STABILITY ANALYSIS OF SOME CLASS OF NONLINEAR TIME DELAY SYSTEMS WITH APPLICATIONS

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Abstract

This paper focuses on the stability analysis of some nonlinear delay models of a congestion control scheme. Some uncertainty interpretations of the nonlinear terms are proposed and a complete Lyapunov-Krasovskii functional is used for deriving local asymptotic stability conditions in both constant and time-varying delay cases.

1 Introduction

In the analysis of congestion control mechanisms, one of the models largely used to describe such a phenomenon, in a fluid modeling setting, is given by the following differential equation including delay:

$$\dot{x}(t) = k\left[w - x(t-h)p(x(t-h))\right], \tag{1}$$

where k, w are positive reals and $p(\cdot)$ is a continuous and differentiable nondecreasing function. To the best of the authors' knowledge, such a model was firstly proposed by Kelly in [7] for describing the dynamics of a collection of flows, all using a single resource, and sharing the same gain parameter k. The delay h represents the round-trip time, and is assumed constant. The function $p(\cdot)$ can be interpreted as the fraction of packets indicating (potential) congestion (presence) [7],[3]. Furthermore, the assumptions on the function $p(\cdot)$ considered above are natural in the context of TCP behavior (see, for instance, [7],[3],[13], and the references therein).

On the other hand, in the context of control theory, the function $p(\cdot)$ can be selected as a classical proportional or proportional-integral controller, as discussed by [6].

In this paper, we consider that function $p(\cdot)$ is a linear increasing function of the form $p(x) = \gamma x$ with $\gamma > 0$. For example, a proportional controller with gain γ, see [6]. Thus, the system (1) can be rewritten as

$$\dot{x}\,(t) = \alpha - \beta x^2(t-h), \tag{2}$$

where $\alpha = kw$ and $\beta = k\gamma$. In order to avoid bad performance of TCP behavior, it is important to ensure, at least, the local asymptotic stability of system (2).

It is well known that the delay in the network is time-varying and it depends on the traffic load, the capacity of the nodes and the number of effective connections. However, most of the recent publications consider only the case when the time-delay is constant, see, e.g. [3] and [6].

The goal of this paper is to analyze the stability of (2), even in the case when the delay is time-varying, by taking advantage of the tools and methods developed in the stability and robust stability analysis of time-delay systems. More explicitly, we shall use the linearization of system (2) and we will interpret the nonlinear part as an uncertainty of the corresponding linear system (see [4] for application of this approach to rational systems without delay, and [12] for the case of rational systems with delay), and then we will apply the Lyapunov-Krasovskii approach in order to obtain local stability conditions for the nonlinear system.

Note that a Lyapunov-Razumikhin approach was proposed by Deb and Srikant in [3] for the stability of (1), and the choices on k and w mentioned above. As seen in [11] (see, e.g., some discussions in chapter 5) in the linear case, the Razumikhin-based approach is more conservative than the Krasovskii's one, where the Lyapunov-Krasovskii functional is generated from the Lyapunov-Razumikhin candidate by adding some (relatively simple) double integral terms.

In this paper instead to use a system transformation of the corresponding linear system, and then propose a particular Lyapunov-Krasovskii functional for the corresponding transformed system in order to obtain delay-dependent stability conditions (see [5] and [10] for the analysis of the conservatism introduced by system transformations), we will use the complete functional introduced in [8] that allows to obtain delay-dependent robust stability conditions depending of a scalar function which satisfies an ordinary differential equation (without delay). Furthermore, we analyze the asymptotic stability of system (2) when there is a time-varying perturbation in the delay value, that is, when there exists a time-varying uncertainty of the round-trip time of the flow. To this aim, we will apply some basic ideas presented in [9] for the application of complete functionals to the case of time-varying delay uncertainties. In particular, an upper delay bound guaranteeing the asymptotic stability is derived.

The paper is organized as follows: Section 2 presents the problem formulation. In Section 3 an explicit construction of a Lyapunov-Krasovskii functional for the stability analysis of (1) is given for constant as well as time-varying delay cases. Numerical examples show the application of the results and some concluding remarks end the paper.

2 Problem formulation

Consider the following nonlinear time delay system:

$$\begin{aligned} \dot{x}(t) &= \alpha - \beta x^2(t-h), \\ x(t) &= \varphi(t), \forall t \in [-h, 0], \end{aligned} \tag{3}$$

where α, β, h are positive constants, and $\varphi(\cdot)$ is the initial function with the following norm

$$|\varphi| = \sup_{\theta\in[-h,0]} |\varphi(\theta)|.$$

It is clear that the equilibrium points of system (3) are $x^* = \pm\sqrt{\frac{\alpha}{\beta}}$.

Let $y(t) = x(t) - x^*$, then system (3) becomes

$$\dot{y}(t) = -2\beta x^* y(t-h) - \beta y^2(t-h). \tag{4}$$

It is well known from the theory of differential-difference equations, see [1], that if the trivial solution of the system

$$\dot{y}(t) = -2\beta x^* y(t-h), \tag{5}$$

is asymptotically stable, then the trivial solution of (4), and hence the equilibrium point x^* of (3) is locally asymptotically stable.

On the other hand, we know that system (5) is asymptotically stable if and only if $2\beta x^* \in \left(0, \frac{\pi}{2h}\right)$, see [11]. Therefore, the unique equilibrium point of system (3) which is asymptotically stable is $x^* = \sqrt{\frac{\alpha}{\beta}}$.

Now, it is clear that the equilibrium point x^* of system (3) is asymptotically stable when the norm of the initial function $|\varphi|$, belongs to a neigborhood of x^*.

The problem arising here is: how large could be the initial function domain in order to guarantee that system (4) remains asymptotically stable for all initial conditions in this domain.

3 Robust stability approach

We will consider a robust stability approach for solving the problem presented above. The main idea is to interpret the nonlinear part of system (4) as an uncertainty of the linear system (5). Next, we will apply the Lyapunov-Krasovskii functional approach in order to derive robust stability conditions that allow to obtain upper bounds for the initial conditions of system (3) such that the stability of the equilibrium point x^* is guaranteed.

Let us to define

$$\delta(t) = \beta y(t-h), \tag{6}$$

satisfying the following inequality:

$$|\delta(t)| \leq \rho, \forall t \geq 0, \tag{7}$$

where ρ is a positive constant.

Then, the nonlinear system (4) can be rewritten as the following perturbed delay linear system

$$\dot{y}(t) = -(a + \delta(t))\, y(t-h), \tag{8}$$

where $a = 2\beta x^*$. Now, it is quite clear, from (6), that if we get an upper bound for ρ then we can give an estimation for the initial function domain such that the asymptotic stability of x^* is guaranteed.

3.1 Constant delay case

In this section, we will apply some recent results presented in [8] (on the construction of complete Lyapunov-Krasovskii functionals) for the case of system (8). Thus, Kharitonov and Zhabko [8] have shown that if the nominal system

$$\dot{y}(t) = -ay(t-h), \tag{9}$$

is asymptotically stable, there exist a functional

$$\begin{aligned} v(y_t) &= mu(0)y^2(t) \\ &-2amy(t)\int_{-h}^{0} u(h+\theta)y(t+\theta)d\theta \\ &+ma^2\int_{-h}^{0}\int_{-h}^{0} u(\theta_1-\theta_2)y(t+\theta_1)y(t+\theta_2)d\theta_1 d\theta_2 \\ &+\int_{-h}^{0}(\mu_1+(h+\theta)\mu_2)\, y^2(t+\theta)d\theta, \end{aligned} \tag{10}$$

which satisfies for some $\alpha_1 > 0$ and $\alpha_2 > 0$ that

$$\alpha_1 y(t)^2 \leq v(y_t) \leq \alpha_2 |y_t|^2, \tag{11}$$

and along to the solutions of system (9)

$$\frac{dv(y_t)}{dt} = -w(y_t), \tag{12}$$

where

$$w(y_t) = \mu_0 y^2(t) + \mu_1 y^2(t-h) + \mu_2 \int_{-h}^{0} y^2(t+\theta)d\theta,$$

for any given $\mu_j > 0, j = 0, 1, 2.$

In (10), we have that $m = \mu_0 + \mu_1 + h\mu_2$ and $u(\cdot)$ is a scalar function which satisfies the following ordinary differential equation

$$\ddot{u}(t) = -a^2 u(t), \tag{13}$$

with the following additional conditions

$$\begin{aligned} \dot{u}(t) &= -au(t-h), \forall t \geq 0, && (14) \\ u(-t) &= u(t), \forall t \geq 0, && (15) \\ u(h) &= \frac{1}{2a}. && (16) \end{aligned}$$

Remark 1 *Observe that functional (10) depends only on the function* $u(\cdot)$ *satisfying an ordinary differential equation (without delay) with appropriate boundary conditions.*

Deriving functional (10) along to the trajectories of the perturbed system (8), see [8], we obtain that the perturbed system (8) remains asymptotically stable, for any perturbation $\delta(t)$ satisfying (7), if $\rho > 0$ is chosen such that

$$\begin{cases} \mu_0 > m\rho u(0), \\ \mu_1 > m\rho\left(u(0) + ahu_0\right), \\ \mu_2 > m\rho a h u_0, \end{cases} \quad (17)$$

where $u_0 = \max_{t\in[0,h]} |u(t)|$.

From (6) we can conclude that if the initial function $\varphi(\cdot)$ belongs to the domain $D = \left\{\varphi : |\varphi| < \frac{\rho}{\beta}\right\}$, then the derivative of the functional $v(y_t)$, along to the trajectories of (4), is definite negative. The positivity of function $v(y_t)$ is guaranteed from the left hand side inequality of (11).

Since our interest is to obtain an estimate of the region of attraction we need to determine the set $U_l = \{\varphi : v(\varphi) < l\} \subset D$. To this aim we will use the right hand side inequality of (11). Thus, we need to compute a positive constant $\alpha_2 > 0$ such that the right hand side inequality of (11) is satisfied.

From (10) direct calculations show that

$$v(y_t) \leq \alpha_2 |y_t|^2,$$

where

$$\alpha_2 = mu_0(1 + ah) + h\left(amu_0(1 + ah) + \mu_1 + h\mu_2\right) \quad (18)$$

Therefore, an appropriate selection for l is

$$l = \alpha_2 \frac{\rho^2}{\beta^2}.$$

Summarizing we arrive to the following result:

Theorem 2 *The trivial solution of the nonlinear system (4) is locally asymptotically stable if* $h\sqrt{\alpha\beta} < \frac{\pi}{4}$ *and if the initial function* φ *belongs to the set*

$$U_l = \{\varphi : v(\varphi) < l\}$$

where $l = \alpha_2 \frac{\rho^2}{\beta^2}$ *with* $\rho > 0$ *satisfies the inequalities (17).*

3.1.1 Construction of $u(\cdot)$

In this section, we will show how to construct the scalar function $u(\cdot)$ which participates in the computation of the lower bound for ρ.

Every solution of equation (13) can be written as

$$u(t) = c_1 \cos(at) + c_2 \sin(at), \quad (19)$$

for some constants $c_1$and c_2. Using the additional conditions (14), (15) and (16), it is not difficult to obtain the constants c_1 and c_2 such that

$$u(t) = \left(\frac{1 + \sin(ah)}{2a\cos(ah)}\right)\cos(at) - \frac{1}{2a}\sin(at), \forall t \in [0, h]. \quad (20)$$

As an example, consider that $a = 1$ and $h = 1$. The function $u(t)$ corresponding to these particular values is plotted in Fig.1 below.

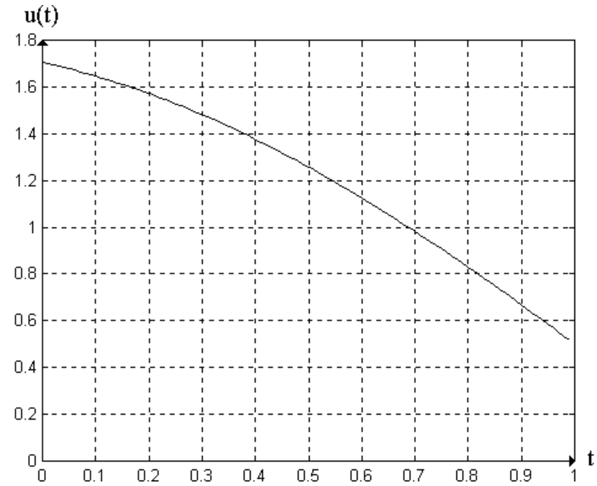

Fig.1 Function $u(t)$

3.2 Time-Varying Delay Case

In this section, we will consider a more complicated case when the nonlinear system includes a time-varying delay perturbation, that is, when there exists a time-varying uncertainty of the round-trip time of the flow. Thus, consider the following nonlinear system

$$\begin{aligned} \dot{x}(t) &= \alpha - \beta x^2(t - h + \eta(t)), && (21) \\ x(t) &= \psi(t), \forall t \in [-2h, 0], \end{aligned}$$

where $\psi(\cdot)$ is the initial function and $\eta(t)$ is a continuous, and bounded function satisfying, for all $t \geq 0$, the following inequalities

$$|\eta(t)| \leq \eta_0 < h \text{ and } \left|\dot{\eta}(t)\right| \leq \eta_1 < 1. \quad (22)$$

Considering that $y(t) = x(t) - x^*$ system (21) becomes

$$\dot{y}(t) = -2\beta x^* y(t-h) - \beta y^2(t - h + \eta(t)). \quad (23)$$

Then, we need to ensure the asymptotic stability of the following perturbed time-varying delay linear system

$$\dot{y}(t) = -(a + \Delta(t))y(t - h + \eta(t)), \tag{24}$$

where $a = 2\beta x^*$, and

$$\Delta(t) = \beta y(t - h + \eta(t)), \tag{25}$$

with

$$|\Delta(t)| \leq \sigma, \forall t \geq 0. \tag{26}$$

As in the case of constant delay, it is clear from (25) that if we get an upper bound for σ, η_0 and η_1, then we can obtain an estimation for the initial function domain implying the local asymptotic stability of the nonlinear system (21).

In [9], robust stability conditions are derived, using complete type Lyapunov-Krasovskii functionals, for the case when there exist only time-varying delay perturbation. To the best of the authors' knowledge, the application of complete type Lyapunov-Krasovskii functional to obtain robust stability conditions in the case of system (24) has not been studied.

Let us to select the following functional

$$\begin{aligned} w(y_t) &= \mu_0 y^2(t) + \mu_1 y^2(t-h) + \mu_2 \int_{-h}^{0} y^2(t+\theta)d\theta \\ &\quad + \mu_3 \int_{-3h}^{0} y^2(t+\theta)d\theta. \end{aligned}$$

Now, the corresponding functional $v(y_t)$ satisfying conditions (11) and (12) is

$$\begin{aligned} v(y_t) &= mu(0)y^2(t) \qquad (27) \\ &-2amy(t)\int_{-h}^{0} u(h+\theta)y(t+\theta)d\theta \\ &+ma^2 \int_{-h}^{0}\int_{-h}^{0} u(\theta_1 - \theta_2)y(t+\theta_1)y(t+\theta_2)d\theta_1 d\theta_2 \\ &+\int_{-h}^{0} (\mu_1 + (h+\theta)\mu_2)\, y^2(t+\theta)d\theta \\ &+\mu_3 \int_{-h}^{0} (3h+\theta)\, y^2(t+\theta)d\theta, \end{aligned}$$

where $m = \mu_0 + \mu_1 + h\mu_2 + 3h\mu_3$ and $u(\cdot)$ is a scalar function which satisfies equation (13) with additional conditions (14),(15) and (16).

Let us rewrite the perturbed system (24) as

$$\begin{aligned} \dot{y}(t) &= -(a+\Delta(t))y(t-h+\eta(t)) \qquad (28) \\ &\quad -(a+\Delta(t))y(t-h) + (a+\Delta(t))y(t-h). \end{aligned}$$

The derivative of functional (27), along to the trajectories of system (28), is

$$\frac{dv(y_t)}{dt} = -w(y_t) + \zeta(y_t, \Delta(t), \eta(t))$$

where

$$\begin{aligned} &\zeta(y_t, \Delta(t), \eta(t)) \\ &= -2m\left(u(0)y(t) - a\int_{-h}^{0} u(h+\theta)y(t+\theta)d\theta\right) \times \\ &\quad \times (\Delta(t)y(t-h) + (a+\Delta(t)) \times \\ &\quad \times [y(t-h+\eta(t)) - y(t-h)]) \end{aligned}$$

Now, we will estimate the terms in the derivative depending of perturbations. First observe that from the Newton-Leibniz formula we have

$$y(t-h+\eta(t)) - y(t-h) = \int_{0}^{\eta(t)} \dot{y}(t-h+\theta)d\theta$$

Substituting the derivative under the integral by the right hand side of equation (24) we get

$$\begin{aligned} &|y(t-h+\eta(t)) - y(t-h)| \\ &\leq (a+\sigma)\int_{0}^{\eta(t)} |y(t-2h+\theta+\eta(t-h+\theta))|\, d\theta \end{aligned}$$

Then, we have:

$$\begin{aligned} &|\zeta(y_t, \Delta(t), \eta(t))| \qquad (29) \\ &\leq mu(0)\left(\sigma + (a+\sigma)^2\eta_0\right)y^2(t) \\ &\quad +m\sigma\left(u(0) + hau_0\right)y^2(t-h) \\ &\quad +mau_0\left(\sigma + (a+\sigma)^2\eta_0\right)\int_{-h}^{0} y^2(t+\theta)d\theta \\ &\quad +m(a+\sigma)^2\left(u(0) + hau_0\right) \times \\ &\quad \times \int_{0}^{\eta(t)} y^2(t-2h+\theta+\eta(t-h+\theta))d\theta \end{aligned}$$

Let $\xi = \theta + \eta(t-h+\theta)$ then

$$\begin{aligned} &\int_{0}^{\eta(t)} y^2(t-2h+\theta+\eta(t-h+\theta))d\theta \\ &= \frac{1}{1+\dot{\eta}(t-h+\theta)} \times \\ &\quad \times \int_{\eta(t-h)}^{\eta(t)+\eta(t-h+\eta(t))} y^2(t-2h+\xi)d\xi \\ &\leq \frac{1}{1-\eta_1}\int_{-\eta_0}^{2\eta_0} y^2(t-2h+\xi)d\xi. \qquad (30) \end{aligned}$$

Considering estimation (30) in (29) and after some simple but tedious algebraic calculations we arrive to the following result:

Theorem 3 *Let system (9) exponentially stable. The perturbed system (24) with perturbations (26), (22) is asymptotically stable if there exist positive constants $\mu_j, j = 1, 2, 3$ such that the following inequalities hold:*

$$\begin{cases} \mu_0 > mu(0)\left(\sigma + (a+\sigma)^2\eta_0\right) \\ \mu_1 > m\sigma\left(u(0) + hau_0\right) \\ \mu_2 > mau_0\left(\sigma + (a+\sigma)^2\eta_0\right) \\ \mu_3 > \frac{m(a+\sigma)^2}{1-\eta_1}\left(u(0) + hau_0\right) \end{cases} \tag{31}$$

Again, as in the constant delay case, equality (25) implies that if the initial function $\psi(\cdot)$ belongs to the domain $E = \left\{\psi : |\psi| < \frac{\sigma}{\beta}\right\}$, then the derivative of the functional $v(y_t)$, along to the trajectories of (23), is definite negative. The left hand side inequality of (11) implies the positivity of the functional $v(y_t)$.

Now we need to determine the set $R_L = \{\psi : v(\psi) < L\} \subset E$ for obtaining an estimate of the region of attraction and the right hand side inequality of (11) help us to obtain such set. Thus, direct calculations derived from (10) yield to the following estimation

$$v(y_t) \leq \alpha_2 |y_t|^2,$$

where

$$\alpha_2 = mu_0(1+ah)+h(amu_0(1+ah)+\mu_1+h\mu_2+3h\mu_3).$$

Therefore an appropriate selection for L is

$$L = \alpha_2 \frac{\sigma^2}{\beta^2}.$$

Summarizing we arrive to the following result:

Theorem 4 *The trivial solution of the nonlinear system (23) with time-varying delay perturbation $\eta(t)$ satisfying (22) is locally asymptotically stable if $h\sqrt{\alpha\beta} < \frac{\pi}{4}$ and if the initial function ψ belongs to the set*

$$R_L = \{\psi : v(\psi) < L\},$$

where $L = \alpha_2 \frac{\sigma^2}{\beta^2}$, and σ, η_0 and η_1 satisfy inequalities (31).

Remark 5 *It is important to mention that the robust conditions (31) remain to be valid even if the perturbation $\Delta(t)$ is a more complicated nonlinear time-varying function depending on $y(t-h+\eta(t))$. The only condition needed is that $\Delta(t)$ satisfies (26).*

Remark 6 *It is possible to obtain better estimations for the lower upper bounds ρ, η_0 and η_1 by means of appropriate selection of parameters μ_0, μ_1 and μ_2 to improve the set of initial conditions with respect to some criterion to be specified. Furthermore, the results could be improved by using different norms, as for example, M^2-norms (see, for instance,[2]).*

4 Example

Consider the following nonlinear system

$$\dot{x}(t) = 1 - 0.25x^2(t-1) \tag{32}$$

The unique equilibrium point of system (32) which is asymptotically stable is $x^* = \sqrt{\frac{\alpha}{\beta}} = 2$. Considering $y(t) = x(t) - x^*$, system (32) becomes

$$\dot{y}(t) = -y(t-h) - 0.25y^2(t-h). \tag{33}$$

Then, the corresponding perturbed linear system is

$$\dot{y}(t) = -(1+\delta(t))\,y(t-1), \tag{34}$$

where

$$\delta(t) = 0.25y(t-h).$$

Function $u(t)$ for the nominal system is given on Fig. 1, and therefore

$$u_0 = u(0) = 1.7041.$$

Let $\mu_0 = 0.75, \mu_1 = 1.5$ and $\mu_2 = 0.75$, then direct calculations derived from (17) show that the perturbed linear system (34) is asymptotically stable if $\rho < 0.1467$. Then, from formula (18) we have that $\alpha_2 = 22.69$. Choosing $\rho = 0.14$ we obtain that $l = 7.18$, and then we arrive to the following estimate for the region of attraction:

$$U_l = \{\varphi : v(\varphi) < 15.99\}.$$

5 Conclusion

In this paper a robust stability approach to study the local asymptotic stability of some class of first-order nonlinear time-delay systems is derived. The results are based on the construction of complete Lyapunov-Krasovskii functionals. The results are applied to the stability analysis of some fluid-flow models used for describing congestion control phenomenons.

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