TRACKING OF PIECEWISE CONSTANT REFERENCES FOR CONSTRAINED NONLINEAR SYSTEMS

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Abstract

The paper addresses tracking of a piecewise constant reference for a nonlinear system subject to control and/or state constraints. The proposed controller, called *dual-mode*, extends to the nonlinear case an approach formerly introduced for linear systems. The dual-mode controller is based on the knowledge of the set of feasible state-setpoint pairs and operates in two different modes: as a regulator if the current state is feasible for the desired set-point, or attempts to recover feasibility as quickly as possible whenever feasibility is lost due to a set-point change. The difficulty of constructing the feasibility set in the state-setpoint space for a nonlinear system is overcome by embedding the original system into a family of uncertain (polytopic) linear systems.

1 Introduction

The tracking of a piecewise constant reference in presence of input and/or state constraints is a challenging control problem. Classical control design techniques, both linear and nonlinear, usually ignore constraints in the design stage and attempt to account for them in the implementation (e.g., anti-windup schemes). Recently there have been a number of successful contributions following the so called *reference governor* approach both for linear [5, 9] and nonlinear [1, 2, 8, 13] systems. The reference governor is essentially a device which manipulates on line, in a state dependent way, a command input to the suitably pre-compensated system so as to satisfy constraints. In [6], an alternative approach called *dual-mode tracking* has been proposed for constrained linear systems. The dual-mode controller operates as a regulator in a suitable neighborhood of the desired equilibrium wherein constraints are feasible, while aims at recovering feasibility as quickly as possible whenever this is lost due to a set-point change. In particular, in the feasibility recovery mode, the controller directly synthesizes the plant control input and hence has more freedom than the reference governor which can only synthesize a command input to a pre-compensated control loop.
In fact, simulation results in [6] demonstrated, for linear systems, the superior tracking speed of the dual-mode controller over the reference governor. The aim of this paper is to extend the dual-mode tracking approach to nonlinear systems. This extension is by no means trivial. In fact the dual-mode strategy requires the off-line construction of a suitable constraint admissible set [6] in the (state, set-point) space and such a construction is not viable for a nonlinear system. To circumvent the above difficulty, we pursued a previous idea, successfully adopted in [7] in the context of predictive regulation of constrained nonlinear systems. The idea is to embed the original nonlinear system into an uncertain LPV model [14, 15, 16] and thus construct the desired admissible set for such a model. The paper is organized as follows. Section 2 formulates the tracking problem of interest. Section 3 describes LPV embedding. Section 4 deals with constraint admissible invariant sets and their construction. Section 5 presents a dual-mode tracking controller for nonlinear systems and discusses its properties. Simulation experiments on a strongly nonlinear system, comparing the proposed controller with reference governors, are reported and discussed in section 6. Finally section 7 draws concluding remarks.

2 Control problem formulation

Consider a discrete-time nonlinear system

$$\begin{aligned} x(t+1) &= f(x(t), u(t)) \\ y(t) &= h(x(t)) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$. The control objective is that

1. the output $y(t)$ track a piecewise constant reference $r(t)$, i.e. a signal switching among different constant set-points;

2. the state $x(t)$ and input $u(t)$ satisfy the linear inequality constraints

$$Lx(t) + Mu(t) \leq b \tag{2}$$

It is assumed that to each constant set-point r there is associated an unique (state,input) equilibrium pair (x_r, u_r) such that

$$x_r = f(x_r, u_r), \qquad r = h(x_r) \tag{3}$$

Clearly the constraints (2) restrict the statically admissible set-points r to those which satisfy $Lx_r + Mu_r \le b$. In order to ensure viability in finite time from one set-point to another [1, 8], the reference $r(t)$ is further restricted to belong to the set

$$R = \{r : Lx_r + Mu_r \le b - \delta \, \mathbf{1}\}$$

where $\mathbf{1}$ is a vector of ones and $\delta > 0$ is arbitrarily small.

3 LPV embedding approach

The approach pursued in this paper will make use of *constraint-admissible invariant sets* [10, 11]. In this respect, note that nonlinearity of (1) makes difficult the construction of such sets. Hence, the constrained nonlinear system (1)-(2) is embedded into an LPV model

$$\left\{ \begin{array}{lcl} i(t+1) & \in & \mathcal{Q}(i(t)) \\ \tilde{x}(t+1) & \in & \mathcal{F}(i(t)) \begin{bmatrix} \tilde{x}(t) \\ \tilde{u}(t) \end{bmatrix} \end{array} \right. \tag{4}$$

subject to linear constraints of the form

$$L_{i(t)}\tilde{x}(t) + M_{i(t)}\tilde{u}(t) + H_{i(t)}r \le b_{i(t)} \tag{5}$$

where: $\tilde{x}(t) \stackrel{\triangle}{=} x(t) - x_r$; $\tilde{u}(t) \stackrel{\triangle}{=} u(t) - u_r$; $i(t) \in \mathcal{I} \stackrel{\triangle}{=} \{1, 2, \ldots, \ell\}$; $\mathcal{F}(i) \subset \mathbb{R}^{n \times (n+m)}$ is a polytope of matrices; $\mathcal{Q}(i)$ is a subset of $\mathcal{I}$. Moreover we shall assume that each index j is reachable from any index i by iterating the dynamics $\mathcal{Q}$. Some remarks on the structure of the model (4)-(5) and its connections with the original system (1)-(2) are in order.

Remarks

- The model (4) consists of a family of ℓ polytopic submodels $\mathcal{F}(i)$ parametrized by the index $i \in \mathcal{I}$, along with a set-valued model $i(t+1) \in \mathcal{Q}(i(t))$ of the index time-evolution.
- The model (4) can be used to *safely* predict the behaviour of the original system (1) in the sense that trajectories of (1) are also trajectories of (4) (the vice-versa is not true, which makes the embedding *conservative* to some extent). More precisely, each polytopic submodel $\mathcal{F}(i)$ is used to predict the future state $x(t+1)$ whenever the current state $x(t)$ belongs to a suitable region $X_i \subset \mathbb{R}^n$. Since the regions X_i need not be disjoint, i.e. are possibly overlapping, let $\mathcal{I}(x(t)) \stackrel{\triangle}{=} \{i : x(t) \in X_i\}$. Clearly all polytopic models $\mathcal{F}(i)$, $i \in \mathcal{I}(x(t))$, are valid for a given state $x(t)$.
- The embedded model (4) is conveniently expressed in terms of the shifted variables $\tilde{x}(t)$ and $\tilde{u}(t)$, instead of $x(t)$ and $u(t)$, consistently with the fact that, irrespectively of r, $\tilde{x}(t)$ and $\tilde{u}(t)$ must be steered to zero.
- The system (1) can be embedded into (4) in many different ways and the embedding procedure, which is the core of gain-scheduling control design, will not be discussed here. The reader is referred to [7, 14, 16] for details. It is important to stress that the choice of the number ℓ of polytopic submodels $\mathcal{F}(i)$ as well as of the size of their validity regions X_i must tradeoff conservatism versus complexity of the model (4).
- Even if x_r and u_r may depend non linearly on r, the form of the constraints (5) is kept linear for the sake of simplicity, at the price of possible conservatism, by suitable linear approximation or embedding of the mappings x_r and u_r.
- The constraint matrices in (5) may depend on the index i to take into account rate constraints on the state of the form $\|T\,(x(t+1) - x(t))\|_\infty \le \delta x$, for a suitable matrix T that selects the variables to which impose rate constraints. Such constraints are typically imposed in gain-scheduling control design [7, 14] so as to guarantee that the state move sufficiently slowly across the regions X_i, i.e. that the dynamics of $i(t)$ is not too fast. ■

4 Admissible sets under gain-scheduling control

Let us consider a gain-scheduling feedback control law

$$\tilde{u}(t) = F_{i(t)}\,\tilde{x}(t) \tag{6}$$

where each gain F_i, $i \in \mathcal{I}$, has been designed so as to stabilize the i-th polytopic submodel $\tilde{x}(t+1) \in \mathcal{F}(i)\,[\tilde{x}(t)',\ \tilde{u}(t)']'$. It is important to establish whether the control law (6) stabilizes the LPV system (4) and satisfies the constraints (5) as in such a case the same properties are also guaranteed for the original nonlinear system (1). Hence, consider the closed-loop system

$$\left\{ \begin{array}{lcl} i(t+1) & \in & \mathcal{Q}(i(t)) \\ \tilde{x}(t+1) & \in & \mathcal{F}(i(t)) \begin{bmatrix} \tilde{x}(t) \\ F_{i(t)}\,\tilde{x}(t) \end{bmatrix} \end{array} \right. \tag{7}$$

Definition - Given μ, $0 \le \mu < 1$, a set $\Sigma \in \mathcal{I} \times \mathbb{R}^n \times \mathbb{R}^p$ is said μ*-contractive* under the closed-loop dynamics (9) if

$$\begin{bmatrix} i(t) \\ \tilde{x}(t) \\ r \end{bmatrix} \in \Sigma \implies \begin{bmatrix} i(t+1) \\ \mu^{-1}\tilde{x}(t+1) \\ r \end{bmatrix} \in \Sigma \tag{8}$$

If in addition all elements $[i', \tilde{x}', r']' \in \Sigma$ satisfy the constraints

$$(L_i + M_i F_i)\,\tilde{x} + H_i r \le b_i, \quad r \in R \tag{9}$$

then Σ is said *constraint-admissible*. ■

The existence of a constraint-admissible μ-contractive set for (7) implies therefore that the gain-scheduling feedback (6) ensures exponential stability with rate of decay $\mu < 1$ and constraint satisfaction. Thus, to check whether (6) solves (locally) our constrained regulation problem and to find the domain of initial state perturbations $\tilde{x} = x - x_r$ and set-points r for which constrained regulation is achieved, one can construct, for some $\mu < 1$, the largest constraint-admissible μ-contractive set Σ_0. For convenience let us represent the set $\Sigma_0 \subset \mathcal{I} \times \mathbb{R}^{n+p}$ as a collection of ℓ sets $\Sigma_0^1, \Sigma_0^2, \ldots, \Sigma_0^\ell \subset \mathbb{R}^{n+p}$ and let $\mathcal{C}_i$ denote the polytope of vectors $[\tilde{x}', r']'$ that satisfy (9). Then, Σ_0 is constructed via the following set recursion:

$$\begin{array}{l} O_0^i \; = \; \mathcal{C}_i \quad i = 1, 2, \ldots, \ell \\ O_k^i \; = \\ \mathcal{C}_i \cap \left\{ \begin{bmatrix} \tilde{x} \\ r \end{bmatrix} : \begin{bmatrix} \frac{\mathcal{F}(i)}{\mu} \begin{bmatrix} \tilde{x} \\ F_i \tilde{x} \end{bmatrix} \\ r \end{bmatrix} \subseteq \bigcap_{j \in Q(i)} O_{k-1}^j \right\} \\ \quad k = 1, 2, \ldots \\ \quad i = 1, 2, \ldots, \ell \end{array} \tag{10}$$

For $k \to \infty$, O_k^i converge to Σ_0^i; moreover the system (7) is exponentially stable with rate of convergence less or equal to μ if and only if Σ_0 has non-empty interior [3, 15]. Finally, under weak conditions [7], Σ_0 is finitely determined i.e. there exists $k^* < \infty$ such that $O_{k^*}^i = O_{k^*+1}^i = \Sigma_0^i$. For the subsequent developments, it will be assumed that the maximal constraint-admissible μ-contractive set for (7), Σ_0, has been constructed and that Σ_0 has non empty interior and is finitely determined. Please note that, since $\mathcal{C}_i$ and $\mathcal{F}(i)$ are polytopic, also Σ_0^i are polytopic and the recursion (10) is easily implemented by stacking linear inequality constraints and possibly removing redundant inequalities by standard linear programming tools.

5 Dual Mode Set-point Tracking

The main difficulty of tracking a piecewise constant reference in presence of constraints is that a large and abrupt set-point change may easily imply large constraint violations. Clearly, the set Σ_0 discussed in the previous section provides valuable information for the design of an effective control law which, on one hand, avoids constraint violations and, on the other hand, yields as fast as possible tracking. In fact, given Σ_0 it is easy to check whether a state x is feasible for a set-point r, or equivalently a set-point r is admissible for a state x; this just requires to check the membership condition

$$\begin{bmatrix} x - x_r \\ r \end{bmatrix} \in \Sigma_0^{\mathcal{I}(x)}, \quad \Sigma_0^{\mathcal{I}(x)} \triangleq \bigcup_{i \in \mathcal{I}(x)} \Sigma_0^i$$

A simple tracking strategy can be obtained by adapting the reference governor policy to the present setup. This yields the following algorithm.

Reference Governor Gain-Scheduling Tracking (RGGST) algorithm - At time t, given the state $x(t)$ and the desired reference $r(t) = r$, let $\mathcal{I}(t) \triangleq \mathcal{I}(x(t)) = \{i : x(t) \in X_i\}$ and assume that a set-point $\overline{r}(t)$ admissible for $x(t)$ is given. Then solve the optimization problem

$$\begin{array}{c} \begin{bmatrix} i(t) \\ \lambda(t) \end{bmatrix} = \arg \min_{i \in \mathcal{I}(t), \lambda} \lambda \\ \textbf{subject to} \\ \left\{ \begin{array}{l} 0 \le \lambda \le 1 \\ \begin{bmatrix} x(t) - x_{\lambda \overline{r}(t) + (1-\lambda) r} \\ \lambda \overline{r}(t) + (1-\lambda) r \end{bmatrix} \in \Sigma^i, \end{array} \right. \end{array} \tag{11}$$

Set $\overline{r}(t+1) = r + \lambda(t)\,[\overline{r}(t) - r]$
Set $u(t) = u_{\overline{r}(t+1)} + F_i[x(t) - x_{\overline{r}(t+1)}]$ ■

The rationale of the above algorithm is, provided that $x(t)$ is feasible for $\overline{r}(t)$, to investigate if $x(t)$ is feasible for a reference $\overline{r}(t+1)$ which is closer to the desired set-point r. Notice that the optimization (11) amounts to linear programming problems, one for each value of $i \in \mathcal{I}(t)$, in the single scalar variable λ.

An effective tracking strategy, consisting of two different modes of operation, could be the following.

- **Regulation Mode** - If the current state is feasible for the desired set-point, use the gain-scheduling feedback control law (6).

- **Feasibility Recovery Mode** - If conversely the current state is infeasible for the desired set-point, choose an input that will make the future state feasible for a set-point as close as possible to the desired one.

This *Dual Mode Tracking* strategy is formalized by the following algorithm.

Dual Mode Gain-Scheduling Tracking (DMGST) algorithm - At time t, given the state $x(t)$, let $\mathcal{I}(t) \triangleq \mathcal{I}(x(t)) = \{i : x(t) \in X_i\}$ and assume that a set-point $\overline{r}(t)$ admissible for $x(t)$ is given. Then the DMGST algorithm operates in dual-mode as follows.

• **Regulation Mode**

If $\begin{bmatrix} x(t) - x_{r(t+1)} \\ r(t+1) \end{bmatrix} \in \Sigma_0^{\mathcal{I}(t)}$ **then** $u(t) = u_{r(t+1)} + F_i\left(x(t) - x_{r(t+1)}\right)$ for some $i \in \mathcal{I}(t)$

• **Feasibility Recovery Mode**

If $\begin{bmatrix} x(t) - x_{r(t+1)} \\ r(t+1) \end{bmatrix} \notin \Sigma_0^{\mathcal{I}(t)}$ **then**

$$\begin{bmatrix} i(t) \\ \lambda(t) \\ u(t) \end{bmatrix} = \arg \min_{i \in \mathcal{I}(t), \lambda, u} \lambda \text{ subject to}$$
$$0 \leq \lambda \leq 1$$
$$Lx(t) + Mu \ \leq b$$
$$\begin{bmatrix} f(x(t), u) - x_{\lambda \overline{r}(t) + (1-\lambda) r(t+1)} \\ \lambda \overline{r}(t) + (1-\lambda) r(t+1) \end{bmatrix} \in \Sigma_0^j, \ \forall j \in \mathcal{Q}(i) \tag{12}$$

Set $\overline{r}(t+1) \ = \ r(t+1) + \lambda(t)(\overline{r}(t) - r(t+1))$ ■

Remarks

- The above DMGST algorithm is a non trivial extension to nonlinear systems of the strategy proposed in [6] for linear systems.
- The DMGST approach requires the determination of Σ_0 which certainly is a computationally expensive task but, luckily, can be carried out off-line. As far as on-line computation is concerned, the most expensive part is the solution of the optimization problem (12). Note that, if h denotes the cardinality of the set $\mathcal{I}(t)$, (12) amounts to h nonlinear optimization problems, one for each value of i, in $m+1$ scalar variables $\lambda \in [0, 1]$ and $u \in \mathbb{R}^m$. Also note that if x_r is linear in r and the nonlinear dynamics $f(x, u)$ is replaced by the LPV dynamics

  $$x_r + \mathcal{F}(i) \begin{bmatrix} x - x_r \\ u - u_r \end{bmatrix}$$

  such problems reduce to linear programming ones.
- The Dual-Mode approach of this paper differs from the Reference Governor (RG) approach [1]. In fact, to recover feasibility the DMGST algorithm fully exploits the plant control input u as degree of freedom, while the RG uses a command input $\overline{r}$.

The proposed DMGST algorithm enjoys the following property.

Theorem - If $x(0)$ is feasible for some $\overline{r}(0) \in R$ and $r(t) = r \in R$ for all $t \geq 0$, the DMGST algorithm guarantees a *finite recovery time* (FRT) i.e. the existence of $\overline{t} < \infty$ such that $x(\overline{t})$ is feasible for the desired set-point r. Further, under the same conditions, the constraints (2) are satisfied for all $t \geq 0$ and the system asymptotically reaches the desired equilibrium i.e. $\lim_{t\to\infty} x(t) = x_r$, $\lim_{t\to\infty} u(t) = u_r$ and $\lim_{t\to\infty} y(t) = r$.

Proof - The proof of the finite recovery time property follows similar lines as in [6] and is omitted here due to space considerations. The convergence to the equilibrium follows directly from the FRT property. In fact, since $x(\overline{t})$ is feasible for r, the gain-scheduling control law (6) will be activated at $\overline{t}$ providing constraint satisfaction for all $t \geq \overline{t}$ and convergence of $x(t) - x_r$ to zero, i.e. of $x(t)$ to x_r. ■

6 Simulation example

In this section, we consider the application of our approach to the strongly nonlinear model of a *continuous stirred tank reactor* (CSTR) [12]. Assuming constant liquid volume, the CSTR for an exothermic, irreversible reaction, $A \longrightarrow B$, is described by the following model

$$\begin{aligned} \dot{T} &= \tfrac{q}{V}(T_f - T) - \tfrac{\Delta H}{\rho C_p} k_o e^{(\frac{-E}{RT})} C_A + \tfrac{UA}{V\rho C_p}(T_c - T) \\ \dot{C}_A &= \tfrac{q}{V}(C_{AF} - C_A) - k_o e^{(\frac{-E}{RT})} C_A \end{aligned} \tag{13}$$

where C_A is the concentration of A in the reactor, T is the reactor temperature and T_c is the temperature of the coolant stream. The objective is to regulate $y = x_1 = T$ and $x_2 = C_A$ by manipulating $u = T_c$. The constraints are $280°K \leq T \leq 400°K$ and $240°K \leq T_c \leq 340°K$. In this example the adopted parameter values are those reported in [12]. It is possible to describe a parametrized family of equilibrium points for (13) through the choice of $x_1 = T$ as scheduling variable.

$$\begin{aligned} C_{A_{eq}}(T) &= \frac{q}{V} \frac{C_{AF}}{\frac{q}{V} + k_o e^{-\frac{E}{RT}}} \\ T_{c_{eq}}(T) &= \tfrac{V\rho C_p}{UA}\left(\tfrac{q}{V}(T - T_f) + \frac{\Delta H k_o}{\rho C_p} C_{A_{eq}}(T)\right) + T \end{aligned} \tag{14}$$

Then defining the new state and control variables [14]

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} T - r \\ C_A - C_{A_{eq}}(T) \end{bmatrix}, \quad \tilde{u} = T_c - T_{c_{eq}}(T) \tag{15}$$

we obtain the quasi-linear description

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{\Delta H}{\rho C_p} p_1 \\ 0 & -\frac{q}{V} + p_3 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \frac{UA}{V\rho C_p} \begin{bmatrix} 1 \\ p_2 \end{bmatrix} \tilde{u} \tag{16}$$

with

$$p_1 = k_o e^{-\frac{E}{Rx_1}},\ p_2 = -\frac{\partial C_{A_{eq}}(x_1)}{\partial x_1} = \frac{qEC_{AF}}{VRx_1^2} \frac{k_o e^{-\frac{E}{Rx_1}}}{\frac{q}{V} + k_o e^{-\frac{E}{Rx_1}}},$$
$$p_3 = -p_1 - \frac{\Delta H}{\rho C_p} p_1 p_2$$

In the new coordinates, the input and state constraints become

$$240 \leq \tilde{u} + T_{c_{eq}}(x_1) \leq 340, \quad 280 \leq \tilde{x}_1 + r \leq 400 \tag{17}$$

Notice that the first constraint exhibit a nonlinear dependence on x_1. From (17) the set of admissible set-points

$T_s = 0.03\ min,\quad \mu = 0.99999,\quad \delta = 0.001,\quad \ell = 17$
$\mathcal{Q}(i) = \{\max\{1, i-1\}, i, \min\{i+1, \ell\}\}$ with $\delta x = 2.75$,
overlapping 45, # of vertices of $\mathcal{F}(i)$ $n_v = 4, \forall\ i$
P_i
$[330.00,\ 340.00]$, $[337.75,\ 342.75]$, $[340.50,\ 345.50]$,
$[343.25,\ 348.25]$, $[346.00,\ 351.00]$, $[348.75,\ 353.75]$,
$[351.50,\ 356.50]$, $[354.25,\ 359.25]$, $[357.00,\ 362.00]$,
$[359.75,\ 364.75]$, $[360.25,\ 370.25]$, $[365.75,\ 375.75]$,
$[371.25,\ 381.25]$, $[376.75,\ 386.75]$, $[382.25,\ 392.25]$,
$[387.75,\ 397.75]$, $[393.25,\ 400.00]$

Table 1: Control parameters

r turns out to be:

$$R = [280 + \delta, 400 - \delta],\quad \delta = 0.001. \qquad (18)$$

We selected 17 regions P_i as indicated in Table 1. For each i, a polytopic embedding of $[A(x_1), B(x_1)]$ with a minimum number of vertices ($n_v = 4$) has been found. Next, for each polytope $\mathcal{F}(i)$ we designed a stabilizing gain F_i with contraction factor $\varrho = 0.96$ using LMI techniques [4]; the parameter dynamic $\mathcal{Q}(i) = \{\max\{1, i-1\}, i, \min\{i+1, l\}\}$ was considered. The successful calculation of a non-empty Σ proves that the corresponding GS controller (6) is stabilizing under the imposed constraints. In this example, the number of constraints n_i defining the sets Σ^i, for a given r, ranges between 8 and 14. Figs 1 and 2 compare the behaviours obtained with DMGST and RGGST; it can be seen that the additional degree of freedom $\tilde{u}$ of DMGST allows to achieve a faster tracking after a set-point change.

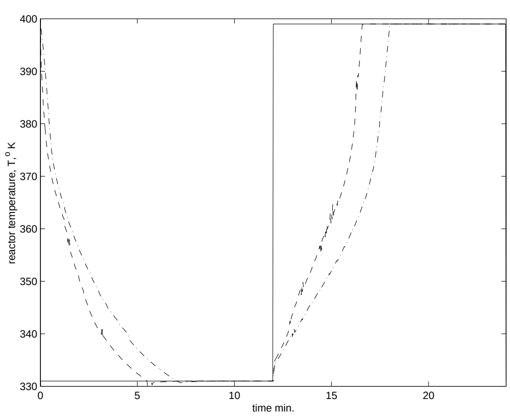

Figure 1: Desired set-point (solid), temperature responses for DMGST (dashed) and RGGST (dash-dotted) algorithms

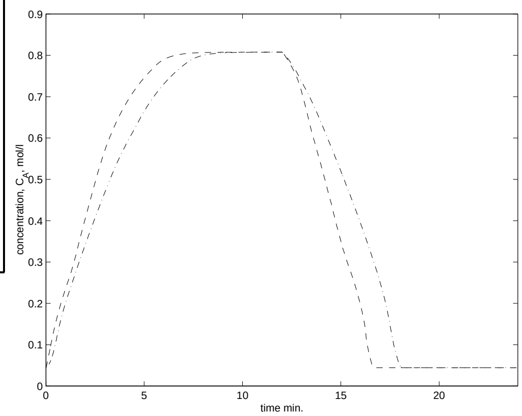

Figure 2: C_A for DMGST (dashed) and for RGGST (dash-dotted) algorithms

7 Conclusions

We proposed a novel control algorithm for making the output of a nonlinear system track a piecewise constant reference in presence of input and/or state constraints. The control algorithm is based on a gain-scheduling policy and makes use of the knowledge of a constraint admissible set in the state-reference space, which has been computed off-line for the nonlinear system in closed-loop with the gain-scheduling controller by means of LPV embedding techniques. Exploiting the valuable information of this constraint-admissible set, the controller can effectively command the transition between two set-points (and the relative gain-scheduled linear controllers) so as to avoid constraint violations and possibly optimize tracking speed.

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