Nonlinear Control and Observation of Induction Motors. Validation on an Industrial Benchmark*

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Abstract

An observer-based controller for tracking rotor flux and rotor speed references of an induction motor is designed, combining the advantages of the singular perturbation methods and sliding modes techniques. This control strategy is implementing on an experimental setup where experimental results are given.

Keywords: Singular Perturbations, Sliding Modes, Observer, Induction Motors.

1. INTRODUCTION

During the last few years, considerable research efforts have been directed toward the control problem of induction motors [1, 3, 4, 7]. Thanks to recent advances in nonlinear control techniques and power electronics, the implementation of powerful nonlinear control laws is possible.

This paper deals with the design of an observer-based controller for tracking rotor flux and rotor speed references for induction motors, combining the advantages of the singular perturbation methods and sliding modes techniques and to implement it on an experimental setup.

It is well-known that many physical systems involve dynamical phenomena occurring in different time scales [6]. Typically, these systems can be modeled using singular perturbation approach which allows to obtain subsystems of reduced dimension in order to design control laws and analyze their stability.

The sliding-mode control as a robust approach has attracted a number of research, see [8]. It is characterized as a high-speed switching controller that provides a robust mean of controlling nonlinear systems by forcing the trajectories to reach a sliding manifold in finite time and stay on the manifold for all time. This leads to theoretical and practical problems. As the controller contains a discontinuous nonlinear term, the existence and uniqueness of solutions should be examined. On the other hand, the implementation of discontinuous controllers yield the phenomenon of chattering which can be avoided by approximating the discontinuous control law by a continuous one.

Furthermore, the observer design for induction motors is an important problem in control theory and of great practical importance as well [9]. The proposed observer is a nonlinear high gain observer.

This paper is organized as follows. In Section 2, the design of a controller based on singular perturbation methods is introduced. In order to overcome the problem of estimating the unmeasurable variables, a nonlinear observer is given in Section 3. In Section 4, we briefly introduce the induction motor considered. In Section 5, the observer-based controller scheme obtained is implemented in a semi-industrial experimental setup and tested with a benchmarck. Finally, conclusions are given.

2. PRELIMINARIES

Consider the class of nonlinear singularly systems described by (see [6]):

$$\dot{x} = f_1(x) + F_1(x)z + g_1(x)u, \qquad x(t_0) = x_0, \quad (1)$$

$$\varepsilon\,\dot{z} = f_2(x) + F_2(x)z + g_2(x)u, \qquad z(t_0) = z_0, \quad (2)$$

where $t_0 \geq 0$, $x \in B_x \subset R^n$ $z \in B_z \subset R^m$ are the *slow state* and *fast state*, respectively; $u \in R^r$ is the *control input* and $\epsilon \in [0, 1)$ is the small *perturbation parameter*. f_1, f_2, the columns of the matrices F_1, F_2, g_1 and g_2 are assumed to be bounded with their components being smooth functions of x. B_x and B_z denote closed and bounded subsets centered at the origin. $F_2(x)$ is assumed to be nonsingular for all $x \in B_x$.

When $\varepsilon = 0$ in (1),(2), the $n-th$ order *slow system* can be obtained as

$$\dot{x}_s = f(x_s) + g(x_s)u_s, \qquad x_s(t_0) = x_0 \quad (3)$$

$$z_s = h(x_s) := -F_2^{-1}(x_s)\left[f_2(x_s) + g_2(x_s)u_s\right] \quad (4)$$

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where x_s, z_s and u_s denote the slow components of the original variables x, z and u, respectively, and $f(x_s) = f_1(x_s) - F_1(x_s)F_2^{-1}(x_s)f_2(x_s)$; $g(x_s) = g_1(x_s) - F_1(x_s)F_2^{-1}(x_s)g_2(x_s)$. In (3) and (4), $u_s(x_s)$ denotes the slow state feedback which only depends on x_s. The fast dynamics is obtained in replacing the slow time scale t by the fast time scale $\tau := (t - t_0)/\epsilon$ and introducing the deviation of z from M_ε, *i.e.* $\eta := z - h_e(x, \epsilon)$. The original system (1),(2) then becomes

$$\frac{d\widetilde{x}}{d\tau} = \varepsilon \left\{ f_1(\widetilde{x}) + F_1(\widetilde{x})[\eta + h_e(\widetilde{x}, \epsilon)] + g_1(\widetilde{x})u \right\} \quad (5)$$

$$\frac{d\eta}{d\tau} = f_2(\widetilde{x}) + F_2(\widetilde{x})[\eta + h_e(\widetilde{x}, \epsilon)] + g_2(\widetilde{x})u - \frac{\partial h_e(\widetilde{x}, \epsilon)}{\partial \widetilde{x}} \frac{d\widetilde{x}}{d\tau} \quad (6)$$

where $\eta(0) = z_0 - h(x_0)$, $\widetilde{z}(\tau) := z(\varepsilon\tau + t_0)$, with $\widetilde{z}(0) = z_0$, and $\widetilde{x}(\tau) := x(\varepsilon\tau + t_0)$, with $\widetilde{x}(0) = x_0$. The *composite control* for the original system (1),(2) is defined by $u(x, \eta, \epsilon) = u_{es}(x, \epsilon) + u_{ef}(x, \eta, \epsilon)$, where u_{es} and u_{ef} denote the *slow* and *fast* control components, respectively. The component u_{ef} is used to make M_ε attractive and vanishes there, *i.e.* $u_{ef}(x, 0, \epsilon) = 0$. If $u_{es}(\widetilde{x}, \epsilon)$ and $\partial h_e(\widetilde{x}, \epsilon)/\partial \widetilde{x}$ are bounded and $\widetilde{x}$ remains relatively constant with respect to τ, then the term $\epsilon(\partial h_e(\widetilde{x}, \epsilon)/\partial \widetilde{x})$ can be neglected for ϵ sufficiently small. Since the equation (6) defines the *fast reduced* subsystem, an $O(\epsilon)$ approximation can be obtained for this subsystem using equation (4) and setting $\varepsilon = 0$ in (5),(6), this is

$$\frac{d\eta_{apx}}{d\tau} = F_2(\widetilde{x})\eta_{apx} + g_2(\widetilde{x})\, u_f \quad (7)$$

where η_{apx}, $h_e(\widetilde{x}, 0) = h(\widetilde{x})$ and u_f are $O(\epsilon)$ approximations for η, $h_e(\widetilde{x}, \varepsilon)$ and u_{ef} during the initial boundary layer and $\eta_{apx}(0) = z_0 - h(x_0, 0)$.

2.1- Sliding-Mode Control Design.

The sliding-mode control for the system (1),(2) is designed in two stages as follows. First, consider the following $(n - r)$-dimensional slow nonlinear switching surface defined by $\sigma_s(x_s, x_{sd}) = col\,(\sigma_{s_1}(x_s, x_{sd}), ..., \sigma_{s_r}(x_s, x_{sd})) = 0$, where $x_{sd} = col(x_{sd_1}, ..., x_{sd_n})$ is a reference vector and each function $\sigma_{s_i} : B_x \times B_x \to R$, $i = 1, ..., r$, is a C^1 function such that $\sigma_{s_i}(0, 0) = 0$. The *equivalent control method* (see [8]) is used to determine the slow reduced system motion restricted to the slow switching surface $\sigma_s(x_s, x_{sd}) = 0$, leading to the *slow equivalent control*

$$u_{se} = -\left[\frac{\partial \sigma_s}{\partial x_s} g(x_s)\right]^{-1} \left[\frac{\partial \sigma_s}{\partial x_s} f(x_s) + \frac{\partial \sigma_s}{\partial x_{sd}} \overset{\bullet}{x}_{sd}\right] \quad (8)$$

where the matrix $[\partial \sigma_s/\partial x_s]g(x_s)$ is assumed to be nonsingular for all $x_s, x_{sd} \in B_x$.

In order to complete the slow control design one sets

$$u_s = u_{se} + u_{sN} \quad (9)$$

where u_{se} is the slow equivalent control (8), which acts when the slow reduced system is restricted to $\sigma_s(x_s, x_{sd}) = 0$, while u_{sN} acts when $\sigma_s(x_s, x_{sd}) \neq 0$. In this work the control u_{sN} is selected as

$$u_{sN} = -[\frac{\partial \sigma_s}{\partial x_s} g(x_s)]^{-1} L_s(x_s)\sigma_s(x_s, x_{sd}) \quad (10)$$

where $L_s(x_s)$ is an $r \times r$ positive definite matrix whose components are C^0 bounded nonlinear real functions of x_s, such that $\|L_s(x_s)\| \leq \rho_s$ for all $x_s \in B_x$ with a constant $\rho_s > 0$. The equation that describes the projection of the slow subsystem motion outside $\sigma_s(x_s, x_{sd}) = 0$ is given by

$$\overset{\bullet}{\sigma}_s (x_s, x_{sd}) = -L_s(x_s)\sigma_s(x_s, x_{sd}). \quad (11)$$

The stability properties of $\sigma_s(x_s, x_{sd}) = 0$ in (11) can be studied by means of the Lyapunov function candidate $V(x_s, x_{sd}) = \frac{1}{2}\sigma_s^T(x_s, x_{sd})\sigma_s(x_s, x_{sd})$, whose time derivative along (11) satisfies $\overset{\bullet}{V} (x_s, x_{sd}) = -\sigma_s^T(x_s, x_{sd})L_s(x_s)\sigma_s(x_s, x_{sd}) < 0$, for all $x_s, x_{sd} \in B_x$, thus assuring the existence of a slow sliding mode.

On the other hand, the *fast control* design for the subsystem (7) can be obtained in a similar way . One considers an $(m - r)$-dimensional fast switching surface defined by $\sigma_f(\eta_{apx}, x_{fd}) = (\sigma_{f_1}(\eta_{apx}, x_{fd}), ..., \sigma_{f_r}(\eta_{apx}, x_{fd}))^T = 0$, where $x_{fd} = (x_{fd_1}, ..., x_{fd_m})^T$ is the reference vector and each function $\sigma_{f_i} : B_z \times B_z \to R$, $i = 1, ..., r$, is also a C^1 function such that $\sigma_{f_i}(0, 0) = 0$. The complete fast control takes the form

$$u_f = u_{fe}(\widetilde{x}, \eta_{apx}, x_{fd}) + u_{fN}(\widetilde{x}, \eta_{apx}, x_{fd}) \quad (12)$$

where u_{fe} is the *fast equivalent control* given by

$$u_{fe} = -\left[\frac{\partial \sigma_f}{\partial \eta_{apx}} g_2(\widetilde{x})\right]^{-1} \left[\frac{\partial \sigma_f}{\partial \eta_{apx}} F_2(\widetilde{x})\eta_{apx} + \frac{\partial \sigma_f}{\partial x_{fd}} \frac{dx_{fd}}{d\tau}\right] \quad (13)$$

and

$$u_{fN} = -\left[\frac{\partial \sigma_f}{\partial \eta_{apx}} g_2(\widetilde{x})\right]^{-1} L_f(\eta_{apx})\sigma_f(\eta_{apx}, x_{fd}), \quad (14)$$

In (13) and (14), the matrix $[\partial \sigma_f/\partial \eta_{apx}]g_2(\widetilde{x})$ is assumed to be nonsingular for all $(\widetilde{x},\ \eta_{apx},\ x_{fd}) \in B_x \times B_z \times B_z$, and $L_f(\eta_{apx})$ is a positive definite matrix of dimension $r \times r$, whose components are C^0 bounded nonlinear real functions of η_{apx}, such that $\|L_f(\eta_{apx})\| \leq \rho_f$, for all $(\widetilde{x}, \eta_{apx}, x_{fd}) \in B_x \times B_z \times B_z$, with a constant ρ_f. The projection of the fast subsystem motion outside $\sigma_f(\eta_{apx}, x_{fd}) = 0$ is described by $d\sigma_f/d\tau = -L_f(\eta_{apx})\sigma_f(\eta_{apx}, x_{fd})$, and arguments similar to the ones used for the slow subsystem motion can be applied to this system to conclude the existence of a fast sliding-mode.

The stability properties of the slow and fast closed-loop systems are in [5].

Based on the reduced order sliding-mode control described above, the original slow and fast state variables are used to construct the composite control, *i.e.*

$$u(x, x_{sd}, \eta, x_{fd}) = u_s(x, x_{sd}) + u_f(x, \eta, x_{fd}) \quad (15)$$

where

$$u_s = -\left[\frac{\partial \sigma_s}{\partial x} g(x)\right]^{-1} \left[\frac{\partial \sigma_s}{\partial x} f(x) + L_s(x)\sigma_s(x, x_{sd})\right], \tag{16}$$

$$u_f = -\left[\frac{\partial \sigma_f}{\partial \eta} g_2(x)\right]^{-1} \left[\frac{\partial \sigma_f}{\partial \eta} F_2(x)\eta + L_f(\eta)\sigma_f(\eta, x_{fd})\right]. \tag{17}$$

When the composite control (15),(16),(17) is substituted in (1),(2), one obtains the closed-loop nonlinear singularly perturbed system

$$\overset{\bullet}{x} = f_c(x, \eta, x_{sd}) \tag{18}$$

$$\varepsilon \overset{\bullet}{\eta} = g_c(x, \eta, x_{fd}) - \epsilon \frac{\partial h}{\partial x}\left[f_c(x, \eta, x_{sd})\right] \tag{19}$$

where $\eta = z - h(x)$, $x(t_o) = x_o$, $z(t_o) = z_o$ and $f_c(x, \eta, x_{sd}) = f(x) + F_1(x)\eta - g(x)\left[\frac{\partial \sigma_s}{\partial x} g(x)\right]^{-1} \times$

$$\left[\frac{\partial \sigma_s}{\partial x} f(x) + L_s(x)\sigma_s(x, x_{sd})\right].$$

The stability properties of the complete closed-loop system can be studied by using, for each subsystem, Lyapunov methods (see [5] for details).

3. OBSERVER SYNTHESIS

Consider the following class of nonlinear systems

$$\begin{aligned} \overset{\bullet}{X}_1 &= A_1(y)X_1 + g_1(u, y, X_1) \\ \overset{\bullet}{X}_i &= A_i(y)X_i + g_i(u, y, X_1, \cdots, X_i) \\ y_i &= C_i X_i; \quad i = 1, \cdots, n. \end{aligned} \tag{20}$$

where $X_i = (x_{i,1}, x_{i,2}, ..., x_{i,r_i})^T \in I\!R^{n_i}$ is the state of system i, $x_{i,j} \in I\!R^{n_i}$ are components of the vector X_i, $\dim X_i = n_i$, for $i = 1, ...n$, $\sum_{i=1}^{n} n_i = N$ is the total dimension of the whole system, $y_i \in I\!R^{p_i}$ is the output vector of the subsystem i, $\dim y_i = p_i$, $\sum_{i=1}^{n} p_i = P$ is the total dimension of the output space, r_i represents the partition of the vector X_i such that $r_i p_i = n_i$. The matrices $A_i \in I\!R^{n_i \times n_i}$, $i = 1, 2, ..., n$; depending on the inputs and outputs are given by

$$A_i(y) = \begin{pmatrix} O_{p_i \times p_i} & A_{1,i}(y) & \cdots & O_{p_i \times p_i} \\ \vdots & \vdots & \ddots & \vdots \\ O_{p_i \times p_i} & O_{p_i \times p_i} & \ddots & A_{r_i - 1, i}(y) \\ O_{p_i \times p_i} & O_{p_i \times p_i} & \cdots & O_{p_i \times p_i} \end{pmatrix}$$

where $A_i \in I\!R^{p_i \times p_i}$ $\forall j = 1, ..., r_i - 1$; are not singular submatrices of constant rank for any input u and output y and $g_i(u, y, X_1, \cdots, X_i) =$

$$\begin{pmatrix} g_{1,i}(u, y, X_1, \cdots, X_{i-1}, x_{i,1}) \\ g_{2,i}(u, y, X_1, \cdots, X_{i-1}, x_{i,1}, x_{i,2}) \\ \vdots \\ g_{r_i,i}(u, y, X_1, \cdots, X_{i-1}, x_{i,1}, ..., x_{i,r_i}) \end{pmatrix} \in I\!R^{p_i},$$

$\forall j = 1, 2, ..., r_i$.

The output matrix of the system i constant and is given by the following form $C_i = (I_{p_i \times p_i}, O_{p_i \times p_i}, \cdots, O_{p_i \times p_i})$.

Now, we introduce the following assumptions:

Assumption B1: *Each subsystem is observable, i.e. the matrices*

$$\Gamma_i(y) = \begin{pmatrix} C_i \\ C_i A_i(y) \\ \vdots \\ C_i A_i^{n_i - 1}(y) \end{pmatrix}$$

for $i = 1, ..., n$; *have full rank for any* y. *More precisely, there exist a class* $\mathcal{U}$ *of bounded admissible inputs, a compact set* $\mathcal{K} \subset I\!R^n$, λ_1, λ_2 *positive constants such that for every* $u \in \mathcal{U}$, *and every* $y(t)$ *associated to* u *and the initial state* $X(0) \in \mathcal{K}$, *the matrices* $A_{j,i} \in I\!R^{p_i \times p_i}$ $\forall j = 1, ..., r_i - 1; i = 1, ..., n$; *satisfy*

$$0 < \lambda_1 I_{p_i \times p_i} \leq A_{j,i}^T(y) A_{j,i}(y) \leq \lambda_2 I_{p_i \times p_i}.$$

Assumption B2: *The matrices* $A_i(y); i = 1, ..., n$; *are of class* C^k , $k > 0$, *with respect to* y.

Assumption B3: *The vector fields* $g_i(u, y, X_1, \cdots, X_i)$, $i = 1, ..., n$; *are globally Lipschitz with respect to* $(X_1, \cdots, X_i)$, *and uniformly with respect to* u *and* y.

The matrix $\Gamma_i(y)$ can be rewritten as *block-diag* $\left\{I_{p_i \times p_i}, A_{1i}(y), ..., \prod_{j=1}^{r_i - 1} A_{ji}(y)\right\}$, for $i = 1, ..., n$. Then, by Assumption B1, the matrices $\Gamma_i(y), i = 1, ..., n$; are invertible.

Then, an observer for systems in cascade (20) is given by

$$\begin{aligned} \overset{\bullet}{Z}_1 &= A_1(y)Z_1 + g_1(u, y, Z_1) + M_1(y)C_1(Z_1 - X_1) \\ \overset{\bullet}{Z}_i &= A_i(y)Z_i + g_i(u, y, Z_1, \cdots, Z_i) \\ &\quad + M_i(y)C_i(Z_i - X_i), \end{aligned} \tag{21}$$

for $i = 2, \cdots, n$; where $M_i(y) = \Gamma_i^{-1}(y)\Delta_{\theta i}^{-1} K_i, i = 1, ..., n$; are the gains of the observer and depend on the input and the output, and $\Delta_{\theta i} = diag\left\{\frac{1}{\theta_i} I_{p_i \times p_i}, \frac{1}{\theta_i^2} I_{p_i \times p_i}, ..., \frac{1}{\theta_i^n} I_{p_i \times p_i}\right\}$ with $\theta_i > 0$, K_i is such that the matrix $(\bar{A}_i - K_i C_i)$ is stable where

$$\bar{A}_i = \begin{pmatrix} O_{p_i \times p_i} & I_{p_i \times p_i} & \cdots & O_{p_i \times p_i} \\ \vdots & \vdots & \ddots & \vdots \\ O_{p_i \times p_i} & O_{p_i \times p_i} & \ddots & I_{p_i \times p_i} \\ O_{p_i \times p_i} & O_{p_i \times p_i} & \cdots & O_{p_i \times p_i} \end{pmatrix}.$$

The proof of the convergence of this observer is given in [5].

Moreover, in [5] a stability analysis of the closed-loop observer-based controller system is given, where sufficient conditions are obtained in order to guarantee the ultimate boundedness of the system's variables. The analysis proves that this scheme can be applied to an experimental induction motor set-up.

4. APPLICATION TO THE INDUCTION MOTOR

The induction motor model considered under the classical assumptions of sinusoidal distribution of magnetic induction in the air-gap, no saturation of the magnetic circuit, the diphasic model $\alpha\beta$ is described by (see [1, 2, 7])

$$\Sigma_{NL}: \quad \overset{\bullet}{X} = \mathcal{F}(X) + \mathcal{G}u \tag{22}$$

where the state of the system is: $X = [\Omega, \phi_{r\alpha}, \phi_{r\beta}, i_{s\alpha}, i_{s\beta}]^T$, Ω is the mechanical speed, $\phi_{r\alpha}, \phi_{r\beta}$ are the rotor fluxes, $i_{s\alpha}, i_{s\beta}$ are the stator currents, and the input is: $u = [u_{s\alpha}, u_{s\beta}]^T$, $u_{s\alpha}, u_{s\beta}$ are stator voltages;

$$\mathcal{F}(X) = \begin{bmatrix} \frac{pM}{JL_r}(\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - \frac{f}{J}\Omega - \frac{1}{J}T_L \\ -\frac{1}{T_r}\phi_{r\alpha} - p\Omega\phi_{r\beta} + \frac{M}{T_r} i_{s\alpha} \\ p\Omega\phi_{r\alpha} - \frac{1}{T_r}\phi_{r\beta} + \frac{M}{T_r} i_{s\beta} \\ \frac{K}{T_r}\phi_{r\alpha} + p\Omega K\phi_{r\beta} - \gamma i_{s\alpha} \\ -p\Omega K\phi_{r\alpha} + \frac{K}{T_r}\phi_{r\beta} - \gamma i_{s\beta} \end{bmatrix},$$

$$\mathcal{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{\sigma L_s} \\ 0 & 0 & 0 & \frac{1}{\sigma L_s} & 0 \end{bmatrix}^T,$$

with M, L_r, L_s are the mutual, rotor, stator inductances respectively, T_L the load torque, J the inertia (motor and load), f the viscous damping coefficient, p the pole pair number, $T_r := \frac{L_r}{R_r}$, $K := \frac{M}{\sigma L_s L_r}$, $\sigma := 1 - \frac{M^2}{L_s L_r}$, $\gamma := \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}$.

We assume that the load torque is constant and unknown and the nominal values of the rotor resistance and the other parameters of the model are known.

4.1. MODEL REPRESENTATION

Consider the following assignment of variables: The *slow variables* $x = [x_1, x_2, x_3]^T = [\Omega, \phi_{r\alpha}, \phi_{r\beta}]^T$, the *fast variables* $z = [z_1, z_2]^T = [i_{s\alpha}, i_{s\beta}]^T$. Taking $\varepsilon = \sigma$, it is possible to write the model (22) in the standard singularly perturbed form

$$\begin{cases} \dot{x} = f_1(x) + F_1(x)z + g_1(x)u \\ \varepsilon\,\dot{z} = f_2(x) + F_2(x)z + g_2(x)u \end{cases}$$

with $f_1(x) = \begin{pmatrix} -\frac{f}{J}x_1 - \frac{1}{J}T_L \\ -\frac{1}{T_r}x_2 - px_1x_3 \\ px_1x_2 - \frac{1}{T_r}x_3 \end{pmatrix}$,

$$F_1(x) = \begin{pmatrix} -\frac{pM}{JL_r}x_3 & \frac{pM}{JL_r}x_2 \\ \frac{M}{T_r} & 0 \\ 0 & \frac{M}{T_r} \end{pmatrix}, \; g_1(x) = 0_{3\times 2},$$

$$f_2(x) = \begin{pmatrix} \frac{\bar{K}}{T_r}x_2 + p\bar{K}x_1x_3 \\ -p\bar{K}x_1x_2 + \frac{\bar{K}}{T_r}x_3 \end{pmatrix},$$

$$F_2(x) = -\bar{\gamma} I_{2\times 2}, \quad g_2(x) = \frac{1}{L_s} I_{2\times 2},$$

$$\bar{\gamma} := \varepsilon\gamma = \frac{R_s}{L_s} + \frac{R_r M^2}{L_s L_r^2} \text{ and } \bar{K} := \varepsilon K = \frac{M}{L_s L_r}.$$

4.2 CONTROL DESIGN

The slow reduced system (3) is described by the vector fields $f(x_s) =$

$$\begin{pmatrix} -\frac{1}{J}[\frac{p^2 M\bar{K}}{L_r\bar{\gamma}}x_{s_1}(x_{s_2}^2 + x_{s_3}^2) + fx_{s1} + T_L] \\ (\frac{M\bar{K}}{T_r\bar{\gamma}} - 1)(\frac{1}{T_r}x_{s_2} + px_{s_1}x_{s_3}) \\ (\frac{M\bar{K}}{T_r\bar{\gamma}} - 1)(\frac{1}{T_r}x_{s_3} - px_{s_1}x_{s_2}) \end{pmatrix},$$

$$g(x_s) = \begin{pmatrix} -\frac{pM}{JL_r}x_{s_3} & \frac{pM}{JL_r}x_{s_2} \\ \frac{M}{T_r} & 0 \\ 0 & \frac{M}{T_r} \end{pmatrix}.$$

Since the control objective is to make the mechanical speed x_{s_1} equal to the reference speed x_{sd_1} and the square rotor flux magnitude $x_{s_2}^2 + x_{s_3}^2$ equal to the reference flux $x_{sd_2}^2 + x_{sd_3}^2$, the following slow switching function is chosen :

$$\sigma_s(x_s - x_{sd}) = S \begin{pmatrix} (x_{s_1} - x_{sd_1}) \\ x_{s_2}^2 + x_{s_3}^2 - (x_{sd_2}^2 + x_{sd_3}^2) \\ \frac{d}{dt}(x_{s_1} - x_{sd_1}) \\ \frac{d}{dt}[x_{s_2}^2 + x_{s_3}^2 - (x_{sd_2}^2 + x_{sd_3}^2)] \end{pmatrix}$$

with $S = \begin{pmatrix} s_1 & 0 & s_2 & 0 \\ 0 & s_3 & 0 & s_4 \end{pmatrix}$, where $s_i > 0$, $i = 1, .., 4$; and $x_{sd} = (x_{sd_1}, x_{sd_2}, x_{sd_3})^T$ is the constant reference.

On the other hand, the fast system is given by

$$\frac{d\eta_{apx}}{d\tau} = F_2(\widetilde{x})\eta_{apx} + g_2(\widetilde{x})\,u_f,$$

where $F_2(\widetilde{x}) = -\bar{\gamma} I_{2\times 2}, \quad g_2(\widetilde{x}) = \frac{1}{L_s} I_{2\times 2}$.

In this application, one just needs to stabilize the fast variables to the origin. Choosing the fast switching function as $\sigma_f = \bar{S}\eta_{apx}$, where $\bar{S} = diag\,(\bar{s}_1, \bar{s}_2)$, with $\bar{s}_1$, $\bar{s}_2 > 0$. Finally, the fast control is given by

$$u_f = -\left[\bar{S}g_2(\tilde{x})\right]^{-1}\left[\bar{S}F_2(\tilde{x})\eta_{apx} + L_f(\eta_{apx})\bar{S}\eta_{apx}\right],$$

where $L_f(\eta_{apx}) = diag\,(l_{f_1}, l_{f_2})$.

4.3 OBSERVER DESIGN

The system (22) can be naturally represented in the form

$$\begin{aligned} \dot{X}_1 &= A_1(y)X_1 + g_1(u, y, X_1) \\ \dot{X}_2 &= A_2(y)X_2 + g_2(u, y, X_1, X_2) \\ y_i &= C_i X_i; i = 1, 2. \end{aligned} \qquad (23)$$

where the electrical subsystem is represented by $X_1 = (x_{1,1}, x_{2,1})^T = (i_{s\alpha}, i_{s\beta}, \phi_{r\alpha}, \phi_{r\beta})^T$ with $x_{1,1} = (i_{s\alpha}, i_{s\beta})^T$ and $x_{2,1} = (\phi_{r\alpha}, \phi_{r\beta})^T$;

$$A_1(y) = \begin{pmatrix} 0_{2\times 2} & KN(\Omega) \\ 0_{2\times 2} & 0_{2\times 2} \end{pmatrix};$$

$$g_1(u, y, X_1) =$$

$$\begin{pmatrix} -\gamma I_{2\times 2} & 0_{2\times 2} \\ \frac{M}{T_r} I_{2\times 2} & -N(\Omega) \end{pmatrix} X_1 + \begin{pmatrix} \frac{1}{\sigma L_s} I_{2\times 2} \\ 0_{2\times 2} \end{pmatrix} u;$$

$$N(\Omega) = \begin{pmatrix} \frac{1}{T_r} & p\Omega \\ -p\Omega & \frac{1}{T_r} \end{pmatrix}, \quad C_1 = \begin{pmatrix} I_{2\times 2} & 0_{2\times 2} \end{pmatrix}.$$

The measurable output is $y_1 = x_{1,1} = (i_{s\alpha}, i_{s\beta})^T$.

The mechanical subsystem is represented by $X_2 = (x_{2,1}, x_{2,2})^T = (\omega, T_l)^T$, $A_2(y) = \begin{pmatrix} 0 & \frac{1}{J} \\ 0 & 0 \end{pmatrix}$,

$$y_2 = \Omega, \text{ and } g_2(u, y, X_1) = \begin{pmatrix} \psi \\ 0 \end{pmatrix},$$

$$\psi = \frac{pM}{JL_r}(\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - \frac{f}{J}\Omega$$

Then, in order to estimate the flux and the load torque, the following observer is designed:

1) For the electrical subsystem:

$$\begin{aligned} \dot{Z}_1 = & \begin{pmatrix} 0_{2\times 2} & m_1 N(\omega) \\ 0_{2\times 2} & 0_{2\times 2} \end{pmatrix} Z_1 \\ & + \begin{pmatrix} -\gamma I_{2\times 2} & 0_{2\times 2} \\ m_2 I_{2\times 2} & -N(\omega) \end{pmatrix} Z_1 + \begin{pmatrix} m_3 I_{2\times 2} \\ 0_{2\times 2} \end{pmatrix} u \\ & - \begin{pmatrix} \theta_1 k_{11} I_{2\times 2} \\ \frac{\theta_1^2 k_{12}}{m_1} N^{-1}(\omega) \end{pmatrix} C_1(Z_1 - X_1). \end{aligned}$$

2) For the mechanical subsystem:

$$\dot{Z}_2 = \begin{pmatrix} 0 & -(1/J) \\ 0 & 0 \end{pmatrix} Z_2 + \begin{pmatrix} \hat{\psi} \\ 0 \end{pmatrix} - \begin{pmatrix} \theta_2 k_{21} \\ -\theta_2^2 k_{22} J \end{pmatrix} C_2(Z_2 - X_2),$$

where $\hat{\psi} = -(f_v/J)\,\omega + (pM_{sr}/JL_r)\left(\hat{\phi}_{r\alpha} i_{s\beta} - \hat{\phi}_{r\beta} i_{s\alpha}\right)$. Notice that in the term $\left(\hat{\phi}_{r\alpha} i_{s\beta} - \hat{\phi}_{r\beta} i_{s\alpha}\right)$, the estimated variables come from the first observer.

5.- EXPERIMENTAL RESULTS

In order to illustrate the performance of the proposed scheme, we now show some experimental results when the controller-observer scheme is implemented in a experimental setup of an induction motor, illustrated in Figure 1. The motor chosen is a 4-pole, 7.5 kW, three-phase induction motor with a squirrel-cage rotor fed by a PWM converter. The converter is built with bipolar transistors and provides a space vector modulation at a frequency up to 1000 Hz. The motor parameters are given in Table I. The induction motor load is simulated by a 7.5 kW DC motor fed by an inverter with current circulation which provides four quadrants operation. The hardware setup for controlling the motor consists of a dSPACE system with a TMS320C30 floating point DSP and interfaces boards for acquisition and measurement of different signals. The TMS320C30 operates at a 1 ms sampling period and the PWM works at 1 kHz.

Description	Param.	Value	Units
Rated power	P	7.5	kW
Rated speed	Ω_{nom}	1450	rev/min
Rated current	I_{nom}	16	A
Stator resistance	R_s	0.63	Ω
Rotor resistance	R_r	0.4	Ω
Mutual inductance	M	0.091	H
Stator inductance	L_s	0.097	H
Rotor inductance	L_r	0.091	H
Inertia	J	0.22	kgm^2
Viscous damping coef.	f	0.001	Ns/rad
Poles paires	p	2	

Tab. I: Motor parameters

In order to validate the controller on a wide operating domain, the control law is tested with the benchmark described in Figure 2. A torque disturbance is applied as shown in Figure 2.

In experiments, the observer parameters that provided the best control are $k_1 = 0.7$, $k_2 = 0.12$, $\theta_1 = 4.5$, $l_1 = 11.0$, $l_2 = 30$ and $\theta_2 = 3$. The parameters of the sliding mode controller are chosen as follows: $s_1 = 100$, $s_2 = 1$, $s_3 = 100$, $s_4 = 1$, $l_{s_1} = 200$, $l_{s_2} = 250$, $\bar{s}_1 = 1$, $\bar{s}_2 = 2$, $l_{f_1} = 10$, and $l_{f_2} = 10$. The sampling period is relatively large, so the discretization of the observer is made by developing the exponential expressions to the 4^{rd} order to improve the precision.

Figure 3 shows the results obtained for the rotor speed and motor torque using the proposed scheme. It includes torque due to acceleration and torque due to load. Figure 4 shows tracking errors on flux and speed. The flux tracking error can be determined using the estimates and the reference, because the real flux is not available for measurement. The tracking of flux norm and speed are given in Figure 5. This information is obtained using wires which are introduced in the stator. The obtained voltage with wires is integrated in order to have some evaluation of stator flux. For technological reasons, it is very difficult to measure flux and electromagnetic torque at low speed.

Figure 6 shows the estimated speed and load torque. Note that the measured flux is only available at nominal speed. In this case, we can see that the influence of rotor speed in the load torque estimation.

Finally, in order to show the robustness of the control strategy under parametric uncertainties in the system, we proceed as follows: First, the nominal value of R_r is augmented of 50%, *i.e.* ($\Delta R_r = +50$) in the induction motor model. Next, the observer and the control law design are done using the modified model. Finally, this control strategy is applied to the induction motor. We proceede in the same way, by decreasing the nominal value of R_r *i.e.* ($\Delta R_r = -50$). Figure 7 shows flux norm and speed, and the corresponding tracking errors are given in Figure 8.

CONCLUSIONS

A nonlinear control-observer strategy based on class of nonlinear systems has been developed and applied to an induction motor. The controller was designed using singular perturbation methods and sliding-modes. Furthermore, a high gain observer for a class of nonlinear systems was designed in order to estimate the rotor flux and the load torque. Using an experimetal set-up of the induction motor, results have been obtained where the good performance of the proposed control strategy has been showed.

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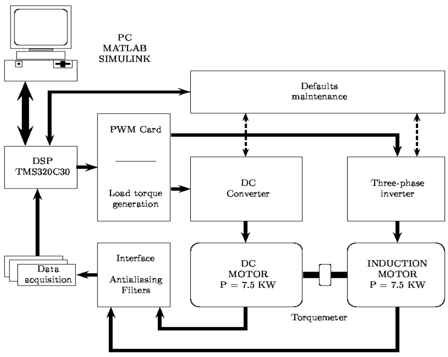

Figure 1. Experimental set-up.

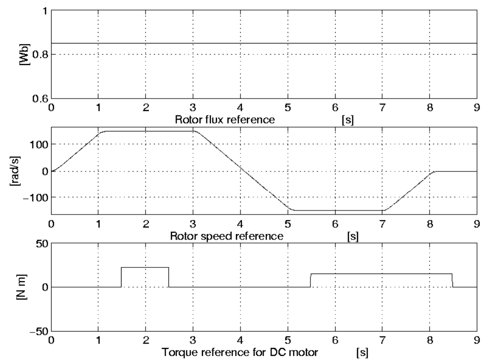

Figure 2. Benchmark: Reference Variables.

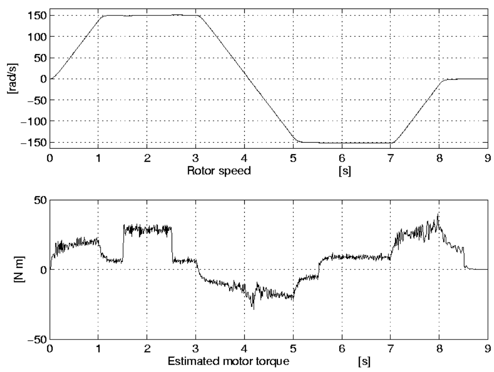

Figure 3. Speed and torque estimation.

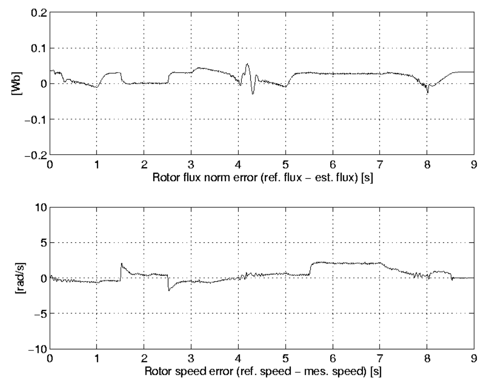

Figure 4. Tracking errors.

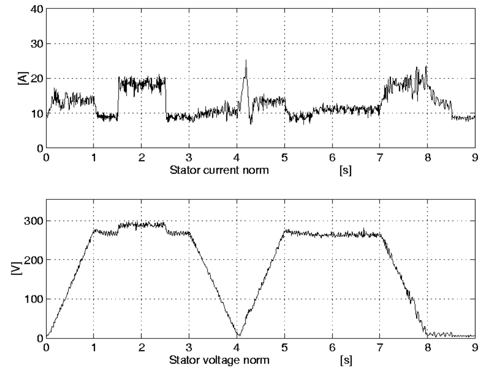

Figure 5. Stator current and stator voltage norms.

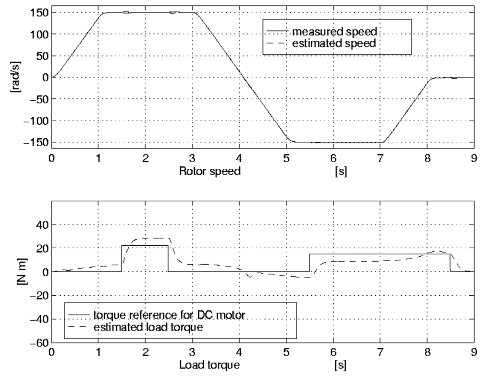

Figure 6. Estimated speed and load torque.

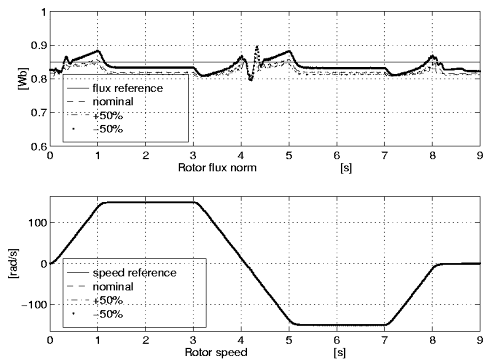

Figure 7. Rotor flux norm and rotor speed

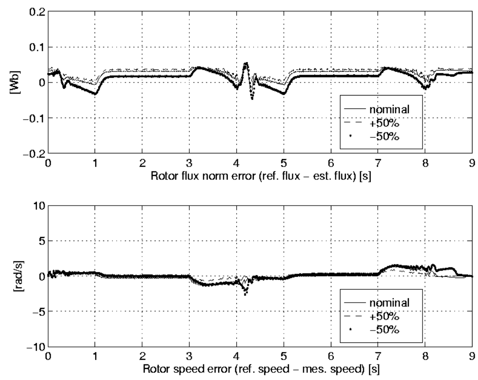

Figure 8. Robustness test: tracking errors.