FAULT DECOUPLING VIA GENERALIZED OUTPUT INJECTION

Cédric Join*, **J-Christophe Ponsart***, **Dominique Sauter*** **and Hicham Jamouli***

Centre de Recherche en Automatique de Nancy (CRAN), CNRS UMR 7039
Université Henri Poincaré, Nancy 1,
BP 239, F-54506 Vandœuvre Cedex, France,
Phone: (33) 3 83 68 44 61 - Fax: (33) 3 83 68 44 62
e-mail:cedric.join@cran.uhp-nancy.fr

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Abstract

This paper deals with the problem of fault (or disturbance) decoupling in nonlinear systems. A new method is proposed to increase dimension of state subspace which is insensitive to fault (or disturbance). This method is based on a nonlinear filter defined by means of the generalized output injection. An example is presented which illustrates results.

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1 Introduction

In this paper, we consider nonlinear model based Fault Detection and Isolation (F.D.I.), that is methods designed on the basis of an explicit mathematical system model described by nonlinear differential equations (or equivalent representations). Considering the control problem, it is now well admit that accounting for nonlinear system behavior yields increased performances. Thus, in the F.D.I. framework, it is expected that such a nonlinear approach may lead as well to an increased process availability.
In this paper, we focus on one of the most challenging problem of F.D.I., namely the fault disturbances decoupling problem. Basically, we aim at designing a F.D.I. procedure that is sensitive to a particular fault while remaining insensitive to disturbances (including the other kinds of faults).
Wonham ([1]), Massoumnia and *al* ([2, 3]), have shown that the addressed problem may be solved in the general setting of linear systems. Isidori and *al* ([4, 5]) extend these works to the decoupling problem by dynamic feedback in nonlinear control scheme while [6, 7] consider the same decoupling problem in the F.D.I. framework. In this framework, we note that decoupling is achieved by an output injection.
In this paper, a new decoupling method is proposed. The main idea of this approach is to use generalized output injection [8, 9] to increase the dimension of a particular state subspace: the insensitive state subspace to a fault (or perturbation).
The paper is organized as follows. In section 2, we formally state the decoupling problem in a nonlinear setting. In section 3, the De Persi and Isidori approach is recalled; a special attention is paid to the particular output injection used. Section 4 gives the main contribution of the paper which is the fault decoupling method by generalized output injection. It is based on a geometrical approach ([1, 7, 4]). The generalized output injection results is an increase of the insensitive state subspace dimension and it is shown that both upper and lower bounds of the subspace dimension increase yielding an improve state estimation. An exemple highlights the interest of the method, in last section.

2 Problem statement

Let us consider the following model for system description:

$$\Sigma_{NL} : \begin{cases} \dot{x} = f_0(x) + \sum_{i=1}^{m} f_i(x)u_i + \sum_{j=1}^{q} P_j(x)w_j \\ y = h(x) \end{cases} \tag{1}$$

where $x(t) \in \mathcal{X} = \mathbb{R}^n$, $u(t) \in \mathcal{U} = \mathbb{R}^m$, $y(t) \in \mathcal{Y} = \mathbb{R}^p$ and $w(t) \in \mathcal{W} = \mathbb{R}^q$ are respectively states, inputs, outputs and faults disturbances. The functions $f_*(\cdot)$, $h(\cdot)$, $P_*(\cdot)$ are matrix-valued differentiable ($\mathcal{C}^\infty$) and all of appropriate dimensions.
We can add that all faults can be written on state differential equation. According to a state transformation, actuator, system and more precisely sensor faults may be included in equation (1). More precisions are given in ([3], [10] and [11]).
The main problem addressed in this paper is fault disturbances decoupling in order to design a filter making possible fault isolation. Such a filter exhibits the following characteristics:

- a part of state is sensitive to fault,
- the other part of state is insensitive to fault.

Let us introduce the following filter:

$$\Sigma_{FD} : \begin{cases} \dot{z} = f_0(z) + \sum_{i=1}^{m} f_i(z)u_i + \Psi(\cdot_x, \cdot_z, \cdot_u) \\ y_z = h(z) \end{cases} \tag{2}$$

where z is an estimation of state vector x and $\Psi(\cdot_x, \cdot_z, \cdot_u)$ is the generalized output injection. We assume:

- $\cdot_x$ is connected with system equation (as for example y_x, $\dot{y}_x$, ...),
- $\cdot_z$ is connected with filter equation (as for example y_z, $\dot{y}_z$, ...),

- $\cdot_u$ is connected with control input (and possibly its derivations).

In order to determine which part of state is considered as sensitive or insensitive to fault, we use the sensitivity definition ([5]) introduced by A. Isidori. A part of state (or state subspace) is defined as insensitive (resp. sensitive) to fault if and only if $x - z = 0,\ \forall w \neq 0$, that is to say that z is close to x (resp. $x - z \neq 0,\ \forall w \neq 0$).
Filter (2) allows to isolate faults by considering a sensitive part and an insensitive part of state. The fault decoupling is obtained according to an output injection. To a particular output injection form is assigned a state subspace insensitive to a fault. With a "$suitable$" choice of output injection, the greater insensitive state subspace to a fault can be obtained. That is our proposition by using derivations of the different signals in the output injection. In the next section, the output injection form De Persis is recalled, and then, our work is detailed.

3 Current decoupling method

This method ([6] and [7]) is based on the following particular output injection form:

$$\Psi(\cdot_x, \cdot_z, \cdot_u) = \Psi(y_x, z, u) - \Psi(y_z, z, u) \tag{3}$$

The minimal invariant state subspace sensitive to the fault w via an output injection is defined by the non-decreasing sequence:

$$\begin{cases} S_0^P = \text{span}\{P\} \\ S_{i+1}^P = \overline{S}_i^P + \displaystyle\sum_{k=0}^{m} [f_k, \overline{S}_i^P \cap \ker\{dh\}] \end{cases} \tag{4}$$

where $\overline{S}_i^P$ denotes the *involutive closure* of the distribution S_i^P, i.e. if $\tau, \sigma \in \overline{S}_i^P$ then $[\tau, \sigma] \in \overline{S}_i^P$ with $[\tau, \sigma]$ the Lie bracket. The stopping conditions of the sequence (4) are:

$$\left.\begin{array}{r} \overline{S}_i^P = S_{i+1}^P \\ \dim(\text{span}\{S_i^P\}) = n \end{array}\right\} \Rightarrow S_*^P = \overline{S}_i^P \tag{5}$$

The greater state subspace insensitive to the fault w denoted by $(S_*^P)^\perp$ is then obtained. A diffeomorphism is defined as $\tilde{x} = \Phi(x)$ with $\frac{\partial \Phi}{\partial x} = \begin{bmatrix} (S_*^P)^T \\ \left((S_*^P)^\perp\right)^T \end{bmatrix}$. Hence, system (1) can be written as:

$$\tilde{\Sigma}_{NL} : \begin{cases} \dot{\tilde{x}}_1 = \tilde{f}_{0,1}(\tilde{x}_1, \tilde{x}_2) + \displaystyle\sum_{i=1}^{m} \tilde{f}_{i,1}(\tilde{x}_1, \tilde{x}_2) u_i \\ \qquad + \tilde{P}(\tilde{x}_1, \tilde{x}_2) w \\ \dot{\tilde{x}}_2 = \tilde{f}_{0,2}(\tilde{x}_1, \tilde{x}_2) + \displaystyle\sum_{i=1}^{m} \tilde{f}_{i,2}(\tilde{x}_1, \tilde{x}_2) u_i \\ y_x = \tilde{h}(\tilde{x}_1, \tilde{x}_2) \end{cases} \tag{6}$$

By definition of S_*^P, it is always possible to find an output injection such that:

$$\begin{aligned} &\tilde{f}_{0,2}(\tilde{x}_1, \tilde{x}_2) + \sum_{i=1}^{m} \tilde{f}_{i,2}(\tilde{x}_1, \tilde{x}_2) u_i = \\ &\bar{f}_{0,2}(\tilde{x}_2) + \sum_{i=1}^{m} \bar{f}_{i,2}(\tilde{x}_2) u_i + \tilde{\Psi}(\tilde{x}_2, y_x, u) \end{aligned} \tag{7}$$

that is to say that $\tilde{\Psi}(\tilde{x}_2, y_x, u)$ expresses the $\tilde{x}_1$ contribution on $\dot{\tilde{x}}_2$ equation.
A nonlinear filter is deduced:

$$\tilde{\Sigma}_{FD} : \begin{cases} \dot{\tilde{z}}_1 = \tilde{f}_{0,1}(\tilde{z}_1, \tilde{z}_2) + \displaystyle\sum_{i=1}^{m} \tilde{f}_{i,1}(\tilde{z}_1, \tilde{z}_2) u_i \\ \dot{\tilde{z}}_2 = \bar{f}_{0,2}(\tilde{z}_2) + \displaystyle\sum_{i=1}^{m} \bar{f}_{i,2}(\tilde{z}_2) u_i + \tilde{\Psi}(\tilde{z}_2, y_x, u) \\ y_z = \tilde{h}(\tilde{z}_1, \tilde{z}_2) \end{cases} \tag{8}$$

We can note that $\tilde{z}_2$ is insensitive to the fault whereas $\tilde{z}_1$ is sensitive to w.
By using output injection (3), S_*^P is overvalued by fault dimension. And related to $\dim(\ker\{dh\})$, insensitive state subspace dimension is undervalued by zero. That is to say:

$$\begin{array}{c} \tilde{\Psi}(\tilde{z}_2, y_x, u) \\ \Downarrow \\ 0 \leq \dim(\text{span}\{(S_*^P)^\perp\}) \leq n - \dim(w) \end{array} \tag{9}$$

In the next section, we propose to use an other output injection form to increase dimension of the insensitive subspace (lower and upper bounds of (9)). This will guarantee a greater dimension of exact state estimation for all faults.

4 Decoupling method via a generalized output injection

The aim of this section is to increase the size of the insensitive state subspace according to a generalized output injection. The design is based on the $reconstructibility$ definition introduced in ([11] and [12]). To facilitate comprehension, we introduce the Δ-reconstructibility definition as follows:

Definition 1 *A signal δ is said Δ-reconstructible if, and only if, it exists a nonlinear fonction Ψ such that:* $\delta = \Psi(\Delta)$.

It is to be noticed that if Δ and δ represent respectively known signals of a process (inputs and outputs) and the fault, it is possible to estimate this fault by output combinations. Our proposition is based on a two steps strategy: the first step is the increase of the upper bound of (9) according to fault estimation; the second one is the increase of the lower bound of (9) according to a novel non-decreasing sequence.

4.1 First step : upper bound increasing

A necessary condition to fault reconstructibility is fault detectability.
If faults can be estimated according to outputs and/or their derivations, then these faults affect outputs and/or their derivations.
A necessary condition to fault estimate is given using the following non-decreasing sequence and the observable space (noted γ_{obs}, defined in [1], [4] and [13]):

$$\begin{cases} C_0^{P_j} = \text{span}\{P_j\} \\ C_{i+1}^{P_j} = \overline{C}_i^{P_j} + \sum_{k=1}^{m} [f_k, \overline{C}_i^{P_j}] \end{cases} \tag{10}$$

The stop conditions of the sequence (10) are:

$$\left.\begin{array}{l} C_i^{P_j} = C_{i+1}^{P_j} \\ \dim(\text{span}\{C_i^{P_j}\}) = n \end{array}\right\} \Rightarrow C_*^{P_j} = C_i^{P_j} \tag{11}$$

$C_*^{P_j}$ expresses the fault w_j propagation within nonlinear states, i.e. the fault w_j affects the state subset $C_*^{P_j}$.
So, if $\gamma_{obs} \nsubseteq (C_*^{P_j})^\perp$ then fault is detectable. More precisions are given in ([14]), and we recall that:

Theorem 1 *The condition $\gamma_{obs} \nsubseteq (C_*^{P_j})^\perp$ $(\forall j)$ is:*

- *a necessary and sufficient condition, if $w_j \in \mathbb{R}^{\mathbf{1}}$*
- *a necessary condition, if $w_j \in \mathbb{R}^{\mathbf{k}}$ (with $\mathbf{k} > \mathbf{1}$) to achieve the detectability of all faults.*

With fault detectability condition, fault estimation (according to known system signals) must be studied in order to increase insensitive state subspace. A general case is first detailled and then, a particular case is considered.

4.1.1 Necessary and sufficient condition for fault estimation via an output injection

Let us define:

$$\Delta_e = \Big[y_{x1}^0, \ldots, y_{x1}^{(\rho_1^w-1)}, \ldots, y_{xp}^0, \ldots, y_{xp}^{(\rho_p^w-1)}, u, \ldots, u^{(max-1)}, \xi_1(Y_x, Y_{x1}), \ldots, \xi_l(Y_x, Y_{xl}), \tilde{x}_{N2}\Big] \tag{12}$$

where ρ_i^w is the characteristic indice of fault w associated to the output i, i.e. the first indice of output derivation showing the fault ($\rho_i^w = min_j(\frac{\partial y_i^{(j)}}{\partial w} \neq 0)$). $Y_x = \Big[y_{x1}^0, \ldots, y_{x1}^{(\rho_1^w-1)}, \ldots, y_{xp}^0, \ldots, y_{xp}^{(\rho_p^w-1)}\Big]$, and $Y_{x1}, \ldots, Y_{xl}$ are signals made up of outputs with derivation indices greater than $\rho_i^w - 1$. $\xi_1, \ldots, \xi_l$ are nonlinear functions where fault and its derivations do not appear. $\tilde{x}_{N2}$ represents the part of state naturally decoupled of fault (i.e., the state part associated with $(C_*^P)^\perp$). Finally, max points out the maximum index of outputs derivation. With these notations, the following conclusion can be given:

Theorem 2 *For all output injections (i.e. $\Psi(\cdot_x, \cdot_z, \cdot_u)$), a fault w is said estimable if, and only if, w is Δ_e-reconstructible.*

Proof of Theorem 1 *Proof is trivial considering generalized output injection use and thus no signals restriction (Δ_e).*

In the case of Δ_e-reconstructibility of w, it exists an output injection $\Psi(\Delta_e)$ such that $w = \Psi(\Delta_e)$. Thus, the following filter can be synthesized:

$$\Sigma_{FD} : \begin{cases} \dot{z} = f_0(z) + \sum_{i=1}^{m} f_i(z)u_i + P(z)\Psi(\Delta_e) \\ y_z = h(z) \end{cases} \tag{13}$$

With the same considerations, theorem 2 can be generalized in the case of $P(x)w$:

Theorem 3 *For all output injections (i.e. $\Psi(\cdot_x, \cdot_z, \cdot_u)$), a fault w effect is said estimable if, and only if, $P(x)w$ is Δ_e-reconstructible.*

Proof of Theorem 2 *The proof is the same one as for theorem 2 by replacing w by $P(x)w$.*

In the Δ_e-reconstructibility of $P(x)w$ case, it exists $\Psi(\Delta_e)$ such that $P(x)w = \Psi(\Delta_e)$. Consequently, the following filter can be designed:

$$\Sigma_{FD} : \begin{cases} \dot{z} = f_0(z) + \sum_{i=1}^{m} f_i(z)u_i + \Psi(\Delta_e) \\ y_z = h(z) \end{cases} \tag{14}$$

where $\Psi(\Delta_e)$ is the output injection.

The following remark shows that the condition of fault effect Δ_e-reconstructibility is less restrictive than fault Δ_e-reconstructibility. Indeed, if some components of $P(x)$ are inobservable, these components of the fault vector w is not Δ_e-reconstructible. But this does not imply the no Δ_e-reconstructibility of fault effect.
Consequently, a necessary and sufficient condition to a totaly fault decoupling system is:

Theorem 4 *A nonlinear system is said totaly fault decoupling if, and only if, the fault effect is Δ_e-reconstructible.*

Proof of Theorem 3 *Proof is given previously and precisely with filter writing (14), according to this equation all components of state vector x (“totaly”) is estimated by z for all fault w.*

If, and only if, theorem 4 is satisfied then the sensitive state subspace is reduced to $\{0\}$. Consequently the upper bound of $(S_*^P)^\perp$ dimension is increased to n and no more to $n-\dim(w)$ as it was in (9).
The generalized output injection used to satisfy this condition is very theorical. In practice, nonlinear functions ξ_i (defined in (12)) are particularly difficult to design. Nevertheless, it is possible to obtain decoupling conditions with a less generalized output injection than above mentioned (but more than output injection (3)). This is the aim of the next paragraph.

4.1.2 Sufficient condition for fault estimation via output injection

Since signals used to filter design are truncated, the necessary and sufficient condition becomes a sufficient condition. Indeed, output injection is not generated with Δ_e (12) but with:

$$\Delta_t = \Big[y_{x1}^0, \dots, y_{x1}^{(\min_j(\rho_1^{w_j})-1)}, \dots, y_{xp}^0, \dots, y_{xp}^{(\min_j(\rho_p^{w_j})-1)}, u, \dots, u^{(\max_i(\min_j \rho_i^{w_j})-1)}, \tilde{x}_{N2}\Big] \quad (15)$$

where $\rho_i^{w_j}$ is the characteristic indice associated with fault j and output i, that is to say that $\rho_i^{w_j} = min_k(\frac{\partial y_i^{(k)}}{\partial w_j} \neq 0)$ and other components are already defined.
Since the output injection is composed by signals Δ_t, then it is possible to give algebraic condition for fault estimation and it is the main objectif of this section.

We introduce a new matrix, named output sensibility matrix, composed by output derivations showing each fault component such that:

$$M^P = \begin{bmatrix} y_{x1}^{(\rho_1^{w_1})} & \dots & y_{x1}^{(\rho_1^{w_q})} \\ \vdots & \ddots & \vdots \\ y_{xp}^{(\rho_p^{w_1})} & \dots & y_{xp}^{(\rho_p^{w_q})} \end{bmatrix} \quad (16)$$

If an output (y_{xi}) is naturally (w_j) fault decoupled, then $\rho_i^{w_j} = \infty$ and $y_{xi}^{(\rho_i^{w_j}-1)}$ is considered as null.
It is to be noticed that by faults detectability assumption, it exists at least one matrix element which is different from zero. That is to say, it exists a pair (i,j) such that:

$$\begin{aligned} M_{(i,j)}^P(x,U,w) &= y_{xi}^{(\rho_i^{w_j})} \\ &= \bar{f}_{0,i}(x,\bar{w}_j) + \bar{f}_{1,i}(x,U,\bar{w}_j) + \bar{P}_i(x,U,\bar{w}_j)w_j \end{aligned} \quad (17)$$

with $U = [u, \dots, u^{(\rho_i^{w_j}-1)}]$ and $\bar{w}_j = (w_1, \cdots, w_{j-1}, w_{j+1}, \cdots, w_q)$.
To simplify the study, we choose to transform the matrix (16) according to the following choice:

- $M_{s(i,j)}^P = 1$ if the both conditions (i) and (ii) below are satisfied,
- $M_{s(i,j)}^P = 0$ if one of both conditions (i) and (ii) below is not satisfied.

Conditions (i) and (ii) are:
(i) the pair $\left(M_{(i,j)}^p(x,U,\bar{w}_j,w_j=0), \frac{\partial M_{(i,j)}^p(x,U,\bar{w}_j,w_j)}{\partial w_j}\right)$ is Δ_t-reconstructible,
(ii) $\rho_i^{w_j} = \min_j(\rho_i^{w_j})$.

The first condition (i) is equivalent to the possibility to estimate a part of fault w_j. However, the objective is the fault vector estimation independently of the other faults (condition (ii)).
To conclude comments, we can add that this simplified sensibility matrix is a binary matrix (with some "0" and "1").
With these considerations, necessary and sufficient condition of theorem 2 is reduced to sufficient condition for fault estimation:

Theorem 5 *If the rank condition:*
$$rank(M_s^P) = \dim(w) = q$$
is satisfied, then, a fault w is said estimable with the particular output injection $\Psi(\Delta_t)$.

Proof of Theorem 4 *This theorem is based on the two conditions (i) and (ii) previously defined but the signals set considered is a truncation of Δ_e. Thus, it is only a sufficient condition.*

If we focus on one fault w_j, and if the condition (i) is satisfied, then it exists two functions ζ_1 and ζ_2 such that $M_{m(i,j)}^p(x,U,\bar{w}_j,w_j) = y_i^{(\rho_{mi}^{w_j})} = \zeta_1(\Delta_t) + \zeta_2(\Delta_t)w_j$. With signals Δ_t, independent estimations are only possible if condition (ii) is satisfied. The aim is to estimate all faults w_j independently, so the conditions (i) and (ii) must be satisfied $\forall j$. It is equivalent to the rank condition of theorem 5.

If theorem 5 is satisfied then the sensitive state subspace is reduced to $\{0\}$. Consequently the upper limit of dimension is increased to n and not $n-\dim(w)$ as it was in (9).

The first step of the decoupling method by generalized output injection has been studied in this section. The second step concerns the decrease of the lower bound of the insensitive subspace and is developed in the next section.

4.2 Second step : lower bound increasing

In order to follow the fault propagation through the state subspace, another non-decreasing sequence (different from 4) is defined. We begin to pose Δ^0 equal to Δ_e. And we consider some components of fault vector which are not Δ^0-reconstructible (or Δ_e-reconstructible) noted $\bar{S}_{m,1}^P$:

$$\bar{S}_{m,1}^P = \bar{S}_{m\overline{\Delta^0}}^P \quad (18)$$

where $\bar{S}^P_{m\overline{\Delta^0}}$ represents the involutive state subspace generated by span$\{P\}$ no Δ^0-reconstructible.
So, this state part propagates fault effect through the state subspace. This propagation can be calculated by a non-decreasing sequence defined by:

$$S^P_{m,k+1} = \bar{S}^P_{m,k} + \sum_{i=0}^{m} \left[\tilde{f}^k_i(\tilde{x}^k_1, \tilde{x}^k_2), S^P_{m\overline{\Delta^k}}\right] \tag{19}$$

It is always possible ([5], according to Frobenius Theorem) to find a diffeomorphism $\tilde{x}^k = \Phi^k(x)$ such as:

$$\frac{\partial \Phi(x)^k}{\partial x} = \begin{pmatrix} (\bar{S}^P_{m,k})^T \\ \left((\bar{S}^P_{m,k})^\perp\right)^T \end{pmatrix} \tag{20}$$

The nonlinear system (1) can be transformed:

$$\tilde{\Sigma}_{NL} : \begin{cases} \dot{\tilde{x}}^k_1 = & f_0(\tilde{x}^k) + \sum_{i=1}^{m} f_i(\tilde{x}^k)u_i + \sum_{j=1}^{q} P(\tilde{x}^k)w_j \\ \dot{\tilde{x}}^k_2 = & f_0(\tilde{x}^k) + \sum_{i=1}^{m} f_i(\tilde{x}^k)u_i \\ y = & h(\tilde{x}^k) \end{cases} \tag{21}$$

where the state subspace associated with $\dot{\tilde{x}}^k_1$ noted $d\dot{\tilde{x}}^k_1$ (equivalent to $\frac{\partial \dot{\tilde{x}}^k_1}{\partial x}$, or $\bar{S}^P_{m,k}$) is the state subspace to express. Thus $S^P_{m\overline{\Delta^k}}$ in (19) represents the subspace part of $d\dot{\tilde{x}}^k_1$ no Δ^k-reconstructible with:

$$\begin{aligned} \Delta^k = \Big[& y^0_{x1}, \ldots, y_{x1}^{(\min_j(\rho^{w_j}_{m1})-1)}, \ldots, \\ & y^0_{xp}, \ldots, y_{xp}^{(\min_j(\rho^{w_j}_{mp})-1)}, u, \ldots, u^{(max-1)}, \\ & \xi_1(Y_x, Y_{x1}), \ldots, \xi_l(Y_x, Y_{xl}), \tilde{x}^k_2 \Big] \end{aligned} \tag{22}$$

After several iterations and the same stop conditions as (5), the smaller state subspace S^P_{m*} sensitive to w via a generalized output injection is deduced. Since $\Psi(\cdot_x, \cdot_z, \cdot_u)$ is more general than (3), then S^P_{m*} $S^P_{m*} \subseteq S^P_*$ and the more interesting result $\dim(\text{span}\{(S^P_{m*})^\perp\}) \geq \dim(\text{span}\{(S^P_*)^\perp\})$.
In the following section, the previous method is applied.

5 Example

In this section, an example (inspired from ([15])) is considered in order to highlight the interest of a generalized output injection in the fault decoupling problem. The system under consideration is represented by the following equations:

$$\Sigma_{NL1} : \begin{cases} \dot{x} = \begin{pmatrix} x_1 x_4 \\ x_3(1-x_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & x_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 + w \end{pmatrix} \\ y_x = \begin{pmatrix} x_1 & x_3 \end{pmatrix}^T \end{cases} \tag{23}$$

with, $x(t) \in \mathcal{X} = \mathbb{R}^4$, $u(t) \in \mathcal{U} = \mathbb{R}^2$, $y(t) \in \mathcal{Y} = \mathbb{R}^2$ and $w(t) \in \mathcal{W} = \mathbb{R}^1$ respectively states, inputs, outputs and the unknown disturbance (an actuator fault in this case). This system is not observable and the inobservable subspace is $\gamma_{inobs} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$.
Using the non-decreasing sequence (10), we obtain:

$$C^P_* = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ x_1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -x_1(1-x_4) \\ x_1 x_4 \\ 0 \end{bmatrix} \right\}$$

It can be deduced that $C^P_* \nsubseteq \gamma_{inobs}$, thus the fault w is detectable. According to output derivation, the fault w can be estimated as follows: $w = (\dot{y}_{x2} - y_{x1}u_2)/y_{x1}$.
A nonlinear filter totaly decoupled of fault w is:

$$\begin{cases} \dot{z} = \begin{pmatrix} z_1 z_4 \\ z_3(1-z_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & z_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_1 \\ 0 \end{pmatrix} \left\{ \frac{\dot{y}_{x2} - y_{x1}u_2}{y_{x1}} \right\} \\ y_z = \begin{pmatrix} z_1 & z_3 \end{pmatrix}^T \end{cases} \tag{24}$$

where $\Psi(\Delta_e) = (\dot{y}_{x2} - y_{x1}u_2)/y_{x1} = w$.
On one hand assuming that $x(0)$ is known the state estimation (z) is always exact ($z = x$) $\forall w$ that is to say (c.f. equation (9) for comparison) :

$$\dim(\text{insensitive state subspace to } w) = n = 4 \tag{25}$$

Particularly we stress on the insensitive to w output estimation of y_{x2}.
On an other hand the convergence of z to x must to be studied, but it is not the paper objectif. However, it can be added that an intuitive method based on contraction analysis [16, 17] can be used for this study.
With current decoupling method (section 3), we obtain:

$$(S^P_*) = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ x_1 \\ 0 \end{bmatrix} \right\} \quad (S^P_*)^\perp = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

A nonlinear filter decoupling fault is:

$$\begin{cases} \dot{z} = \begin{pmatrix} y_{x1} z_4 \\ z_3(1-z_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & z_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ y_z = \begin{pmatrix} z_1 & z_3 \end{pmatrix}^T \end{cases} \tag{26}$$

where $\Psi(z, y_x, u) - \Psi(z, y_z, u) = y_{x1}z_4 - y_{z1}z_4$ (c.f. equation (3)). Compared with (24), nonlinear filter (26) results are less impressive, because only a reduced state vector is estimated without errors. Indeed, even if $x(0) = z(0)$ only state estimations $[z_1\ z_2\ z_4]^T$ are always exact $\forall w$ (to compare with (25)):

$$\dim(\text{insensitive state subspace to } w) = n - \dim(w) = 3 \tag{27}$$

Particularly we stress on the sensitive to w output estimation of y_{x2}.

To conclude this example, results are summed up as follows with the table TAB. 1.

Method using \ Size of	Insensitive states	insensitive outputs
generalized output injection	4	2
classic output injection	3	1

Table 1: Results comparaison

6 Conclusion

This paper focus on fault decoupling method for nonlinear system. A decoupling method is proposed by means of a generalized output injection in order to increase the dimension of the decoupling state subspace. This output injection is generated from known signals and their derivations. Thus the decoupling state part sensitive to a fault is decreased. This ensures a correct estimation of a larger part of the state for all faults. Moreover, an example emphasizes the interest of this work.

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