OPTIMAL SENSOR LOCATION FOR FAULT DETECTION AND ISOLATION IN LINEAR STRUCTURED SYSTEMS

C. Commault, J-M. Dion

Laboratoire d'Automatique de Grenoble ,
ENSIEG-BP 46, 38402 Saint Martin d'Hères Cedex, FRANCE
email : {Christian.Commault, Jean-Michel.Dion}@inpg.fr

Keywords: Linear systems, Fault detection and isolation, Sensor location, Graph

Abstract

We consider here the Fault Detection and Isolation (FDI) problem for linear systems. We design a set of observer-based residuals, in such a way that the transfer from the faults to the residuals is diagonal. We deal with this problem when the system under consideration is structured, that is, the entries of the system matrices are either fixed zeros or free parameters. This problem was solved recently in terms of the graph that one can associate in a natural way to a structured system. We are concerned in this paper with the case where the FDI solvability conditions are not satisfied. We consider that some internal states can be measured at some cost and wonder if the problem is solvable with these new measurements. If this is possible we try to reach the solvability conditions at minimal cost. These problems are solved using the bipartite graph of the system.

1 Introduction

This paper is concerned with the Fault Detection and Isolation (FDI) problem for linear systems. This problem has received considerable attention in the past ten years [3, 4, 10, 16].
In this paper we consider the observer based FDI problem using structured residual sets that allow fault isolation. We are interested in obtaining a transfer from faults to residuals with a diagonal structure (i.e. a dedicated structured residual set). We consider for these problems intrinsic solvability conditions depending on the internal structure of the system and not on the specific values of the parameters. We look for internal structures which are well suited for diagnosis. An interesting tool for this purpose is the notion of structured system [14]. The solvability conditions for the bank of observers-based FDI problem were given recently in terms of the graph that can can be associated in a natural way to a structured system [6].
In general the FDI solvability conditions are not satisfied. These conditions are restrictive and we consider that some new sensors can be implemented at some cost and wonder if the problem is solvable with these new measurements. If this is possible we try to reach the solvability conditions at minimal cost. These problems are solved through the bipartite graph of the system. The results are expressed in standard combinatorial optimization terms. Note that the sensor location problem has received a lot of attention in system observation, supervision and abrupt changes detection, see for example [2, 1]. The structural and graphical approaches have also been used in fault detection problems but in a context which is different from ours, see [17].

The outline of this paper is as follows. The problem is formulated in section 2. The linear structured systems are presented in section 3. First results on the possibility to reach the FDI conditions by adding new measurements are given in section 4. In section 5 we consider that new measurements have a cost and solve the minimal cost sensor location problem for FDI. Some concluding remarks end the paper.

2 Problem formulation

2.1 Observer-based FDI problem

Let us consider the following linear time-invariant system :

$$\Sigma \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f(t) \in \mathbb{R}^r$ the fault vector and $y(t) \in \mathbb{R}^p$ the measured output vector. A, C, L and M are matrices of appropriate dimensions.
Note that the control input effects are not considered here as, for any observer-based FDI problem, it is well known that these can be taken into account in the observer structure without loss of generality.
A dedicated residual set is designed using a bank of r observers for system (1), according to the dedicated observer scheme [3]. Each residual will be designed to be sensitive to a single fault while remaining insensitive to the other faults. The ith observer of this bank of r observers is designed for a system of type (1) as follows:

$$\dot{\hat{x}}^i(t) = A\hat{x}^i(t) + K^i(y(t) - C\hat{x}^i(t)) \tag{2}$$

where $\hat{x}^i(t) \in \mathbb{R}^n$ is the state of the ith observer, K^i is the observer gain to be designed such that $\hat{x}^i(t)$ asymptotically converges to $x(t)$, when no fault is considered.
The residuals are defined as :

$$r_i(t) = Q^i(y(t) - C\hat{x}^i(t)), \text{ for } i = 1, \dots, r \tag{3}$$

where Q^i is a $1 \times p$ matrix.

Definition 1 *The bank of observers-based FDI problem consists in finding, if possible, matrices K^i and Q^i, such* *that, for $i = 1, 2, \ldots, r$, $A - K^i C$ is stable, and the fault to residual transfer matrix is non zero, proper and diagonal, i.e. the transfer form the faults to the residuals has the form*

$$r(s) = \begin{bmatrix} t_{11}(s) & 0 & \cdots & 0 \\ 0 & t_{22}(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & t_{rr}(s) \end{bmatrix} f(s) \quad (4)$$

where $t_{ii}(s) \neq 0$ for $i = 1, 2, \ldots, r$.

The solvability conditions for this problem will be detailed further. These conditions express in particular that there must exist a sufficient number of measured outputs to be able to detect and isolate the faults.

2.2 Sensor location for FDI

Consider again the system (1). In general the above defined FDI problem has no solution using only the existing sensors on the system. In this case we consider new sensors which could be implemented on the system with some cost. Define the new output vector z which collects the potential new measurements:

$$z(t) = Dx(t) + Nf(t), \quad (5)$$

$z(t) \in \mathbb{R}^q$, where $z_i(t)$ is the measure obtained from the i-th additional sensor with cost c_i. The cost of the measured outputs $y_1(t), \ldots, y_p(t)$ is assumed to be null. Define now the composite system denoted by Σ^c.

$$\Sigma^c \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \\ z(t) = Dx(t) + Nf(t) \end{cases} \quad (6)$$

In the next sections we will consider the following optimal sensor location problem for FDI:

- First check the solvability of the FDI problem on the system Σ. If it is solvable, the optimal sensor location problem has clearly a zero cost solution.
- When the FDI problem on system Σ has no solution, check the solvability of the FDI problem on the composite system Σ^c with additional sensors. When this problem is solvable find a minimal cost solution, which will give us the sensors to be actually implemented.

Our study will be achieved in the framework of structured systems that we introduce now.

3 Linear structured systems

In this part we recall some definitions and results on linear structured systems. More details can be found in [5, 8].

We consider linear systems as described in (1), but with parameterized entries and denoted by Σ_Λ

$$\Sigma_\Lambda \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \end{cases} \quad (7)$$

This system is called a linear structured systems if the entries of the composite matrix $J = \begin{bmatrix} A & L \\ C & M \end{bmatrix}$ are either fixed zeros or independent parameters (not related by algebraic equations). $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ denotes the set of independent parameters of the composite matrix J. For the sake of simplicity the dependence of the system matrices on Λ will not be made explicit in the notation. A structured system represents a large class of parameter dependent linear systems. The structure is given by the location of the fixed zero entries of J. This structure often comes from physical particularities of the system (for example interconnection of subsystems); thus the only exact knowledge on the system is the the structure, i.e. the absence of direct relations between variables as state variables for example (see [9] for a detailed discussion on internal structure representation).

For such systems one can study generic properties i.e. properties which are true for almost all values of the parameters collected in Λ [15, 20]. More precisely a property is said to be generic (or structural) if it is true for all values of the parameters (i.e. any $\Lambda \in \mathbb{R}^k$) outside a proper algebraic variety of the parameter space, i.e. the zero set of a finite number of nontrivial polynomials in the parameters. A directed graph $G(\Sigma_\Lambda) = (Z, W)$ can be easily associated to the structured system Σ_Λ of type (7) where the matrix $\begin{bmatrix} A & L \\ C & M \end{bmatrix}$ is structured:

• the vertex set is $Z = F \cup X \cup Y$ where F, X and Y are the fault, state and output sets given by $\{f_1, f_2, \ldots, f_r\}$, $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_p\}$ respectively,

• the arc set is $W = \{(f_i, x_j) | L_{ji} \neq 0\} \cup \{(x_i, x_j) | A_{ji} \neq 0\} \cup \{(x_i, y_j) | C_{ji} \neq 0\} \cup \{(f_i, y_j) | M_{ji} \neq 0\}$, where A_{ji} (resp. C_{ji},L_{ji},M_{ji}) denotes the entry (j, i) of the matrix A (resp. C,L,M).

Moreover, recall that a directed path in $G(\Sigma_\Lambda)$ from a vertex $i_{\mu 0}$ to a vertex $i_{\mu q}$ is a sequence of arcs $(i_{\mu 0}, i_{\mu 1}), (i_{\mu 1}, i_{\mu 2}), \ldots, (i_{\mu q-2}, i_{\mu q-1}), (i_{\mu q-1}, i_{\mu q})$ such that $i_{\mu t} \in Z$ for $t = 0, 1, \ldots, q$ and $(i_{\mu t-1}, i_{\mu t}) \in W$ for $t = 1, 2, \ldots, q$. The length of a path is the number of its arcs, each arc being counted the number of times it appears in the sequence. For the last sequence, the path has length q. Occasionally, we denote the path P by the sequence of vertices it consists of, i.e. by :

$$P = (i_{\mu 0}, i_{\mu 1}, \ldots, i_{\mu q-1}, i_{\mu q})$$

Moreover, if $i_{\mu 0} \in F$ and, $i_{\mu q} \in Y$, P is called a fault-output path. A path which is such that $i_{\mu 0} = i_{\mu q}$ is called a circuit.

A set of paths with no common vertex is said to be a vertex disjoint. A k-linking is a set of k vertex disjoint fault-output paths, it is also called a linking of size k. A linking is maximal when k is maximal.

All the previous definitions can be extended to a composite structured system Σ^c_Λ with associated graph $G(\Sigma^c_\Lambda)$ where Σ^c_Λ is defined as

$$\Sigma^c_\Lambda \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \\ z(t) = Dx(t) + Nf(t) \end{cases} \quad (8)$$

Using their associated graphs many important results have been obtained for these systems on structural controllability, decoupling, disturbance rejection, ... [5, 8, 14]. As a first example of these results, recall the graph characterization of the structural observability, which will be useful later [14, 15].

Proposition 1 *Let Σ_Λ be the linear structured system defined by (7) with its associated graph $G(\Sigma_\Lambda)$. The system (in fact the pair(C,A)) is structurally observable if and only if:*

- *there exists a state-output path starting from any state vertex in X,*
- *there exists a set of vertex disjoint circuits and state-output paths which cover all state vertices.*

Consider now system Σ_Λ defined in (7) which transfer matrix is $T_\Lambda(s) = C(sI-A)^{-1}L + M$.
We can calculate the generic rank of $T_\Lambda(s)$ by using the following result [8, 18].

Theorem 1 *Let Σ_Λ be the linear structured system defined by (7) with its associated graph $G(\Sigma_\Lambda)$. The generic rank of $T_\Lambda(s)$ is equal to the size of a maximal linking in $G(\Sigma_\Lambda)$.*

Example 1 *Let us now present an example to illustrate the previous notions and results. Consider the following structured system Σ_Λ which is of type (7) with three faults and two outputs:*

$$A = \begin{bmatrix} 0 & 0 & 0 \\ \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \end{bmatrix}, L = \begin{bmatrix} \lambda_3 & 0 & 0 \\ 0 & \lambda_4 & 0 \\ 0 & 0 & \lambda_5 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & \lambda_6 & 0 \\ 0 & 0 & \lambda_7 \end{bmatrix}, M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The entries of these matrices are the free parameters $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_7)$. The associated graph $G(\Sigma_\Lambda)$ is given in Figure 1.

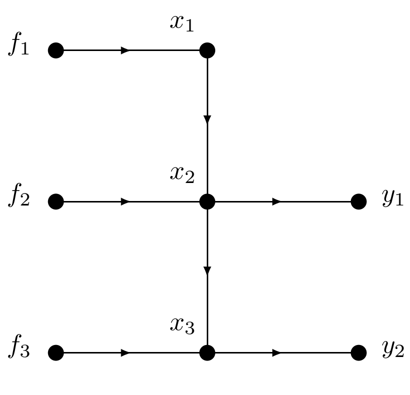

Figure 1. Graph $G(\Sigma_\Lambda)$

This system is structurally observable as can be seen from Proposition 1. Indeed there is a state-output path starting from any state vertex and the set of vertex disjoint paths (x_1, x_2, y_1) and (x_3, y_2) covers all state vertices. The system has clearly rank two since there exists a linking of size two in the graph.

Give now the result concerning the diagonal FDI problem by using a bank of observers which was stated first in [6].

Theorem 2 *Consider the structurally observable system with r faults Σ_Λ as defined in (7) and the associated graph $G(\Sigma_\Lambda)$. The bank of observers-based diagonal FDI problem of Definition 1, is generically solvable if and only if:*

$$k = r \tag{9}$$

where k is the size of a maximal linking in $G(\Sigma_\Lambda)$

4 System decomposition and first results

We have seen that the solvability of the FDI problem is based on the maximal size of fault-output linkings. In this section we will consider some properties of these maximal linkings and derive some useful consequences for the FDI problem.

4.1 Basic notions

Most of the basic material of this subsection is based on [19]. First consider again the graph $G(\Sigma_\Lambda) = (Z, W)$ of a structured system of type (7) with vertex set Z and edge set W. A separator S is a set of vertices such that any fault-output path has at least one vertex in S. Separators with a minimal number of vertices are called minimal. A classical result is that the minimal size of a separator is the maximal size of a fault-output linking. The set of essential vertices Z_{ess} is the set of vertices which belong to any maximal size linking. Construct now the set of vertices which contains for any fault-output path the first vertex which is in Z_{ess}, call this set S^*. It can be shown that S^* is a minimal separator. S^* is indeed the first bottleneck between faults and outputs. S^* may contain fault, state and output vertices. It follows from this definition that the maximal size of a linking is r if and only if $S^* = F$. Define $F_1 = F/(F \bigcap S^*)$, $Y_1 = S^*/(F \bigcap S^*)$ and X_1 is the set of state vertices in any fault-output path from F_1 to Y_1. Now think of a new structured system defined by its graph with input set F_1, output set Y_1, state set X_1; the set of edges corresponds to the edges in any path from F_1 to Y_1. The corresponding structured system is denoted $\Sigma_{1\Lambda}$, its graph is denoted by $G(\Sigma_{1\Lambda})$.
A directed graph $G(\Sigma_\Lambda^c)$ can be easily associated with the structured system Σ_Λ^c of type (6). In fact $G(\Sigma_\Lambda^c)$ is obtained from $G(\Sigma_\Lambda)$ by adding q output vertices $z_1, \dots, z_q$ and edges from $F \bigcup X$ to Z in $G(\Sigma_\Lambda^c)$ corresponding to non null parameters of matrices D and N.

4.2 The sensor location problem and some simple results

We assume that the observability condition of Theorem 2 is satisfied for the initial system (7), adding new edges will preserve the property, therefore Σ_Λ^c is observable. In the following we will then concentrate on the rank condition (9). In graph terms, we try to get a size r linking thanks to the addition of new output vertices and new edges which connect state or fault vertices to them. A variable w_i is said to be *measurable with additional sensors* if we can add a new sensor with corresponding output z_j such that in the graph of the composite system $G(\Sigma_\Lambda^c)$, there is an edge (w_i, z_j) between this variable and the new output. Let us denote by W_m the set of measurable fault and state variables. .

From the previous subsection if the FDI conditions are met we have $S^* = F$ and the system $\Sigma_{1\Lambda}$ will not exist. This is of course not the case of interest for us here, therefore from now on we will assume that $\Sigma_{1\Lambda}$ is not empty, i.e. the FDI problem without additional sensors has no solution.

Proposition 2 *Consider a linear structured system Σ_Λ with graph $G(\Sigma_\Lambda)$. Assuming that the FDI problem without additional sensors has no solution, the FDI problem with additional sensors has a solution only if*

$$(F_1 \bigcup X_1) \bigcap W_m \neq \varnothing \tag{10}$$

Proof Since the FDI problem without additional sensors has no solution we have $F_1 \neq \varnothing$. If the condition (10) is not satisfied, there is no edge connecting $F_1 \bigcup X_1$ to the additional vertices of Z, then $S^*(\Sigma_\Lambda^c) = S^*(\Sigma_\Lambda)$. This implies that S^* remains the first bottleneck between F and Y and then that the size of a maximal fault-output linking is the same in Σ_Λ and Σ_Λ^c. Then by Theorem 2 the FDI problem with additional sensors has no solution. $\square$

To check if supplementary measurements allow to solve the problem we could simply add the new output vertices, the new edges, and verify on the modified graph that the condition is satisfied. However, another result which may drastically reduce the dimension of the problem is the following.

Proposition 3 *The FDI problem with additional sensors has a solution for the system Σ_Λ if and only if the FDI problem with additional sensors has a solution for the system $\Sigma_{1\Lambda}$.*

Proof

only if

Assume that the FDI problem with additional sensors has a solution for the system Σ_Λ, then on $G(\Sigma_\Lambda^c)$ there exists a maximal linking of size r from F to $Z \bigcup Y$ in Σ_Λ^c. Consider a sub-linking of this linking containing the paths with initial vertex in F_1. This sub-linking contains a maximal linking from F_1 to $Z \bigcup Y_1$ of size $dim(F_1)$. Therefore the FDI problem with additional sensors has a solution for $\Sigma_{1,\Lambda}$.

if

Assume now that the FDI problem with additional sensors has a solution for the system $\Sigma_{1\Lambda}$. Then there exists a maximal linking of size $dim(F_1)$ from F_1 to $Z \bigcup Y_1$ in $\Sigma_{1\Lambda}^c$. Since S^* is a minimal separator such that the vertices of F/F_1 lie in S^*, there exists a maximal linking from S^* to Y of size $dim(S^*)$. From these two linkings it is easy to construct a size r linking from F to $Z \bigcup Y$. Then the FDI problem with additional sensors has a solution for Σ_Λ. $\square$

This result is of practical interest since we can restrict the analysis to the subsystem $\Sigma_{1,\Lambda}$ which can be of much lower dimension than Σ_Λ.

Consider again our example. Using Theorem 2 one can see that the FDI problem without additional sensors has no solution because $r = 3$ and $k = 2$. Add now two additional sensors z_1 and z_2 with matrices $D = \begin{bmatrix} \lambda_8 & \lambda_9 & 0 \\ 0 & \lambda_{10} & \lambda_{11} \end{bmatrix}, N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda_{12} \end{bmatrix}$.

From the construction of $\Sigma_{1,\Lambda}$ we have $F_1 = \{f_1, f_2\}$, $X_1 = \{x_1\}$, $Y_1 = \{x_2\}$. The corresponding graph $G(\Sigma_{1\Lambda}^c)$ is given in Figure 2.

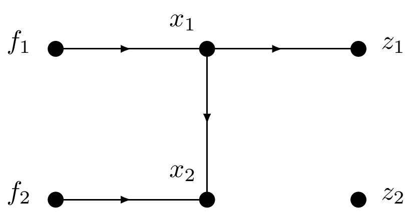

Figure 2. Graph $G(\Sigma_{1\Lambda}^c)$

It is clear that the sensor z_2 will not help for solving the FDI problem. In fact the FDI problem with additional measurements has a solution since the condition of Theorem 2 is satisfied on $\Sigma_{1\Lambda}^c$. $k = r = 2$, there is a linking of size 2 from F_1 to $(Y_1 \bigcup Z)$, namely (f_1, x_1, z_1) and (f_2, x_2).

5 Bipartite graph and optimal sensor location

In section 3 we have presented a graph $G(\Sigma_\Lambda)$ which can be naturally associated with a structured system Σ_Λ. This graph gives a visual representation of the internal structure and the solvability of several structural problems can be stated in a very pedagogical way in terms of this graph. We will present now another representation in terms of a bipartite graph, although probably less appealing in terms of visualization, this representation is better suited for efficient computations. We will now introduce this graph.

5.1 Bipartite graph of a system

We consider a linear structured system Σ_Λ of type (7) as previously. The bipartite graph of this system is $B(\Sigma_\Lambda) = (V, V', E)$, where we give a new meaning to V and E. The

sets V and V' are two disjoint vertex sets and E is the edge set. The vertex set V is given by $F \cup X^1$, the vertex set V' is given by $X^2 \cup Y$, with $F = \{f_1, \dots, f_r\}$ the set of fault vertices, $X^1 = \{x_1^1, \dots, x_n^1\}$ the first set of state vertices, $X^2 = \{x_1^2, \dots, x_n^2\}$ the second set of state vertices and $Y = \{y_1, \dots, y_p\}$ the set of output vertices. Notice that here we have split each state vertex x_i of G into two vertices x_i^1 and x_i^2. Denoting (v, v') for an edge from the vertex $v \in V$ to the vertex $v' \in V'$, the edge set E is newly described by $E_A \cup E_L \cup E_C \cup E_M$ with $E_A = \{(x_j^1, x_i^2) | A_{ij} \neq 0\}$, $E_L = \{(f_j, x_i^2) | L_{ij} \neq 0\}$, $E_C = \{(x_j^1, y_i) | C_{ij} \neq 0\}$ and $E_M = \{(f_j, y_i) | M_{ij} \neq 0\}$. In the latter, for instance $A_{ij} \neq 0$ means that the (i, j)-th entry of the matrix A is a parameter (a nonzero). We complete this graph by all the "horizontal edges" of the type (x_i^1, x_i^2) for $i = 1, \dots, n$.
To the composite system Σ_Λ^c defined in (6) we can associate the bipartite graph B^c using the rules given for $B(\Sigma_\Lambda)$.
Let us consider a general bipartite graph $B = (V, V', E)$ as follows. The sets V, V' are two disjoint vertex sets and E is the edge set, where all edges have the form (v, v') with $v \in V$ and $v' \in V'$. These graphs received a considerable attention in the literature on combinatorics. A *matching* in a bipartite graph $B = (V, V', E)$ is an edge set $M \subseteq E$ such that the edges in M have no common vertex. The cardinality of a matching, i.e. the number of edges it consists of, is also called its size. The maximal matching problem is the problem of just finding a matching of maximal cardinality. This problem can be solved using very efficient algorithms based on alternate augmenting chains or ideas of maximum flow theory [12]. This notion allows allows a simple characterization of the generic rank of a structured system in terms of its bipartite graph [7].

Theorem 3 *Consider a linear structured system of type (7) with bipartite graph $B(\Sigma_\Lambda)$ and transfer matrix $T_\Lambda(s)$. The generic rank of $T_\Lambda(s)$, g-rank $T_\Lambda(s)$, is equal to the size of a maximal matching in $B(\Sigma_\Lambda)$ minus n.*

Let us now consider a *weighted* bipartite graph, i.e. a bipartite graph $B = (V, V', E)$, for which a real number $w(e)$ is associated to each edge $e \in E$. The weight of a matching $M \subseteq E$ is defined as $w(M) = \sum_{e \in M} w(e)$. The *optimal μ-matching problem* consists of finding a matching M of size μ such that $w(M)$ is maximal (or minimal). Again, there exist a lot of efficient algorithms to solve this problem, among them there is the famous Hungarian method [11, 13]. The classical software packages in operations research contain optimized versions of these algorithms.

5.2 Optimal sensor location for FDI

Let us come back to our main problem. We assume that the FDI problem on the composite system Σ_Λ^c in (8) with additional sensors has a solution. We look for a minimal cost solution to be actually implemented on the system. To any edge of terminal vertex z_i we associate the weight c_i which is the cost of the i-th sensor. The other edges have weight 0. We get a weighted graph denoted by $B_w(\Sigma_\Lambda^c)$. Then we have the following result.

Theorem 4 *Consider a structurally observable linear structured system of type (8) with r faults, n states, q additional sensors, and the associated weighted bipartite graph $B_w(\Sigma_\Lambda^c)$. The optimal additional sensors for the FDI problem are given by the z_i's belonging to a minimal cost matching of size $n + r$ in $B_w(\Sigma_\Lambda^c)$, if any.*

Proof
When the FDI problem with additional sensors has a solution, from Theorem 2 there exists a size r fault-output linking in $G(\Sigma_\Lambda^c)$. Among all such possible linkings, consider one of minimal weight. The cost of such a minimal weight linking corresponds to the sum of the costs of the sensors z_i actually used. Notice that for any sensor z_i there is at most one edge with terminal vertex z_i appearing in a linking. Therefore, the cost c_i of the sensor z_i appears only once in the total cost if z_i is a vertex of the chosen linking. The cost c_i of z_i does not appear in the total cost in the case where z_i does not belong to the linking.
Consider now the bipartite graph $B_w(\Sigma_\Lambda^c)$ and a minimal cost matching of size $n + r$ in $B_w(\Sigma_\Lambda^c)$. It is proved in [7] that any matching of size $n + r$ in $B_w(\Sigma_\Lambda^c)$ corresponds in $G(\Sigma_\Lambda^c)$ to:

- a fault-output linking of size r,
- a set of circuits joining state vertices,
- a set of fault-state paths,
- a set of state-output paths.

From our definition of weights, the above mentioned circuits, fault-state paths and state-output paths with final vertex in Y, have zero cost. Consider now a state-output path $(x_{i1}, x_{i2}, \dots, x_{il}, z_i)$ with final vertex in Z, which is in one to one correspondence with the following set of edges in $B_w(\Sigma_\Lambda^c)$: $(x_{i1}^1, x_{i2}^2), (x_{i2}^1, x_{i3}^2), \dots, (x_{il}^1, z_i)$ belonging to an optimal matching of size $n + r$ in $B_w(\Sigma_\Lambda^c)$. Replacing these edges by the set: $(x_{i1}^1, x_{i1}^2), (x_{i2}^1, x_{i2}^2), \dots, (x_{il}^1, x_{il}^2)$ would lead to another maximal matching of lower cost, which is a contradiction. Then in a minimal cost solution no state-output path with final vertex in Z would appear.
We have proved that an optimal matching of size $n + r$ in $B_w(\Sigma_\Lambda^c)$ corresponds to an optimal fault-output linking of size r in $G(\Sigma_\Lambda^c)$ with the same cost. This matching provides us with the optimal additional sensors to be implemented, in fact the z_i's which belong to this optimal matching. $\square$
Let us consider again our example. The bipartite graph corresponding to the system $\Sigma_{1\Lambda}^c$ of Figure 2, is given in Figure 3. Notice that here x_2 plays the role of an output for $\Sigma_{1\Lambda}^c$.

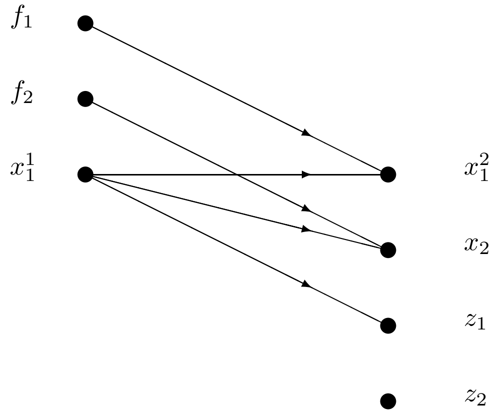

Figure 3. Bipartite graph $B(\Sigma^c_{1\Lambda})$

The size of a maximal matching in this graph is clearly 3. Using Theorem 3 we get $n + r = 3$, and from Theorems 1 and 2 it appears that the FDI problem with additional sensors has a solution because $k = 3 - 1 = r = 2$. In this case the optimal sensor location is trivial since z_1 belongs to any size 3 matching, then this sensor must be implemented whatever its cost, while the additional sensor z_2 is useless.

6 Concluding remarks

In this paper we have considered a particular FDI problem and we concentrated on the case when this problem has no solution using the measurements available on the system. We Considered that new sensors could be implemented on the system with some cost. Then we solved the problem of the optimal choice of additional sensors to be implemented to achieve the FDI at a minimal cost. The result was expressed on the bipartite graph of the system and amounts to solve a very classical problem of combinatorial optimization.
Extensions of the previous ideas are under investigation for more complex cases of the FDI problem.

References

[1] M. Bassville, A. Benveniste, G.V. Moustakides, and A Rougée. Optimal sensor location for detecting abrupt changes in dynamical behaviour. *IEEE Trans. on Autom. Control*, 32(12):1067–1075, 1987.

[2] T. Carpentier. *Placement de capteurs pour la surveillance des procédés complexes*. PhD thesis, Univeristé des Sciences et Technologies de Lille, 1999.

[3] J. Chen and R.J. Patton. *Robust model-based fault diagnosis for dynamic systems*. Kluwer academic publishers, 1999.

[4] J. Chen, R.J. Patton, and H.Y. Zhang. Design of unknown input observers and robust fault detection filters. *Int. Journ. of Control*, 63(1):85–105, 1996.

[5] C. Commault, J.M. Dion, and A. Perez. Disturbance rejection for structured systems. *IEEE Trans. Automat. Control*, 36:884–887, 1991.

[6] C. Commault, J.M. Dion, O. Sename, and R. Motyeian. Observer-based fault detection and isolation for structured systems. *IEEE Trans. Automat. Control*, 47(12):2074–2079, 2002.

[7] C. Commault, J.M. Dion, and J. van der Woude. Characterization of generic properties of linear structured systems for efficient computations. *Kybernetika*, 38(5):503–520, 2002.

[8] J.M. Dion and C. Commault. Feedback decoupling of structured systems. *IEEE Trans. Automat. Control*, 38:1132–1135, 1993.

[9] J.M. Dion, C. Commault, and J. van der Woude. Generic properties and control of linear structured systems: a survey. *Automatica*, 39(7):1125–1144, 2003.

[10] P.M. Frank. Analytical and qualitative model-based fault diagnosis - a survey and some new results. *European Journal of Control*, 2:6–28, 1996.

[11] M. Gondran and M. Minoux. *Graphs and algorithms*. Wiley, Chichester, 1984.

[12] J.E. Hopcroft and R.M. Karp. An $n^{5/2}$ algorithm for maximum matchings in bipartite graphs. *SIAM J. on Comp.*, 2:225–231, 1973.

[13] H.W. Kuhn. The hungarian method for the assignment problem. *Nav. Res. Log. Quat.*, 2:83–97, 1955.

[14] C.T. Lin. Structural controllability. *IEEE Trans. Automat. Control*, 19:201–208, 1974.

[15] K. Murota. *Matrices and Matroids for System Analysis*. Algorithms and Combinatorics. Springer-Verlag, New-York, 2000.

[16] R.J. Patton. Robust model-based fault diagnosis : the state of the art. In *IFAC Safeprocess Symposium*, pages 1–24, Espoo, Finland, 1994.

[17] M. Staroswiecki and J.P. Cassar. Approche structurelle pour la conception des systèmes de surveillance. In *Ecole d'Eté d'Automatique*, Grenoble, France, 1996.

[18] J.W. van der Woude. A graph theoretic characterization for the rank of the transfer matrix of a structured system. *Math. Control Signals Systems*, 4:33–40, 1991.

[19] J.W. van der Woude. The generic number of invariant zeros of a structured linear system. *SIAM Journ. of Control*, 38:1–21, 2000.

[20] W.M. Wonham. *Linear multivariable control : a geometric approach*. Springer-Verlag, New-york, 1985.