then, from (12) the residual can be written, according to Chen [9], as:

$$\boldsymbol{r}(s) = \boldsymbol{W}(s)\Delta\boldsymbol{G}_{yu}(s,\boldsymbol{\theta})\boldsymbol{u}(s) \quad (13)$$

where: $\Delta\boldsymbol{G}_{yu}(s,\boldsymbol{\theta}) = \boldsymbol{G}(s,\boldsymbol{\theta}) - \boldsymbol{G}(s,\boldsymbol{\theta}_o)$ with $\boldsymbol{\theta}_o$ being the nominal parameters and $\boldsymbol{W}$(s) a post-filter.

The threshold value for $\|\boldsymbol{r}\|_e$ using (10) and (13) will be:

$$J_{th} = \sup_{\boldsymbol{\theta}} \left\|\Delta\boldsymbol{G}_{yu}(s,\boldsymbol{\theta})\boldsymbol{u}(s)\right\|_e \quad (14)$$

in case $\boldsymbol{W}(s)=1$. Then:

$$J_{th}^2 = \sup_{\boldsymbol{\theta}} \frac{1}{2\pi}\int_{\omega_l - W}^{\omega_l} \left|\Delta\boldsymbol{G}_{yu}(\omega,\boldsymbol{\theta})\boldsymbol{u}(\omega)\right|^2 d\omega \quad (15)$$

3.1 Adaptive thresholding using H_∞

In the H_∞ framework, it is assumed that

$$\left\|\Delta\boldsymbol{G}_{yu}(\omega,\boldsymbol{\theta})\right\|_\infty < \delta_u \quad (16)$$

according to Chen [9]. Then (15) can be computed as:

$$J_{th}^2 < \frac{\delta_u^2}{2\pi}\int_{\omega_l - W}^{\omega_l} |\boldsymbol{u}(\omega)|^2 d\omega \quad (17)$$

providing an upper bound of (14):

$$J_{th} < \delta_u^2 \|\boldsymbol{u}(\omega)\|_e \quad (18)$$

3.2 Adaptive thresholding using Kharitonov polynomials

On the other hand, the computation (15) requires to determine:

$$\sup_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \left|\Delta\boldsymbol{G}_{yu}(\omega,\boldsymbol{\theta})\right| \quad (19)$$

according to Rambeaux [30]. Moreover, (19) can be computed using Kharitonov polynomials associated with the numerator and the denominator of $\Delta\boldsymbol{G}_{yu}(\omega,\boldsymbol{\theta})$ assuming that the uncertain parameters θ are independent, according to Hamelin [18].

4 Relating time and frequency approaches

Once time and frequency domain approaches to adaptive threshold generation have been presented, they will be related through Parseval's Theorem [2].

4.1 Parseval's Theorem

In Fourier signal analysis, the ***Parseval's Theorem*** establishes a link between the evaluation of the signal's energy in the time and frequency domains, stating:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{r}\|_{2,t} = \|\boldsymbol{R}\|_{2,\omega} \quad (20)$$

where:

$$\|\boldsymbol{r}\|_{2,t} = \sqrt{\int_0^\infty \boldsymbol{r}'(t)\boldsymbol{r}(t)dt} \quad (21)$$

$$\|\boldsymbol{R}\|_{2,\omega} = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{+\infty} \boldsymbol{R}'(\omega)\overline{\boldsymbol{R}}(\omega)d\omega} \quad (22)$$

However, since (21) and (22) are unrealizable in practice due to the residual signal should be known in the interval $t\in[0,\infty]$, the residual evaluation function (20) is, instead, usually realized according to (9).Then:

$$\|\boldsymbol{r}\|_{2,t,T} = \sqrt{\int_{\tau-T}^{\tau} \boldsymbol{r}'(t)\boldsymbol{r}(t)dt} = \sqrt{\int_0^\infty \boldsymbol{r}_T'(t)\boldsymbol{r}_T(t)dt} = \|\boldsymbol{r}_T\|_{2,t} \quad (23)$$

where: $\boldsymbol{r}_T(t)$ is the truncated version of $\boldsymbol{r}(t)$ in the time interval $t\in[t-T,t]$. Then, applying Parseval's Theorem to (23):

$$\sqrt{\int_0^\infty \boldsymbol{r}_T'(t)\boldsymbol{r}_T(t)dt} = \sqrt{\frac{1}{\pi}\int_0^\infty \boldsymbol{R}_T'(\omega)\overline{\boldsymbol{R}}_T(\omega)d\omega} = \|\boldsymbol{R}_T\|_{2,\omega} \quad (24)$$

where: $\boldsymbol{R}_T(\omega)$ is the Fourier Transform of $\boldsymbol{r}_T(t)$. Therefore:

$$\|\boldsymbol{r}_T\|_{2,t} = \|\boldsymbol{R}_T\|_{2,\omega} \quad (25)$$

However, $\|\boldsymbol{R}_T\|_{2,\omega}$ is still unrealizable due to only an evaluation over a finite frequency band W is possible in practice:

$$\|\boldsymbol{R}_T\|_{2,\omega,W} = \sqrt{\frac{1}{\pi}\int_{\omega_l - W}^{\omega_l} \boldsymbol{R}_T'(\omega)\overline{\boldsymbol{R}}_T(\omega)d\omega} \quad (26)$$

what is equivalent to band-limiting the residual by a filter of bandwidth W. Then:

$$\sqrt{\frac{1}{\pi}\int_0^{+\infty} \boldsymbol{R}'_{T,W}(\omega)\overline{\boldsymbol{R}}_{T,W}(\omega)d\omega} = \left\|\boldsymbol{R}_{T,W}\right\|_{2,\omega} \quad (27)$$

being: $\boldsymbol{R}_{T,W}(\omega)$ the Fourier Transform of $\boldsymbol{r}_T(t)$ band-limited to a bandwidth W. Then, according to Parseval's Theorem applied to (27):

$$\left\|\boldsymbol{R}_{T,W}\right\|_{2,\omega} = \left\|\boldsymbol{r}_{T,W}\right\|_{2,t} = \sqrt{\int_{\tau-T}^{\tau} \boldsymbol{r}'_{T,W}(t)\boldsymbol{r}_{T,W}(t)dt} \quad (28)$$

So, adaptive thresholds computed in time and frequency domain would be equivalent even if they were evaluated as in (28), i.e, considering that the residual would be evaluated in a finite time window *T* and in a finite frequency band *W*.

4.2 An example

In this section an example proposed by Ding [12] will be used in order to compare time and frequency domain approaches. Let us consider that the expression for the residual to be evaluated is:

$$R(s) = \Delta G_{yu}(s,\boldsymbol{\theta})U(s) = \frac{1}{\tau s+1} \quad (29)$$

where: $\boldsymbol{\theta} = \tau$ with $\tau \in [\tau_{min}, \tau_{max}]$. Or, alternatively in time domain:

$$r(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}u(t) \quad (30)$$

where: $u(t) = 0$ if $t<0$ and $u(t) = 1$ if $t\geq 0$. Then, applying residual evaluation function (9) in time-domain:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{r}\|_{2,t,T} = \sqrt{\int_{\tau-T}^{\tau} \boldsymbol{r}'(t)\boldsymbol{r}(t)dt} = \\ = \sqrt{\frac{1}{2\tau}\left[(1-e^{-2\frac{t}{\tau}})u(t) - (1-e^{-2\frac{t-T}{\tau}})u(t-T)\right]} \quad (31)$$

Using (31) the optimal threshold will be presented from Fig. 1 with , $\tau_{min} = 2$, $\tau_{max} = 4$ and time window $T = 10$.

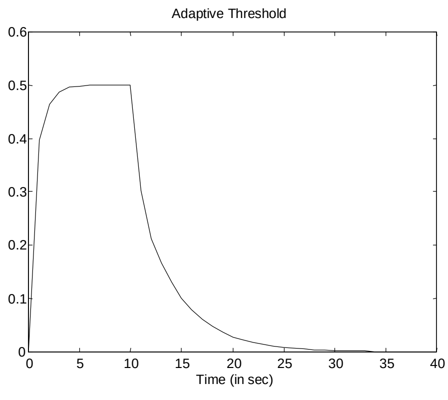

Figure 1: Adaptive threshold in time domain

In case of considering the limits of the integral in interval $t \in [0,\infty)$ as in Parseval's Theorem:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{r}\|_{2,t,T} = \sqrt{\int_0^{\infty} \boldsymbol{r}'(t)\boldsymbol{r}(t)dt} = \frac{1}{\sqrt{2\tau}} \quad (32)$$

Analogously, applying evaluation function (6) in frequency domain:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{R}\|_{2,\omega,W} = \sqrt{\frac{1}{\pi\tau}\int_{\omega_l-W}^{\omega_l} \boldsymbol{R}'(\omega)\overline{\boldsymbol{R}}(\omega)d\omega} = \\ \frac{1}{\pi\tau}(\arctan(\tau W) + \arctan(\tau(\omega_l - W)) \quad (33)$$

assuming that the signal $\boldsymbol{r}(t)$ is known in the time interval: $t \in [0,\infty)$. Comparing this result with (31) it is clear that both evaluation functions provide different results due to the limitation introduced by the frequency window W. However, taking infinite window length in frequency domain:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{R}\|_{2,\omega,W} = \sqrt{\frac{1}{\pi\tau}\int_0^{\infty} \boldsymbol{R}'(\omega)\overline{\boldsymbol{R}}(\omega)d\omega} = \frac{1}{\sqrt{2\tau}} \quad (34)$$

what is not surprising due to Parseval's Theorem.

But in practice, to determine adaptive threshold using (11) some limitations in the time window used to compute $\boldsymbol{R}(\omega)$ should be introduced. Using a time window of length *T* as in (31), (11) provides (assuming than the bandwidth *W* is infinity in order to be comparable with (31))[2]:

- for t<T:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{R}\|_{2,\omega,W} = \sqrt{\frac{1}{\pi}\int_0^{\infty} \boldsymbol{R}'_T(\omega)\overline{\boldsymbol{R}}_T(\omega)d\omega} = \\ = \sqrt{\frac{1}{2\tau}(1-e^{-\frac{2t}{\tau}})(u(t)-u(t-T))} \quad (35)$$

  where: $\boldsymbol{R}_T(\omega) = \int_0^T \boldsymbol{r}(t)e^{-j\omega\tau}d\tau$. Obviously, this expression tends to (32) if the time window used to compute $\boldsymbol{R}\ (\omega)$ tends to infinity.

- for t>T:

$$\|\boldsymbol{r}\|_e = \|\boldsymbol{R}\|_{2,\omega,W} = \sqrt{\frac{1}{\pi}\int_0^{\infty} \boldsymbol{R}'_T(\omega)\overline{\boldsymbol{R}}_T(\omega)d\omega} = \\ = \sqrt{\frac{1}{2\tau}(e^{-2\frac{t-T}{\tau}} - e^{-2\frac{t}{\tau}})u(t-T)} \quad (36)$$

  where: $\boldsymbol{R}_T(\omega) = \int_{t-T}^{t} \boldsymbol{r}(\tau)e^{-j\omega\tau}d\tau$

[2] Otherwise, a band-limited version of the residual should used in computing (31) in order to obtain the same results in frequency and time domain according to (28).

Comparing (35) and (36) with (31) it can be seen that they are completely equivalent. Then, the optimal threshold represented in Fig. 2, with , $\tau_{min} = 2$, $\tau_{max} = 4$ and time window $T = 10$, is the same than the one presented in Fig. 1.

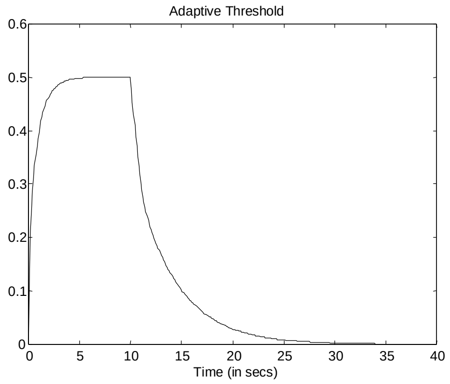

Figure 2: Adaptive threshold in frequency domain

5 Application example

The application example deals with an industrial smart actuator consisting of a flow servovalve driven by a smart positioner, proposed as an FDI benchmark in the European DAMADICS project. The smart actuator consists of a control valve, a pneumatic servomotor and a smart positioner [5]. Using physical modelling [5] and a mixed optimisation-identification algorithm, the following linear discrete-time interval model for booster, E/P transducer, servomotor and displacement transducer has been derived using a sample time of 1s:

$$\begin{aligned} &\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) \\ &y(k) = x_3(k) \end{aligned} \tag{37}$$

with: $\boldsymbol{x}(k) = [x_1(k) \quad x_2(k) \quad x_3(k)]^t$,

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\theta_3 & -\theta_2 & -\theta_1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \theta_4 \end{bmatrix} \quad \text{and}$$

$$\boldsymbol{u}(k) = CVP(k-2)$$

where: $x_3(k)$ is the position of the valve, $y(k)$ is this position measured by the displacement transducer (in Volt), $CVP(k)$ is the command pressure (in Pascal) and the uncertain parameters are bounded by their confidence intervals according to: $\left[\bar{\theta}_i - \alpha\sigma_{\theta_i}, \bar{\theta}_i + \alpha\sigma_{\theta_i}\right]$ with α=0.05, being:

$\bar{\theta}_i$	σ_{θ_i}
-1.1444	0.0549
-0.4049	0.1078
0.5510	0.0530
0.2182e-3	0.0030e-3

Using this interval model, a linear interval observer, as (12), with $L=[-0.9852 \quad 1.9166 \quad -0.9087]$ will be used to detect faults f_1 and f_{10} of the DAMADICS benchmark.

5.1 Fault f_{10} ("diaphragm perforation")

In this case, a fault in the pneumatic servomotor is introduced. The fault consists in servomotor's diaphragm perforation caused by fatigue of diaphragm material. In the DAMADICS benchmark this fault is named as f_{10}. In the present experiment the fault scenario that will be used corresponds to the abrupt big size [5]. The fault appears at time instant $t=600s$. In Fig. 3, results from residual evaluation using an adaptive threshold evaluated in time-domain (solid line) according to (9) and in frequency domain (dashed line) according to (11) are presented. Comparing the results from time-domain and frequency-domain evaluation it can be seen that both provide very similar values. The little difference observed between the two approaches is due to when residual signal present high frequency frequencies (oscillations) the sample time of 1 s is not enough to have a good discretisation. Frequency domain results would improve if sample time could be decreased, been closer to time-domain results[3]. The residual energy goes out the threshold but due to the partial measure correction comes back inside the envelope after some time instants. However, the persisting fault during 70 seconds (from t=600s to t=670s) is enough to detect a faulty behaviour of the system. Moreover, is almost instantaneously detected, which is a good performance for an earlier fault detection of this fault.

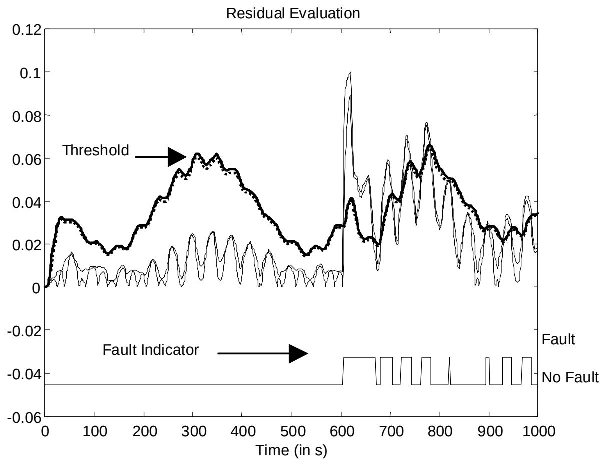

Figure 3: Residual Evaluation in Presence of Fault f_{10}

5.2 Fault f_1 ("valve clogging")

In this case, a fault in the control valve is introduced. The fault is a "valve clogging". It consists in blocking servomotor rod displacement by an external mechanical nature event. In the DAMADICS benchmark this fault is named as f_1. In the

3 In fact, when implementing the spectral evaluation of signal in discrete-time the signal is band-limited to π/T_s, where T_s is the sampling time. Then, in order to obtain the same results in time and frequency domain, the residual in time-domain should be filtered to be band-limited to a band compatible with this sampling time.

present experiments the fault scenario that will be used corresponds to the abrupt big size [5]. The fault appears at time instant $t=600$. According to Fig. 4, fault f_1. is indicated almost instantaneously and is persistently indicated during 300 seconds (from $t=600s$ to $t=900s$). These results allow to confirm conclusions from experiments in presence of fault f_{10}.

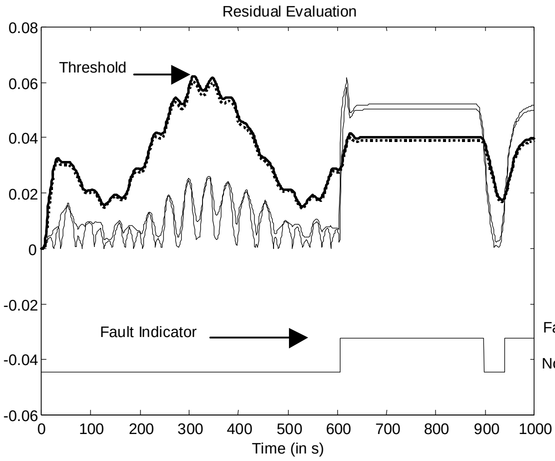

Figure 4: Residual Evaluation in Presence of Fault f_1

6. Conclusions

In this paper, interval models are proposed as means to produce adaptive thresholds in robust fault detection using techniques in time and frequency domains. In the literature, interval models have been used to produce such adaptive thresholds using one or the other technique but never before has been put together defining a common evaluation index and comparing them. Thanks to Parseval's Theorem, it can be concluded that from the theoretical point of view both approaches produce the same results, but in practice some difficulties appear when implementing them that prevent from reaching this ideal results. Finally, an application example based on DAMADICS benchmark is proposed as the test case to compare results coming from the application of both approaches.

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