A NOVEL APPROACH TO LINEAR DECENTRALIZED ROBUST PERFORMANCE STABILIZATION OF LARGE-SCALE SYSTEMS

B. Labibi, Y. Bavafa-Toosi, A. Khaki-Sedigh, B. Lohmann
labibi@eetd.kntu.ac.ir K.N. Toosi University of Technology, Iran
ybavafat@yahoo.com 2/71 Abuzar 11, Ahmadabad Ave., Mashhad, Iran
sedigh@eetd.kntu.ac.ir K.N. Toosi University of Technology, Iran
bl@iat.uni-bremen.de University of Bremen, FRG

Keywords: Linear decentralized robust performance stabilization

Abstract: The existing methods of decentralized control suffer from two major restrictions. First, almost all of them hinge on Lyapunov's method, and second, they do not address the problem of performance robustness. A novel methodology to overcome the above defects is presented in this paper. Central to this approach is the notion of a finite-spectrum-equivalent descriptor system in the input-output decentralized form. By way of this notion, a new formulation of the interaction which introduces some degrees of freedom into the design procedure is offered. The main result, i.e. a sufficient condition for decentralized performance stabilization in a desirable performance region and maximal robustness to unstructured uncertainties in the controller and plant parameters, nevertheless, is in terms of regular systems. Based on minimal sensitivity design of isolated subsystems via eigenstructure assignment, an analytic method for the satisfaction of the aforementioned sufficient condition is also presented.

1 INTRODUCTION

Many real-world large-scale systems, such as urban traffic networks, digital communication networks, cooperating robotic systems, power systems and economic systems, comprise a number of small interconnected subsystems. For such systems, a centralized controller is difficult to design and very costly to implement. As a result, decentralized control theory emerged in the 1960s and developed to a pole of attraction for the system and control community thereafter.

In the broad sense, existing results on large-scale systems appear in two main directions, see [2-9,11] and the references therein. On the one hand some structural properties have been explored and on the other hand stabilization methods have been developed. With reference to stabilization, the complex nature of the problem has encouraged the use of nonlinear control and/or Lyapunov's stability criteria in almost all of the existing methods. Lyapunov's method, none the less, provides only a sufficient condition for stability and one may search in vain for a stabilizing control. Moreover, the stability property of the system is highly dependent on the choice of the Lyapunov functions for the subsystems. Thus, linear controllers which have the advantage of being simpler, more feasible and economical than the nonlinear ones, are of particular importance in decentralized control, especially if they can be designed without using Lyapunov's method. This is one of the underlying motivations for this research work.

A central issue in control systems design is that of robustness. The existing decentralized robust control schemes mostly address the problem of robust stabilization, not robust performance, and hinge on Lyapunov's method. A step towards performance robustness was taken in [1] by the introduction of the so-called guaranteed cost control. This, although being used in decentralized control methods, provides only an upper bound on a given performance index, does not address the uncertainties in the controller itself, and is based on Lyapunov's stability criteria as well. The above mentioned shortcomings are other motivations for this work.

It is well-known that especially for large-scale systems state estimation is often infeasible and may even result in the curse of dimensionality. Thus, output feedback control is of special significance for high-order systems. This is the third motivation for this research work.

A number of existing results have some other special restrictions. For instance, in [2,3] it is assumed that the system is generically (i.e. in almost all cases) stable, minimum-phase and square. There are also some H_∞-based methods which are all iterative. It should also be noted that there are only few output-feedback linear design methods for decentralized control. In particular some results can be found in [4-6] which all pivot on Lyapunov's method. In addition, in [4] all the isolated subsystems were restricted to be invertible and have their transmission zeros in the open left half plane; local controllers were then synthesized using observer-based high-gain feedback strategy.

Very recently the above-mentioned defects were partly rectified in [7-9]. In [7,8], without using Lyapunov's method, a sufficient condition for output feedback linear decentralized stabilization of large-scale systems was introduced. Then, based on [7,8,10], the problem of linear output-feedback decentralized robust exponential stabilization was addressed in [9]. The method provides a desirable rate of decay and maximal robustness to unstructured perturbations in the system and controller parameters. However, the system is restricted to be in the input-output decentralized form.

Motivated by the aforementioned arguments, the results of [9] are extended to performance robustness of generic systems, i.e. systems not in the input-output decentralized form. This paper is organized as follows: In Section 2, in order to simplify the design procedure and to get rid of the interaction due to input-output centralization, for a given large-scale system a *finite-spectrum-equivalent* descriptor system [7] in the input-output decentralized form is introduced. The proposed formulation enjoys some flexibility (degrees of freedom) which is exploited in the design procedure. In Section 3, it is proved that this descriptor system is regular, impulse-free, and its finite spectrum is exactly the same as the spectrum of the original system. Thus, stability of the finite spectrum of this descriptor system is (necessarily and sufficiently) equivalent to the stability (of the spectrum) of the original system. The design procedure will be based on this descriptor system. However, the final result is in terms of nonsingular systems - the descriptor system vanishes. In Section 4, the problem of performance stabilization is defined and a necessary and sufficient condition for that of descriptor systems is presented. A sufficient condition for decentralized performance stabilization of large-scale systems is then derived. To incorporate maximal robustness (minimal sensitivity) into the above condition, the newly developed analytic approach of [9,10], resulting in a compact-form sufficient condition, is utilized. The main result of the paper, i.e. a sufficient condition for performance stabilization in a desirable performance region and maximal robustness to unstructured uncertainties in the controller and plant parameters by decentralized linear output feedback of generic large-scale systems, is established thereafter. An analytic solution to the problem arising from the aforementioned sufficient condition is hence available.

Throughout the paper it is assumed that the desirable closed-loop eigenvalues are distinct, since they possess better robustness properties than the repeated ones. In addition, because the design of a linear dynamic controller can be reduced to that of a linear static one [9], and also for notational implicity, only static controllers are addressed. All the results are presented for output feedback; state feedback thus follows directly.

2 PROBLEM FORMULATION

Consider a large-scale system G with the state-space equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where $A \in \mathbf{R}^{n \times n}$, $B \in \mathbf{R}^{n \times m}$ and $C \in \mathbf{R}^{p \times n}$ are the system state, input and output matrices. The system is partitioned into N linear-time-invariant subsystems $G_i(s)$ described by

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i + B_{ii}u_i + \textstyle\sum_{j=1}^{N} A_{ij}x_j + \sum_{j=1}^{N} B_{ij}u_j \\ y_i &= C_{ii}x_i + \textstyle\sum_{j=1}^{N} C_{ij}x_j \quad, j \neq i \end{aligned} \tag{2}$$

where $A_{ij} \in \mathbf{R}^{n_i \times n_j}$, $B_{ij} \in \mathbf{R}^{n_i \times m_j}$, $C_{ij} \in \mathbf{R}^{p_i \times n_j}$, $\sum_1^N n_i = n$, $\sum_1^N m_i = m$ and $\sum_1^N p_i = p$. The terms $\sum_{j=1}^N A_{ij}x_j$, $\sum_{j=1}^N B_{ij}u_j$ and $\sum_{j=1}^N C_{ij}x_j$ describe the interactions with other subsystems. In this work, in contrast to the literature, the isolated subsystems, i.e. the triples (A_{ii}, B_{ii}, C_{ii}) $i = 1,...,N$, are not restricted to be minimal (see Remark 4.1).

To simplify the design procedure and to get rid of the interaction due to input-output centralization, for the above system a finite-spectrum-equivalent descriptor system in the input-output decentralized form is introduced. To this end, similar to the behavioral approach [12], the augmented state vector is defined by [7]

$$\tilde{x} = (y_1^T, x_1^T, u_1^T, \ldots, y_N^T, x_N^T, u_N^T) \tag{3}$$

by which Equatios (1) are transformed to

$$\begin{aligned} \tilde{E}\dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}u \\ y &= \tilde{C}\tilde{x} \end{aligned} \tag{4}$$

where

$$\tilde{E} = \begin{bmatrix} 0_{p_1 \times p_1} & 0_{p_1 \times n_1} & 0_{p_1 \times m_1} & \cdots & 0_{p_1 \times p_N} & 0_{p_1 \times n_N} & 0_{p_1 \times m_N} \\ 0_{n_1 \times p_1} & I_{n_1 \times n_1} & 0_{n_1 \times m_1} & \cdots & 0_{n_1 \times p_N} & 0_{n_1 \times n_N} & 0_{n_1 \times m_N} \\ 0_{m_1 \times p_1} & 0_{m_1 \times n_1} & 0_{m_1 \times m_1} & \cdots & 0_{m_1 \times p_N} & 0_{m_1 \times n_N} & 0_{m_1 \times m_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0_{p_N \times p_1} & 0_{p_N \times n_1} & 0_{p_N \times m_1} & \cdots & 0_{p_N \times p_N} & 0_{p_N \times n_N} & 0_{p_N \times m_N} \\ 0_{n_N \times p_1} & 0_{n_N \times n_1} & 0_{n_N \times m_1} & \cdots & 0_{n_N \times p_N} & I_{n_N \times n_N} & 0_{n_N \times m_N} \\ 0_{m_N \times p_1} & 0_{m_N \times n_1} & 0_{m_N \times m_1} & \cdots & 0_{m_N \times p_N} & 0_{m_N \times n_N} & 0_{m_N \times m_N} \end{bmatrix} \tag{5}$$

$$\tilde{C} = \begin{bmatrix} I_{p_1 \times p_1} & 0_{p_1 \times n_1} & 0_{p_1 \times m_1} & \cdots & 0_{p_1 \times p_N} & 0_{p_1 \times n_N} & 0_{p_1 \times m_N} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0_{p_N \times p_1} & 0_{p_N \times n_1} & 0_{p_N \times m_1} & \cdots & I_{p_N \times p_N} & 0_{p_N \times n_N} & 0_{p_N \times m_N} \end{bmatrix} \tag{6}$$

$$\tilde{A} = \begin{bmatrix} -I_{p_1\times p_1} & C_{11} & 0_{p_1\times m_1} & \cdots & 0_{p_1\times p_N} & C_{1N} & 0_{p_1\times m_N} \\ 0_{n_1\times p_1} & A_{11} & B_{11} & \cdots & 0_{n_1\times p_N} & A_{1N} & B_{1N} \\ 0_{m_1\times p_1} & 0_{m_1\times n_1} & -I_{m_1\times m_1} & \cdots & 0_{m_1\times p_N} & 0_{m_1\times n_N} & 0_{m_1\times m_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0_{p_N\times p_1} & C_{N1} & 0_{p_N\times m_1} & \cdots & -I_{p_N\times p_N} & C_{NN} & 0_{p_N\times m_N} \\ 0_{n_N\times p_1} & A_{N1} & B_{N1} & \cdots & 0_{n_N\times p_N} & A_{NN} & B_{NN} \\ 0_{m_N\times p_1} & 0_{m_N\times n_1} & 0_{m_N\times m_1} & \cdots & 0_{m_N\times p_N} & 0_{m_N\times n_N} & -I_{m_N\times m_N} \end{bmatrix} \quad (7)$$

$$\tilde{B} = \begin{bmatrix} 0_{p_1\times m_1} & \cdots & 0_{p_1\times m_N} \\ 0_{n_1\times m_1} & \cdots & 0_{n_1\times m_N} \\ I_{m_1\times m_1} & \cdots & 0_{m_1\times m_N} \\ \vdots & \vdots & \vdots \\ 0_{p_N\times m_1} & \cdots & 0_{p_N\times m_N} \\ 0_{n_N\times m_1} & \cdots & 0_{n_N\times m_N} \\ 0_{m_N\times m_1} & \cdots & I_{m_N\times m_N} \end{bmatrix}. \quad (8)$$

Extracting a diagonal part of $\tilde{A}$ as in

$$\tilde{A} = \tilde{A}^d + \tilde{H} \quad (9)$$

Where

$$\tilde{A}^d = diag\{\tilde{A}_{ii}^d\} \quad (10)$$

$$\tilde{H}_{ii} = \tilde{A}_{ii} - \tilde{A}_{ii}^d \quad , \quad \tilde{H}_{ij} = \tilde{A}_{ij} \quad i \neq j \quad (11)$$

$$\tilde{A}_{ii} = \begin{bmatrix} -I_{p_i\times p_i} & C_{ii} & 0_{p_i\times m_i} \\ 0_{n_i\times p_i} & A_{ii} & B_{ii} \\ 0_{m_i\times p_i} & 0_{m_i\times n_i} & -I_{m_i\times m_i} \end{bmatrix} \quad (12)$$

$$\tilde{A}_{ij} = \begin{bmatrix} 0_{p_i\times p_j} & C_{ij} & 0_{p_i\times m_j} \\ 0_{n_i\times p_j} & A_{ij} & B_{ij} \\ 0_{m_i\times p_j} & 0_{m_i\times n_j} & 0_{m_i\times m_j} \end{bmatrix} \quad i \neq j \quad (13)$$

and defining

$$\tilde{E}_{ii} = \begin{bmatrix} 0_{p_i\times p_i} & 0_{p_i\times n_i} & 0_{p_i\times m_i} \\ 0_{n_i\times p_i} & I_{n_i\times n_i} & 0_{n_i\times m_i} \\ 0_{m_i\times p_i} & 0_{m_i\times n_i} & 0_{m_i\times m_i} \end{bmatrix} \quad (14)$$

$$\tilde{B}_{ii} = \begin{bmatrix} 0_{p_i\times m_i} \\ 0_{n_i\times m_i} \\ I_{m_i\times m_i} \end{bmatrix} \quad (15)$$

$$\tilde{C}_{ii} = \begin{bmatrix} I_{p_i\times p_i} & 0_{p_i\times n_i} & 0_{p_i\times m_i} \end{bmatrix} \quad (16)$$

for $i = 1,...,N$, the descriptor system $(\tilde{E}, \tilde{A}^d, \tilde{B}, \tilde{C})$, denoted by $\tilde{G}$, will be in the input-output decentralized form. As will be proved, $\tilde{G}$ is regular and impulse-free, and its finite spectrum is exactly the same as the spectrum of $\tilde{G}$; this is also valid for their isolated subsystems. Therefore, stability of the finite spectrum of $\tilde{G}$ is equivalent to the stability of (the spectrum of) G, and $\tilde{G}$ is called a finite-spectrum-equivalent descriptor system for G. Hence, the design procedure is simplified and based on each isolated subsystem $\tilde{G}_i$ given by

$$\begin{aligned} \tilde{E}_{ii}\dot{\tilde{x}}_i &= \tilde{A}_{ii}^d \tilde{x}_i + \tilde{B}_{ii} u_i \\ y_i &= \tilde{C}_{ii} \tilde{x}_i \end{aligned} \quad (17)$$

provided a sufficient condition, which is derived in Section 4.4, is satisfied. Thus, the objective of this paper is to design, for each isolated subsystem $\tilde{G}_i$ $(i = 1,...,N)$, a local static output-feedback controller K_i

$$U_i = K_i(R_i - \tilde{C}_{ii} X_i) \quad (18)$$

where R_i is the ith reference input vector, such that the overall system is stabilized in a desirable performance region with maximal robustness to unstructured uncertainties in the controller and plant parameters.

By the application of the linear decentralized output-feedback controller $K = diag\{K_i\}$ to system (4), i.e. $\tilde{G}$ plus uncertainty $\tilde{H}$, the closed-loop state matrix will be $\tilde{A}^d + \tilde{B}K\tilde{C} + \tilde{H} = \tilde{A}_{cl} + \tilde{H}$ where $\tilde{A}_{cl} = diag\{\tilde{A}_{cl_i}\}$ in which $\tilde{A}_{cl_i} = \tilde{A}_{ii}^d - \tilde{B}_{ii} K_i \tilde{C}_{ii}$ denotes the closed-loop state matrix of the ith isolated subsystem.

Remark 2.1: In the design procedure $\tilde{H}$, which embodies the interactions, will be treated as an uncertainty in $\tilde{A}^d$. The definition of $\tilde{H}$ (interaction measure) in decomposition (9) introduces some flexibility into the design procedure by the freedom in choosing $\tilde{A}_{ii}^d$ $(i = 1,...,N)$ up to the following structure

$$\tilde{A}_{ii}^d = \begin{bmatrix} D_{p_i\times p_i} & C_{ii}^x & X \\ 0_{n_i\times p_i} & A_{ii}^x & B_{ii}^x \\ 0_{m_i\times p_i} & 0_{m_i\times n_i} & D_{m_i\times m_i} \end{bmatrix} \quad (19)$$

in which $(A_{ii}^x, B_{ii}^x, C_{ii}^x)$ forms any arbitrary-element minimal triple, X represents arbitrary elements, and $D_{a_i\times a_i}$ denotes any full rank diagonal matrix. It is clear that there are always infinite number of choices for $\tilde{H}$.

Remark 2.2: In [13] it has been shown that for a minimal representation (1) a generic (i.e. for almost all systems) sufficient condition for linear output feedback pole assignment is that $mp > n$. Thus, eigenvalues of $\tilde{A}_{cl_i}$ $(i = 1,...,N)$ are assignable if $m_i p_i > m_i + p_i + n_i$. Consequently, the order (and number) of the subsystems is dictated by this condition.

3 A FINITE-SPECTRUM-EQUIVALENT DESCRIPTOR SYSTEM

It is well known that the existence and uniqueness of (classical) solutions to a descriptor system $(\tilde{E}, \tilde{A}^d, B, C)$ is guaranteed if the pair $(\tilde{E}, \tilde{A}^d)$ is regular, i.e., if $\det(\lambda\tilde{E} - \tilde{A}^d)$ is not identically zero. In addition, the system is called impulse-free if $\deg\ \det(\lambda\tilde{E} - \tilde{A}^d) = rank(\tilde{E})$ where $\lambda \in \mathbf{C}$.

Theorem 3.1 [7]**:** *The descriptor system $\tilde{G}$ has (classical) unique solutions.* (*Proof:* Left to the reader.)

Theorem 3.2 [7]**:** *The descriptor system $\tilde{G}$ is impulse-free*. (*Proof:* Left to the reader.)

The problem of controllability, observability and duality in descriptor systems has been extensively studied. There are several controllability concepts with different meanings, namely, c-controllability, r-controllability, i-controllability and s-controllability. Observability is the dual of controllability, and thus similar concepts exist for observability of descriptor systems.

Theorem 3.3 [7]**:** *All the isolated subsystems G_i are strongly controllable*. (*Proof:* Left to the reader.)

Theorem 3.4 [7]**:** *All the isolated subsystems G_i are strongly observable.* (*Proof:* Left to the reader.)

Theorem 3.5 [7]**:** *The decentralized controller K stabilizes G iff it stabilizes the finite spectrum of $\tilde{G}$*.
Proof: Clearly, there exists a similarity transformation by which Equations (4) are transformed to

$$\hat{E}\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$$
$$y = \hat{C}\hat{x}$$

where

$$\hat{E} = \begin{bmatrix} 0_{p\times p} & 0_{p\times n} & 0_{p\times m} \\ 0_{n\times p} & I_{n\times n} & 0_{n\times m} \\ 0_{m\times p} & 0_{m\times n} & 0_{m\times m} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} -I_{p\times p} & C & 0_{p\times m} \\ 0_{n\times p} & A & B \\ 0_{m\times p} & 0_{m\times n} & -I_{m\times m} \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} 0_{p\times m} \\ 0_{n\times m} \\ I_{m\times m} \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} I_{p\times p} & 0_{p\times n} & 0_{p\times m} \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} y \\ x \\ u \end{bmatrix}.$$

Since similarity transformations do not affect the eigenvalues, finite poles of $\tilde{G}$ are given by the roots of $\det(\lambda\hat{E} - \hat{A} + \hat{B}K\hat{C}) = 0$ where $\lambda \in \mathbf{C}$. On the other hand, it is easily seen that

$$\det(\lambda\hat{E} - \hat{A} + \hat{B}K\hat{C}) = \det\begin{bmatrix} I_{p\times p} & -C & 0_{p\times m} \\ 0_{n\times p} & \lambda I - A & -B \\ K & 0_{m\times n} & I_{m\times m} \end{bmatrix}$$
$$= \det(\lambda I - A + BKC)$$

and because the reverse of the above argument is also valid, stability of the finite spectrum of $\tilde{G}$ is (necessarily and sufficiently) equivalent to the stability of (the spectrum of) G.
Δ.

Evidently, the above argument is also valid for all the isolated subsystems of $\tilde{G}$ and G. Thus, $\tilde{G}$ is called a finite-spectrum-equivalent descriptor system for G.

4 DECENTRALIZED ROBUST PERFORMANCE STABILIZATION

4.1 PERFORMANCE STABILIZATION

Performance stabilization of a system refers to assigning the poles of the system in some prescribed region which represents the requirements on the stability and performance. From among the common desirable performance regions, i.e. sector, elliptical, vertical strip and parabolic regions, sector region is adopted in the sequel. The proceeding analysis and synthesis, nevertheless, is applicable to all of the abovementioned regions.

Ω *region:* This represents the whole part of the left-half complex s-plane left to both lines

$$y = \pm\text{cotan}(\delta)(x + \alpha) \tag{20}$$

where x and y denote $\Re(s)$ and $\Im(s)$ in the complex s-plane, respectively, $0 < \delta < \pi/2$ and $\alpha > 0$.

Theorem 4.1: *The decentralized controller K assigns (stabilizes) the finite spectrum of system (4) into Ω region iff it assigns (stabilizes) the finite spectrum of its associated* *augmented system given by (21) into the open left-half complex s-plane.* (*Proof:* Left to the reader.)

Augmented System: The system described by

$$\Theta(\delta)\otimes\tilde{E}\dot{z}=\Theta(\delta)\otimes(\tilde{A}_{cl}+\tilde{H}+\alpha\tilde{E})z \tag{21}$$

is called the augmented system associated with system (4), where

$$\Theta(\delta)=\begin{bmatrix}\cos\delta & -\sin\delta\\ \sin\delta & \cos\delta\end{bmatrix} \tag{22}$$

with $\alpha>0$, $0<\delta<\pi/2$ and where $\otimes$ is the Kronecker product of matrices.

Corollary 4.1: *The decentralized controller K stabilizes the performance of system G into Ω region iff it assigns (stabilizes) the finite spectrum of system (21) into the open left-half complex s-plane.*

4.2 DECENTRALIZED PERFORMANCE STABILIZATION

A sufficient condition for decentralized performance stabilization is presented in the following Theorem.

Theorem 4.2: *The decentralized controller K stabilizes the performance of system G into Ω region, if*

$$\max_{\substack{i\\ i=1,\dots,N}} \lambda_{\max}\left(\frac{\Theta(\delta)\otimes\tilde{A}_{cl_i}+\Theta(\delta)^T\otimes\tilde{A}_{cl_i}{}^T}{2}\right) < -\lambda_{\max}\left(\frac{\Theta(\delta)\otimes\tilde{H}+\Theta(\delta)^T\otimes\tilde{H}^T}{2}\right) \tag{23}$$

in which $\lambda_{\max}$ denotes the maximum eigenvalue of $(.)$.
(*Proof:* Left to the reader.)

4.3 MAXIMAL ROBUSTNESS DESIGN

Let the condition number of a matrix be defined as the ratio of its greatest singular value to its smallest one.

Problem $\mathcal{P}$ [9,10]**:** *The problem of minimal sensitivity (maximal robustness) of eigenvalues in linear output feedback is to find an analytic solution for the static output feedback gain such that: a) condition number of the modal matrix of the closed-loop state matrix be at its minimum, i.e. one, and b) pole assignment be accomplished in some admissible region Ω which represents the requirements on the stability and performance. Region Ω is restricted to produce nondefective (completely diagonalizable) closed-loop system matrices, since such matrices exhibit better sensitivity properties than the defective ones.*

A compact-form solution to Problem $\mathcal{P}$ is given in the subsequent Theorem.

Theorem 4.3 [9,10]**:** *Let a linear-time-invariant multivariable plant be described by Equations (1) with the output-feedback linear controller $u=-Ky$. If a real matrix Y can be found such that*

$$BB^{+}(A^T+Y)C^{+}C=A^T+Y \tag{24}$$

and b) be satisfied, then problem $\mathcal{P}$ is solved and the solution is given by

$$K=-B^{+}(A^T+Y)C^{+} \tag{25}$$

where $(.)^{+}$ denotes the pseudo-inverse of $(.)$.

A natural method for finding Y is to use a random-number generator. Yet, a better and faster approach is to invoke a genetic algorithm. This way, part b) of the objective along with some other design criteria - e.g. reliability, decoupling and low actuator gain [14], and the flexibility in the decomposition (9) which helps satisfy conditions (24), (27) and (29) - can easily be incorporated in the design procedure.

Remark 4.1: No assumption is made on the controllability and observability of the system, because condition number minimization is accomplished by eigenvector assignment which is possible for all poles. Hence, region Ω must include the uncontrollable and unobservable modes. With reference to Sections 4.2,4.4, region Ω represents part of the complex s-plane described by (23), (26), (27) and (29).

4.4 DECENTRALIZED ROBUST PERFORMANCE STABILIZATION

Utilizing Theorems 4.2,4.3, a sufficient condition for decentralized performance robustness is established in the following Theorem.

Theorem 4.4: *If the eigenstructure of each isolated closed-loop subsystem is assigned such that its eigenvectors compose a set of orthonormal vectors and*

$$\max_{\substack{i\\ i=1,\dots,N}} \lambda_{\max}\left(\frac{\Theta(\delta)\otimes\tilde{A}_{cl_i}+\Theta(\delta)^T\otimes\tilde{A}_{cl_i}{}^T}{2}\right) < -\lambda_{\max}\left(\frac{\Theta(\delta)\otimes(\tilde{H}+\Delta\tilde{H})+\Theta(\delta)^T\otimes(\tilde{H}^T+\Delta\tilde{H}^T)}{2}\right) \tag{26}$$

where $\Delta\tilde{H}$ is the 2-norm bounded uncertainty in $\tilde{H}$, then performance stabilization in Ω region with maximal

robustness to unstructured perturbations in the controller and plant parameters of system G is assured.
(*Proof:* Left to the reader.)

The sequel Corollary follows from the above Theorem directly. The analytical method for the satisfaction of its condition is readily available by Theorem 4.3.

Corollary 4.2: *If the decentralized controller K is designed such that all closed-loop subsystems have symmetric state matrices and*

$$\max_{\substack{i \\ i=1,\dots,N}} \lambda_{\max}(\Theta(\delta)\otimes\tilde{A}_{cl_i}) < -\lambda_{\max}\left(\frac{\Theta(\delta)\otimes(\tilde{H}+\Delta\tilde{H})+\Theta(\delta)^T\otimes(\tilde{H}^T+\Delta\tilde{H}^T)}{2}\right) \tag{27}$$

then performance stabilization in Ω region with maximal robustness to unstructured uncertainties in the controller and plant parameters of system G is guaranteed.

Remark 4.2: *If only an upper bound of the interaction is known, since*

$$\lambda_{\max}\left(\frac{M+M^T}{2}\right) \le \sigma_{\max}(M) \tag{28}$$

where $\sigma_{\max}(.)$ denotes the maximum singular value of $(.)$ and M is any square matrix, condition (27) can be substituted by

$$\max_{\substack{i \\ i=1,\dots,N}} \lambda_{\max}(\Theta(\delta)\otimes\tilde{A}_{cl_i}) < -\sigma_{\max}(\Theta(\delta)\otimes(\tilde{H}+\Delta\tilde{H}))\,. \tag{29}$$

5 CONCLUSIONS

The literature on decentralized control is lacking a method which is not based on Lyapunov's stability criteria and/or addresses the problem of performance robustness. This paper offers a solution to the above defects. Central to the methodology is the concept of a finite-spectrum-equivalent descriptor system in the input-output decentralized form. The main result, i.e. a sufficient condition for decentralized performance stabilization in a desirable performance region and maximal robustness to unstructured perturbations in the controller and plant parameters, non the less, is in terms of regular systems. Based on maximal robustness design of isolated subsystems through eigenstructure assignment, an analytic method for the satisfaction of the aforementioned sufficient condition is also presented. In addition to addressing the abovementioned shortcomings, the proposed methodology has the sequel distinctions: a) minimal sensitivity to unstructured uncertainties in the controller and plant parameters, b) some flexibility introduced by a new formulation of the interaction, c) applicability to nonminimum-phase and nonsquare systems, and d) noniterativeness.

REFERENCES

[1] I.R. Petersen, D.C. McFarlane. "Optimal Guaranteed Cost Control and Filtering for Uncertain Linear Systems", IEEE Transactions on Automatic Control, 39, pp. 1971-1977, (1994).

[2] P. Grosdidier, M. Morari. "Interaction Measure for Systems under Decentralized Control", *Automatica,* 22(3), pp. 309-319, (1986).

[3] C.N. Nett, J.A. Uthgenannt. "An Explicit Formula and an Optimal Weight for 2-Block Structured Singular Value Interaction Measure", *Automatica*, 24(2), pp. 261-265, (1988).

[4] A. Saberi, H. Khalil. "Decentralized Stabilization of Interconnected Systems using Output Feedback", *International Journal of Control*, 41(6), pp. 1461-1475, (1985).

[5] J.C. Geromel, A. Yamakami. "Stabilization of Continuous and Discrete Linear Systems subjected to Control Structure Constraints", *International Journal of Control*, 36, pp. 429-444, (1982).

[6] D.-Z. Zheng. "Decentralized Output Feedback Stabilization of a Class of Nonlinear Interconnected Systems", *IEEE Transactions on Automatic Control,* 34(12), pp. 1297-1300, (1989).

[7] B. Labibi. "Stability and Robustness in Decentralized Control of Large-Scale Systems", *PhD Dissertation,* University of Tehran, Iran, (2001).

[8] B. Labibi, B. Lohmann, A. Khaki-Sedigh, P. Jabedar-Maralani. "Sufficient Condition for Stability of Decentralized Control", *Electronics Letters*, 36(6), pp. 588-589, (2000).

[9] B. Labibi, Y. Bavafa-Toosi, A. Khaki-Sedigh, B. Lohmann. "A Novel Method for Decentralized Robust Exponential Stabilization of Large-Scale Systems", *Journal of Soft Computing and Intelligent Automation* Special Issue on *Developments in Intelligent Adaptive and Robust Techniques for Automation and Control systems*, 9(2), (2003, to appear).

[10] Y. Bavafa-Toosi, A. Khaki-Sedigh. "Minimum Sensitivity in Linear Output Feedback Design", *Robotics, Manufacturing, Automation and Control*, 14, (2002), NM: TSI Press.

[11] B. Labibi, B. Lohmann, A. Khaki-Sedigh, P. Jabedar Maralani. "Output Feedback Decentralized Control of Large-Scale Systems using Weighted Sensitivity Functions Minimization", *Systems & Control Letters*, 47, pp. 191-198, (2002).

[12] J.C. Willems. "Paradigms and Puzzles in the Theory of Dynamical Systems", *IEEE Transactions on Automatic Control*, 36(3), pp. 259-294, (1991).

[13] X.A. Wang. "Grassmannian, Central Projection, and Output Feedback Pole Assignment in Linear Systems", *IEEE Transactions on Automatic Control*, 41(6), pp. 786-794, (1996).

[14] A. Khaki-Sedigh, Y. Bavafa-Toosi. "Design of Static Linear Multivariable Output Feedback Controllers using Random Optimization Techniques", *Journal of Intelligent and Fuzzy Systems,* 10(3,4), pp. 185-195, (2001).