USING THE BICAUSALITY CONCEPT TO BUILD REDUCED ORDER OBSERVERS IN LINEAR TIME INVARIANT SYSTEMS MODELLED BY BOND GRAPH

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Abstract

In this paper a bond graph approach to build reduced order observers for linear time invariant systems is shown. This approach uses the bicausality concept to simplify the construction and the calculation of the observer. The method is based on the Luenberger's algebraic method for the design of reduced order observers; some classical matrix calculations are simplified and the calculation of the inverse of matrices is not needed, which is an important improvement mainly for large-scale systems. The calculation of the observer gain is based on the pole placement techniques for linear systems modelled by bond graph. As application, one example with multiple outputs is developed.

1 Introduction

In some applications of automatic control, it is assumed that the entire state vector is measurable an thus available for feedback. Often, however, this is not the case; it is either impractical to measure some or all of the states, or the quality of the measurement, in terms of signal to noise ratio, is poor enough to reduce the utility of the compensator. In these instances we must estimate the unavailable state variables, using an observer. This approach will normally enable us to implement the compensator with acceptable performance.
For linear systems, one of the most used observers is the Luenberger observer [4]. When implemented in an observable system, Luenberger observers can be designed in a way that the difference between the actual system states and the states of the observer can be driven to zero.
If an observer is used to provide estimates of all state variables, it is referred to as a full-order state observer. Sometimes, however, it is possible to obtain a satisfactory measurement of some, but not all, states. In these instances a reduced-order state observer may be implemented, targeting the estimation of only the inaccessible states. In this paper, we will consider the design of reduced order linear state observers.

The main advantage for implementing a reduced order observer is that it will estimate only the state variables that cannot be accessible by the measurements; the order of the observer model will be lower than the order of a complete order observer, and thus the computational cost to estimate these variables is also lower.
From an algebraic point of view, the Luenberger's method [4] considers an observable linear time invariant system modelled by the following state equation:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{1}$$

with $A \in \Re^{n\times n}$, $B \in \Re^{n\times p}$, $C \in \Re^{m\times n}$, y regroups the outputs of the system.
The method for building reduced order observers consists in dividing the state variables of the model into accessible variables and non-accessible variables. The accessible variables are the state variables that can be directly measured from a sensor or can be calculated from the measurement of the sensor. With this classification, the state equation of the system can be written as a function of the accessible ($x_a \in \Re^m$) and non-accessible ($x_b \in \Re^{n-m}$) variables as follows:

$$\begin{aligned}\begin{bmatrix}\dot{x}_a \\ \dot{x}_b\end{bmatrix} &= \begin{bmatrix}A_{aa} & A_{ab} \\ A_{ba} & A_{bb}\end{bmatrix}\begin{bmatrix}x_a \\ x_b\end{bmatrix} + \begin{bmatrix}B_a \\ B_b\end{bmatrix}u \\ y &= \begin{bmatrix}C_a & C_b\end{bmatrix}\begin{bmatrix}x_a \\ x_b\end{bmatrix}\end{aligned} \tag{2}$$

Afterwards, with a linear transformation, the state equation is written as a function of the output and the non-accessible state

$$\begin{bmatrix}\dot{y} \\ \dot{x}_b\end{bmatrix} = \begin{bmatrix}\overline{A}_{aa} & \overline{A}_{ab} \\ \overline{A}_{ba} & \overline{A}_{bb}\end{bmatrix}\begin{bmatrix}y \\ x_b\end{bmatrix} + \begin{bmatrix}\overline{B}_a \\ \overline{B}_b\end{bmatrix}u \tag{3}$$

where:

$$\begin{aligned}\overline{A}_{aa} &= \left(C_a A_{aa} + C_b A_{ba}\right)C_a^{-1} \\ \overline{A}_{ab} &= \left(C_a A_{ab} + C_b A_{bb} - \overline{A}_{aa} C_b\right) \\ \overline{A}_{ba} &= A_{ba} C_a^{-1} \\ \overline{A}_{bb} &= A_{bb} - A_{ba} C_a^{-1} C_b \\ \overline{B}_a &= C_a B_a + C_b B_b \\ \overline{B}_b &= B_b\end{aligned} \tag{4}$$

With this representation the inverse of the C_a matrix is needed. Because of that, after the selection of x_a, the condition: rank(C_a) = m must be verified to guarantee the existence of C_a^{-1}.

From the state equation (3), the equation for $\hat{x}_b$ is:

$$\dot{\hat{x}}_b = \overline{A}_{ba}\, y + \overline{A}_{bb}\, \hat{x}_b + \overline{B}_b\, u + K\left(\overline{A}_{ab}\, x_b - \overline{A}_{ab}\, \hat{x}_b\right) \quad (5)$$

with $K \in \Re^{(n-m)\times m}$.

Thus, the dimension of the observer is equal to $(n - m)$, where n is the total number of state variables and m is the number of outputs of the system.

As x_b is non-accessible, in equation (5) the term $\overline{A}_{ab} x_b$ must be substituted by an expression derived from equation (3), which is a function of y and u as shown in equation (6):

$$\overline{A}_{ab} x_b = \dot{y} - \overline{A}_{aa} y - \overline{B}_a u \quad (6)$$

Afterwards, to avoid the time derivation of the output, the estimated state is calculated by means of an auxiliary variable, $\hat{z}$:

$$\hat{z} = \hat{x}_b - Ky \quad (7)$$

With this variable and an algebraic manipulation, the state equation for the estimated states is:

$$\dot{\hat{z}} = \overline{A}_{ba}\, y + \overline{A}_{bb} (\hat{z} + Ky) + \overline{B}_b\, u - K(\overline{A}_{aa}\, y + \overline{A}_{ab}\, (\hat{z} + Ky) + \overline{B}_a\, u) \quad (8)$$

The observer gives the following dynamics for the estimation error, $e = x_b - \hat{x}_b$:

$$\dot{e} = (\overline{A}_{bb} - K\overline{A}_{ab})e \quad (9)$$

Luenberger [4] showed that if the model in equation (1) is observable, the pair $(\overline{A}_{bb}, \overline{A}_{ab})$ is also observable, therefore, the eigenvalues of $(\overline{A}_{bb} - K\overline{A}_{ab})$ can be arbitrary selected to define the error dynamics.

The bond graph implementation is based on the Luenberger's method for reduced order observers in linear system [4]. Because of that, it is necessary for the bond graph model to be equivalent to equation (1). That means that in the bond graph model there are nor causal loops between R-elements neither derivative causalities that could generate an implicit state equation.

Taking advantage of the structural properties of bond graphs, in this work the technique showed by [9] is used to verify the structural observability of the model, which is a necessary condition for building the Luenberger's observers.

The computational capabilities of the bicausality concept have shown that this is an adequate tool for solving the problem of inverse systems [2]. From the observability point of view, Gawthrop [2] showed how to determine the initial conditions of the states in a model with given inputs, outputs and parameters. Ngwonpo et al. [6] used bicausality to derive the inverse system state equation from a bond graph model. With respect to the inverse of the C_a submatrix, Pichardo-Almarza et al. [7] have shown how to determine the invertibility of the C_a submatrix and how to calculate it directly from the bond graph using the bicausality concept.

The main objective of the present work is to use the results shown by [7] to generate a new procedure to build the observers in which some calculations are simplified; for example the calculation of C_a^{-1} to build the reduced order observer is not needed anymore, which is an important simplification mainly in large scale systems.

As an application, one example with three outputs is studied where the measurements are dependents on several state variables.

This paper is organized as follows: section 2 shows the bond graph implementation of reduced order observers. Section 3 shows the principles to design the observer starting from a bond graph model. Section 4 proposes the implementation of the method on one example. Finally Section 5 gives the conclusions of this work.

2 Bond Graph Implementation

As it has been shown until now, the Luenberger's method for reduced order observers in linear systems needs the algebraic manipulation of matrices, which includes the calculation of the inverse of C_a submatrix. In this paper a graphical method is shown that can be applied over a bond graph model directly to build the observers without generation or any manipulation of the state equations of the system.

The bond graph implementation is based on the Luenberger's method and it needs the calculation of the estimated state by means of the auxiliary variable, $\hat{z}$, as it is shown in equation (7). Then, equation (8) to calculate the estimated states can be written as:

$$\dot{\hat{z}} = \overline{A}_{ba}\, y + \overline{A}_{bb} (\hat{z} + Ky) + \overline{B}_b\, u + \phi \quad (10)$$

where: $\phi = -K(\overline{A}_{aa}\, y + \overline{A}_{ab}\, (\hat{z} + Ky) + \overline{B}_a\, u)$ (11)

In the present work some results of [7] are taken regarding the structure of the observers and the use of the bicausality concept in this procedure is extended to build the reduced order observers and to avoid the calculation of C_a^{-1}.

2.1 Bg-rank of C_a submatrix

As it is shown in equation (5) the $\overline{A}_{ij}$ ($i, j = a, b$) matrices depend on C_a^{-1}. The rank of the C matrix can give information about the invertibility of C_a.

From a bond graph point of view, the definition of *bond graph rank* (bg-rank) can be used to verify the rank of the model's matrices. The bg-rank is a structural rank in the sense of graph theory, but correspond in fact to the numerical rank because it takes into account parameter dependency through the causality. Sueur and Dauphin-Tanguy [9] have shown the following property about the bg-rank[C]:

Property 1. The bg-rank[C] is equal to the number of detectors in a bond graph model that can be dualized without creating causality conflicts and while accepting the change of causality of the dynamical elements in integral causality.

This property also gives information about the existence of redundant outputs. Some redundant outputs might exist and might be neglected; thus the dimension of vector x_a always has to be equal to the bg-rank[C]:

$$\dim(x_a) = \text{bg-rank}[C] = m$$

Taking only the outputs in the detectors that can be dualized without any causality conflict, Property 1 implies that there exists an invertible submatrix of C with dimensions $m \times m$, called C_a, corresponding to a selection of components x_a.

After applying Property 1 to select the non-redundant outputs, the bicausality concept can be appropriately used to select the x_a vector guaranteeing the existence of C_a^{-1}. The bicausality allows fixing or imposing at the same time a variable and its conjugate as bicausal bonds decouple the effort and flow causalities. In the context of the inversion problem, imposing the output variable without modifying the energy structure (or constraint equations) of the system can be carried out by an SS element having a flow source /effort source causality [6]. When this procedure is made, the conjugate variable on the detectors is equal to zero, because the detector is supposed to be ideal (no power dissipated or stored). Then, to obtain information about C_a^{-1}, the detectors can be substituted by SS elements leading to a null power flow on that bond. From this analysis Theorem 1 can be introduced.

Theorem 1. The inverse of C_a submatrix exists (or the bg-rank$[C_a] = m$), if the following operations can be made in the bond graph model without introducing any causality conflict:

(i) All detectors are substituted by SS elements as shown in Figure 1.

(ii) The dynamical elements associated with x_a change its integral causality into a bicausality as it is shown in Figure 2.

(iii) The dynamical elements associated with x_b remain in integral causality.

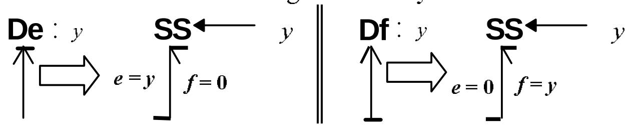

Figure 1. Substitution of detectors and bicausality of the SS elements.

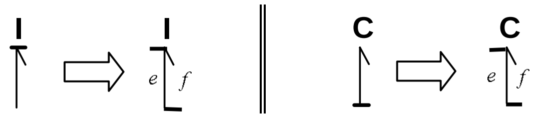

Figure 2. Bicausality of the elements associated with x_a. □

2.2 Proposition of a procedure to build the reduced order observers

After applying Property 1 to select only the non-redundant outputs, the proposed procedure to build the reduced order observers from a bond graph model can be summarized as follows:

Step 1. Verification of the structural observability of the model.

Step 2. Selection of x_a, verification of rank$(C_a) = m$ and calculation of C_a and C_b.

Step 3. Suppression of the dynamical elements associated with x_a.

Step 4. Sum of the term Ky (equation (7)).

Step 5. Sum of the term ϕ (equation (10)).

The relevant steps in this procedure are 3 and 5, because they show how to use the bicausality concept in the bond graph implementation of the observer. Step 3 shows how to avoid the calculation of C_a^{-1}, whereas in step 5 two lemmas are proposed to facilitate the observer building.

A brief description of each step is given now:

Step 1. The verification of structural observability of the bond graph model is made with the technique proposed by [9]. The authors showed that if all dynamical elements in integral causality are causally connected with a detector and all the I-C elements in integral causality are in derivative causality when a derivative assignment is performed over the initial bond graph, then the model is structurally state observable by the detectors.

Step 2. From a bond graph point of view any state variable of a dynamical element with a direct causal path with a detector can be defined as an accessible state variable. The selection of x_a must consider the conditions of Theorem 1 to verify rank$(C_a) = m$. Then, the calculation of C_a and C_b is directly derived from the initial bond graph model by calculating the gain of the causal path of length one (1) from the I or C-elements associated with the time derivative of x_a and x_b to the output y.

Afterwards, the bond graph model of the observer is made from the bond graph model with bicausality generated to analyse the invertibility of C_a. Steps 3, 4 and 5 specify the changes that are needed to conclude the construction of the observer.

Step 3. This step is made using Theorem 1. When the causality of the dynamical elements associated with x_a changes to a bicausality (Figure 2) and the detectors are substituted by SS elements (Figure 1), the complementary variables Z_a and the derivative of the state variables $\dot{x}_a$ can be calculated knowing the output y. Then in the observer model these dynamical elements are removed and substituted by 0-junctions or 1-junctions as it is shown in Figure 3, and because of that, in this step occurs the order reduction and the calculation of C_a^{-1} is avoided.

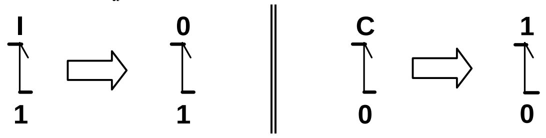

Figure 3. Suppression of dynamical elements in the bond graph model of the observer

Step 4. Sum of the term Ky.

In the dynamical elements associated with $\hat{x}_b$, the term Ky is summed to calculate the state:

$$\hat{x}_b = \hat{z} + Ky \tag{12}$$

This operation is equivalent to add modulated sources in the dynamical elements (I or C) of the observer as it is shown in Figure 4.

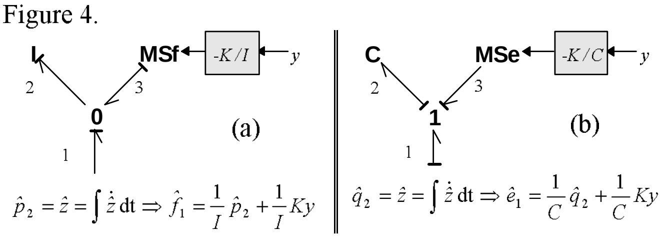

Figure 4. Bond Graph model to sum Ky in: (a) an I-element; (b) a C-element

Step 5. Sum of the term ϕ.

As in equation (11) to calculate $\dot{\hat{z}}$ the term ϕ is required. Using equation (2), the term ϕ is equal to:

$$\phi = -K\left[C_a\, \hat{\varphi}_a + C_b\, \hat{\varphi}_b\,\right] \tag{13}$$

where,

$$\hat{\varphi}_a = A_{aa}\, C_a^{-1} y + \left(A_{ab} - A_{aa} C_a^{-1} C_b\right)(\hat{z} + Ky) + B_a\, u \quad (14)$$

$$\hat{\varphi}_b = A_{ba}\, C_a^{-1} y + \left(A_{bb} - A_{ba} C_a^{-1} C_b\right)(\hat{z} + Ky) + B_b\, u \quad (15)$$

According to equation (13), the term ϕ can be added with a modulated source in the bond graph of the observer as it is shown in Figure 5. The term ϕ is a flow when the state variable x_b is associated with a C-element (or an effort when x_b is associated with an I-element).

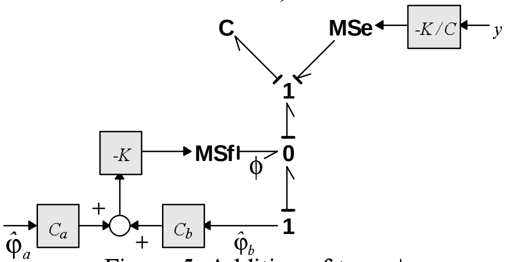

Figure 5. Addition of term ϕ

In the bond graph model of the observer, $\hat{\varphi}_a$ and $\hat{\varphi}_b$ will be flows when the state variables x_a and x_b are associated with a C-element (or efforts when x_a and x_b are associated with an I-element). To identify the variables $\hat{\varphi}_a$ and $\hat{\varphi}_b$ directly in a bond graph model, the following two lemmas are proposed.

Lemma 1. The variables $\hat{\varphi}_a$ are equal to the output variables in the junctions (0 or 1) with bicausality that substitute the dynamical elements associated with x_a as it is shown in Figure 6.

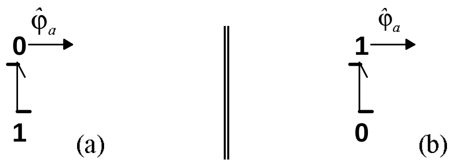

Figure 6. Variable $\hat{\varphi}_a$ in the bond graph model of the observer associated with (a) I-element; (b) C-element □

Proof. When the causality of the dynamical elements associated with x_a change to a bicausality (Figure 2) and the detectors are substituted by SS elements (Figure 1), the complementary variables Z_a and the derivative of the state variables $\dot{x}_a$ can be calculated knowing the output y. Then the calculus of $\dot{x}_a$ corresponds to:

$$\dot{x}_a = A_{aa}\, C_a^{-1} y + \left(A_{ab} - A_{aa} C_a^{-1} C_b\right) x_b + B_a\, u \quad (16)$$

If the dynamical elements associated with x_a are substituted by 0-junctions or 1-junctions according to the Figure 3, then the equation of the output variable of these junctions, ν_a, has to be equal to equation (21).

$$\nu_a = A_{aa}\, C_a^{-1} y + \left(A_{ab} - A_{aa} C_a^{-1} C_b\right) x_b + B_a\, u \quad (17)$$

Then, as the bond graph model of the observer keeps the same causality relations, the output variable of the junctions associated with x_a must have the same form as equation (22).

$$\hat{\varphi}_a = A_{aa}\, C_a^{-1} y + \left(A_{ab} - A_{aa} C_a^{-1} C_b\right)(\hat{z} + Ky) + B_a\, u \quad ■$$

Lemma 2. In the bond graph model of the observer, when x_b is associated with a C-element, $\hat{\varphi}_b$ is the flow before the addition of ϕ (in the case where the dynamical element associated with x_b is an I-element, then $\hat{\varphi}_b$ is the effort before the addition of ϕ).

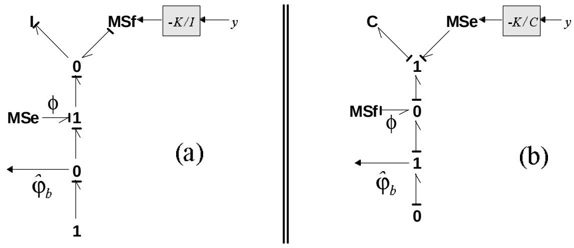

Figure 7. Variable $\hat{\varphi}_b$ in the bond graph model of the observer associated with (a) I-element; (b) C-element

Proof. By equation (11): $\dot{\hat{z}} = \bar{A}_{ba}\, y + \bar{A}_{bb}(\hat{z} + Ky) + \bar{B}_b\, u + \phi$

Then in the bond graph model of the observer, if the effort variable before the addition of ϕ is taken (when x_b is associated with an I-element) or the flow variable before the addition of ϕ is taken (when x_b is associated with a C-element), then this effort or flow variable has to be $\hat{\varphi}_b$ and has to be equal to:

$$\hat{\varphi}_b = \bar{A}_{ba}\, y + \bar{A}_{bb}(\hat{z} + Ky) + \bar{B}_b\, u$$

Then, using equation (5):

$$\hat{\varphi}_b = A_{ba}\, C_a^{-1} y + \left(A_{bb} - A_{ba} C_a^{-1} C_b\right)(\hat{z} + Ky) + B_b\, u \quad ■$$

In conclusion, using the last two lemmas, the addition of the term ϕ can be represented as it is shown in Figure 8. In this figure, according to Lemma 1 and Lemma 2, $\hat{\varphi}_a$ is the effort in the 0-juntion associated with x_a (in the case where the dynamical element associated with x_a is a C-element, then $\hat{\varphi}_a$ would be the flow in the 1-junction associated with x_a and $\hat{\varphi}_b$ is the flow in the observer bond graph before the addition of ϕ).

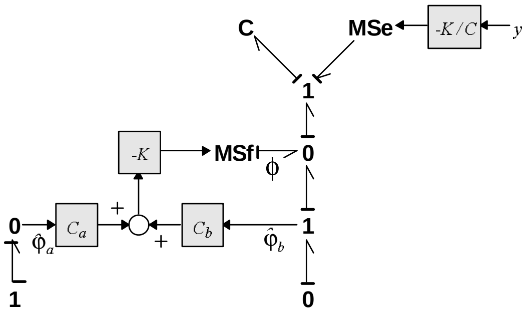

Figure 8. Addition of term ϕ

3 Observer design

The observer design is based on the pole placement techniques proposed in [8] like in the approach proposed in [7]. The characteristic polynomial of the reduced observer ($P_{(\bar{A}bb-K\bar{A}ab)}(s)$) is selected and then the calculation of K is based on the polynomial coefficients. This calculation is possible considering the information signals associated with K and applying the proposed Theorem 2. Although in this case the bond graph model of the observer contains bicausalities, the Theorem 2 can be applied.

Theorem 2. The value of each coefficient of the characteristic polynomial $P_{(\bar{A}bb-K\bar{A}ab)}(s)$ is equal to the total gain of the ith-order families of causal cycles in the bond graph model:

$$P_{(\bar{A}bb-K\bar{A}ab)}(s) = s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1} s + \alpha_n$$

The gain of each involved family of causal cycles must be multiplied by $(-1)^d$ if the family is constituted by d disjoint causal cycles.

Thus, the causal analysis to calculate K is made only with the family of causal cycles in the observer's bond graph.

4 Example

Model with three outputs. In this example, a simple model to design seat belts and to generate crash test simulations was selected [3]. The model is composed by the car (M), the crash test dummy (m), his seat belts (k_1, b_1) and the shock-absorbing bumper of the car (k_2, b_2). The model considers the car when it hits a wall (Figure 9). The bond graph model is shown in Figure 10. In the model, the state variables are the variables associated with the dynamical elements and the outputs are the force in the seat belts, the force in the bumper and the velocity of the car.

$$x = [x_1, x_2, x_3, x_4]^T = \left[p_{I_m}, p_{I_M}, q_{C_1}, q_{C_2}\right]^T$$

$$y = [y_1, y_2, y_3]^T = [F_1, F_2, V_M]^T = \left[e_{I_m}, e_{Sf}, f_{I_M}\right]^T$$

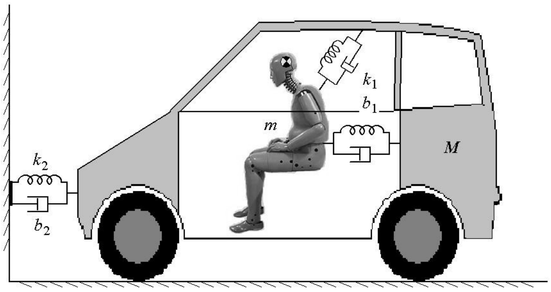

Figure 9. Example. Vehicle crash test

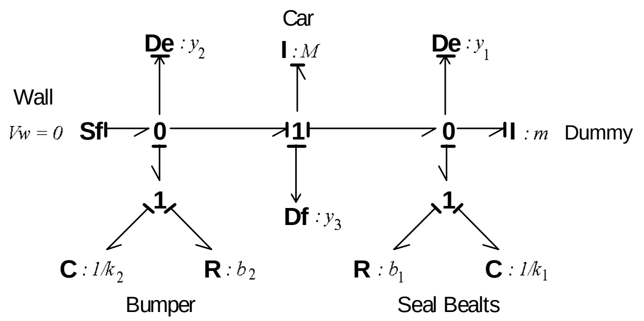

Figure 10. Bond graph model of the example.

After applying Property 1 it is possible to determine that y_1, y_2 and y_3 are non-redundant outputs, then applying the procedure 2.2 for building the reduced order observer:

Step 1. The structural analysis [9] for this model reveals that the system is structurally observable by the detectors.

Step 2. For this system, the momentum in the I-element of the car (M) and the displacements in the C-elements are selected as accessible state variables.

$$x_a = \left[p_{I_M}, q_{C_1}, q_{C_2}\right]^T$$

By Theorem 1 the new bond graph shown in Figure 14 is generated:

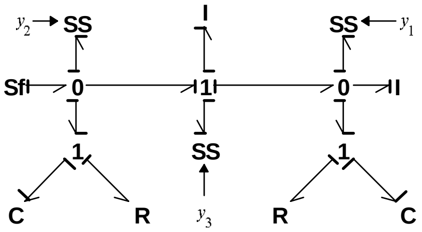

Figure 11. Bond graph to analyse the invertibility of C_a

Figure 11 shows that there are no causality conflicts, then:

bg-rank$[C_a]$ = 3 = m

Finally, the values of C_a and C_b derived from Figure 10 are:

$$C_a = \begin{bmatrix} b_1/M & 1/(1/k_1) & 0 \\ -b_2/M & 0 & 1/(1/k_2) \\ 1/M & 0 & 0 \end{bmatrix}; C_b = \begin{bmatrix} -b_1/m \\ 0 \\ 0 \end{bmatrix}$$

Step 3. The suppression of the dynamical elements associated with x_a is made directly from the bond graph model with bicausality shown in Figure 11, obtaining the bond graph model shown in Figure 12.

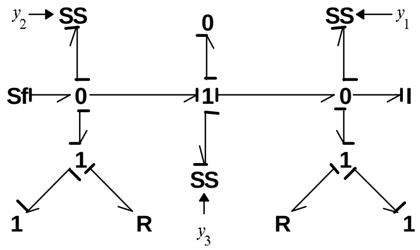

Figure 12. Suppression of the dynamical elements associated with x_a.

Step 4. The sum of the term $Ky = K_1y_1 + K_2y_2 + K_3y_3$ is made as it is shown in Figure 13.

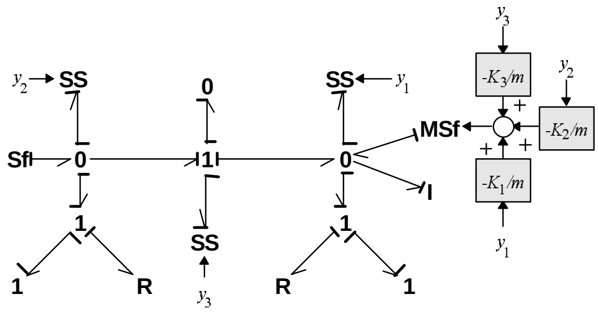

Figure 13. Sum of term Ky in the model of the observer.

Step 5. Sum of the term ϕ:

With this operation, the observer bond graph is complete as it is shown in Figure 14.

Observer design

Applying Theorem 2 in the observer model (Figure 14), only one first order causal cycle is found; therefore the calculation of the desired coefficient α_1 is a simple algebraic operation, because only one of the components of K matrix (K_1) is associated with this causal cycle.

Figure 14 is a first order model, then selecting α_1 as the desired coefficient of the characteristic polynomial:

$$P_{(\bar{A}_{bb}-K\bar{A}_{ab})}(s) = s + \alpha_1$$

The calculus of K is directly derived from α_1, because:

$$\alpha_1 = G_{(a)} \Rightarrow \alpha_1 = -K_1(1/m)(k_1) \quad (18)$$

Then, K_1 is calculated directly from equation (18):

$$\Rightarrow K_1 = -m\,(\alpha_1/k_1) \qquad (19)$$

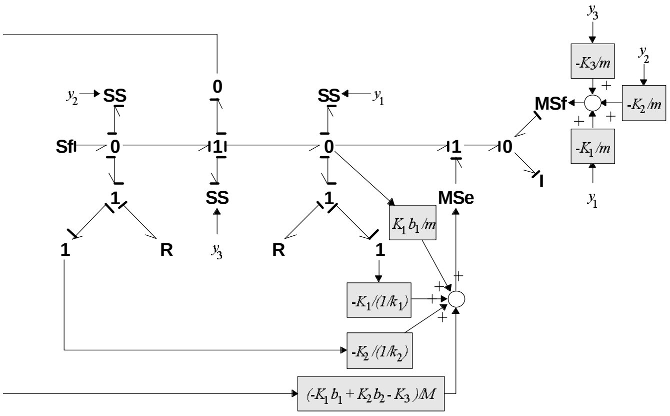

Figure 14. Bond graph model of the observer

Simulations

Simulations have been made in 20-Sim software version 2.3 [1]. As this software does not accept the bicausality assignment directly, it was needed to program new elements that allow the calculation of flows and efforts following the causality rules shown in the bond graph model of Figure 14. Figure 15 shows the estimation error dynamic.

The values of the parameters in this case are: Sf = 0 m/s, m = 100 Kg, M = 1500 Kg, $k_1 = 1 \times 10^4$ N/m, $k_2 = 3 \times 10^5$ N/m, b_1 = 1200 Ns/m, b_2 = 16000 Ns/m, α_1 = 25, and the initial conditions are: $x_{a1}(0) = 33530$, $x_{a2}(0) = 0$, $x_{a3}(0) = 0$, $x_b(0) = 2235$ and $\hat{z}(0) = 0$; the desired coefficient is: $\alpha_1 = 31.25$. With these parameters and the desired coefficient, the value of K_1 is –0.3125.

For the implementation, the values of the parameters of the observer with respect to the model of the system have a variation of –10%.

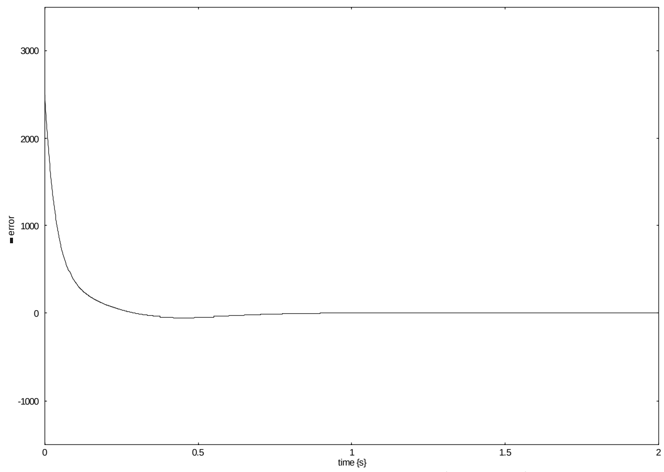

Figure 15. Estimation error $\left(x_b - \hat{x}_b\right)$

Figure 15 shows that the steady state estimation error has a value that represents 10% of the real value of x_b. This result is associated with the sensitivity of the Luenberger observers that depends heavily on the precise setting of the parameters and the precise measurement of the output vector. Any disturbance (noise) in the measurement, parameter differences, or internal noises (which can include such things as different timing on the power stage of the amplifier as opposed to the model) can make the observer unusable [5].

5 Conclusions

In this paper, a bond graph method to design reduced order observers has been shown. The new method presented depends on the output matrix C, as in the algebraic methods. However, the observer building for LTI systems modelled by bond graphs does not need the calculation of C_a^{-1}, showing the advantages of using the bicausality concept in the state estimation problem. The work presented in this paper represents a control tool for people who develop bond graph models, since it is possible to build a reduced order observer directly from a bond graph model where the matrix calculations can be directly derived from the observer's graph (including the observer gain) and avoiding the algebraic manipulation of matrices. From the procedure of the present work it is possible to build an observer with a structure simpler than the structure of the observer shown in [7] and because of that, the calculation of the observer's gain is also simpler.

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