

# ROBUST PERFORMANCE CONTROLLER DESIGN FOR VEHICLE LANE KEEPING

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## Abstract

In this paper the problem of vehicle lateral control in highway experimental conditions is addressed. The vehicle under consideration is equipped with an electric motor acting on the steering angle (the command input) and a vision system providing two measurements (the two outputs): the lateral displacement and the angular orientation of the vehicle with respect to the lane centerline. We show how, exploiting properties of single-input two-outputs systems, our original SITO control problem can be simplified to the design of a SISO controller. The design of such a controller is performed through  $\mu$ -synthesis techniques in order to obtain robust performance in the face of model uncertainties. Experimental results obtained testing the designed controller on highways are reported.

## 1 Introduction

Driver assistance systems received a growing attention in the last twenty years due to its acknowledged ability to reduce the driver's workload and to enhance the driving safety (see, e.g., [9]). Many researches have been done facing a number of problems related to the subject (see, e.g., the overview [7]). In the last years remarkable efforts have been focused on the solution of the vehicle lateral dynamics control problem on highway scenery. The objective is to design a steering controller able to keep the vehicle inside the lane on the basis of the measurements provided by the sensors system. Two main approaches have been investigated in the literature, the so-called *look-down* and *look-ahead* sensing schemes (see, e.g., [6]). Relevant contributions to the application of advanced linear, nonlinear and robust control techniques to the design of lateral dynamics controllers were provided by Tomizuka and coworkers (see, e.g., [12], [10]) and by Ackermann and coworkers (see, e.g., [1]). In this paper we address the lane keeping problem through automatic steering in highway conditions using a vision system which provides the measurement of two output quantities: the vehicle lateral displacement  $q$  and the vehicle orientation  $m$  with respect to a suitable approximation of the centerline of the lane. We focus on the single-input two-output (SITO) structure of the considered system. Recently some attention has been paid to the analysis of properties of single-input two-output systems, motivated by the large number of control problems which exhibit a SITO structure (see [3], [11] and the references

therein). Lu and Tomizuka in [5] faced the problem of vehicle lateral control with combined use of laser scanning radar sensor and rear magnetometers. Using some of the properties introduced in [3], they propose a procedure for designing a two-input single-output (TISO) controller based on the minimization of the effects of the disturbance associated with one channel on the other one and *vice-versa*. In this work, exploiting a result introduced in [3], we show that our original SITO control problem can be simplified to the controller design of a SISO control system. Such a design is performed using  $\mu$ -synthesis techniques in order to obtain robust performances in the face of model uncertainties. Experimental results obtained along highways are presented.

## 2 Plant description and modeling

The plant to be controlled, provided by Centro Ricerche Fiat, consists of a Fiat Brava 1600 ELX equipped with a vision system and a steering actuator. The vision system comprises a single CCD videocamera and related image processing algorithms. The steering actuator system is a locally controlled DC brush-less electric motor. Both control and vision algorithms are processed by an INTEL 486 microprocessor based Personal Computer for industrial applications. The sampling time of the whole system is  $T_s = 40$  ms. The described given hardware is not supposed to be modified by the control designer.

The mathematical modeling of such a plant was discussed in detail in paper [2] in which a simplified model, able to describe the vehicle behaviour in highway experimental conditions, was presented. The equations of such a model are here recalled for self-consistency of the paper. The interaction between the vehicle lateral dynamics and the vision system can be modeled by the following state space equations parameterized by the longitudinal velocity  $v_x$

$$\begin{bmatrix} \dot{v}_y \\ \dot{\psi} \\ \dot{q} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \frac{-a_1}{m_v v_x} & \frac{-m_v v_x^2 + a_2}{m_v v_x} & 0 & 0 \\ \frac{a_3}{I_\psi v_x} & \frac{-a_4}{I_\psi v_x} & 0 & 0 \\ -1 & 0 & 0 & v_x \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_y \\ \psi \\ q \\ m \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix} \delta_v + \begin{bmatrix} 0 \\ 0 \\ -L v_x \\ v_x \end{bmatrix} K_L \quad (1)$$

where  $L$  is the so-called look-ahead distance, taken along the longitudinal axis, from the vehicle center of gravity to a suit-

able point in the road between 3 and 20 meters ahead the vehicle (see Figure 1);  $K_L(t)$  is the road curvature, i.e., the inverse of the instantaneous curve radius at the look-ahead point;  $\delta_v$  is the steering-wheel angle;  $m$  and  $q$  are the measurements supplied by the vision system about the vehicle location on the lane, as shown in Figure 2. The meaning of the symbols involved in the equations is the following

- $m_v$  : vehicle mass;
- $v_x$  : longitudinal component of center of gravity(CG) velocity;
- $v_y$  : transverse component of CG velocity;
- $\psi$  : vehicle yaw angle;
- $I_\psi$  : inertial vehicle moment around center of gravity referred to the vertical axis;
- $\delta_v$  : steering-wheel angle;
- $k$  : front wheels angle/steering-wheel angle ratio, expressed in rad/degrees;
- $\delta_f = \delta_v k$  : front wheels angle;
- $c_r$  : cornering stiffness of front tires;
- $c_f$  : cornering stiffness of rear tires;
- $l_r$  : distance between the rear axle and the center of gravity;
- $l_f$  : distance between the front axle and the center of gravity;
- $l = l_r + l_f$  : wheelbase;
- $a_1 = c_f + c_r$ ;
- $a_2 = c_r l_r - c_f l_f$ ;
- $a_3 = -l_f c_f + l_r c_r$ ;
- $a_4 = l_f^2 c_f + l_r^2 c_r$ ;
- $b_1 = \frac{c_f}{m} k$ ;
- $b_2 = \frac{l_f c_f}{I_\psi} k$ .

The closed loop steering actuator, given by *Centro Ricerche Fiat* is described by the following transfer function:

$$\frac{\delta_v}{\theta} = \frac{1580}{s^2 + 75.5s + 1580} \quad (2)$$

where  $\theta$  is the reference steering signal provided by the lateral controller to be designed.

The nominal values of the parameters  $\gamma_{nom} = [m_{v_0} \ I_{\psi_0} \ c_{r_0} \ c_{f_0} \ v_{x_0}]$  and the corresponding parameter uncertainty intervals  $PUI_i = [\gamma_i^{min}, \gamma_i^{max}]$  are summarized in Table 1.

Table 1: Values of model parameters

	Parameter	$\gamma_{nom}$	PUI
$\gamma_1$	$m_v$ (kg)	1226	[1226, 1626]
$\gamma_2$	$I_\psi$ (kgm <sup>2</sup> )	1900	[1900, 2520]
$\gamma_3$	$c_r$ (N/rad)	96000	[81600, 110400]
$\gamma_4$	$c_f$ (N/rad)	60000	[51000, 69000]
	$l_r$ (m)	1,506	–
	$l_f$ (m)	1,034	–
$\gamma_5$	$v_x$ (km/h)	95	[60, 130]
	$k$ (rad/degrees)	$\pi/(180 \cdot 18)$	–

The uncertain system generated by the given parameter uncertainty intervals can be represented by the following model

set:

$$\mathcal{M}_P = \{P(s, \gamma) \in \mathcal{C} : \gamma \in \Gamma\} \quad (3)$$

where  $P(s, \gamma) = [P_1(s, \gamma) \ P_2(s, \gamma)]^T$ ;  $P_1(s, \gamma)$  is the transfer function from  $\theta$  to the lateral displacement  $q$ ;  $P_2(s, \gamma)$  is the transfer function from  $\theta$  to the vehicle orientation  $m$ ;  $\gamma = [m_v \ I_\psi \ c_r \ c_f \ v_x]$  is the vector of the uncertain parameters and  $\Gamma = \{\gamma \in \mathbb{R}^5 : \gamma_i \in PUI_i, i = 1, 2, 3, 4, 5\}$ .

As far as the curvature disturbance modeling is concerned, italian legislation on roads building states that straight sections must be connected to bends through a given class of geometric curves [4] and that at the velocity of 130 km/h, the maximum velocity allowed on italian highways, the radius of the bends must be greater than 800 m. Thus, using such curves building rules we can evaluate the corresponding curvature variation  $K_L(t)$ . Such an analysis highlights that the worst-case disturbance  $K_{L_{ws}}(t)$  is a step of size  $1/800 = 1.25 \cdot 10^{-3} \text{ m}^{-1}$  which correspond to a straight section directly connected to a circle with constant radius of 800 m. The magnitude of such a worst-case disturbance in the frequency domain is shown in Figure 3.

### 3 Control problem formulation

The problem we are dealing with in this paper is the design of a controller able to keep the vehicle inside the lane along typical highway paths. Thus, the objective of the control problem is to keep the value of the lateral displacement of the vehicle with respect to the centerline of the lane within a prescribed tolerance, i.e.,  $|q(t)| \leq \rho$ , where  $\rho = 0.2$  m. As a matter of fact, although specifications are explicitly given only on the value of  $|q|$ , the minimization of such a quantity is also strictly related to the regulation of the orientation of the vehicle measured by the value of  $m$ . Thus, the addressed design control problem requires the rejection of the effect of the disturbance  $K_L$  on both the two measured outputs  $q$  and  $m$  through the design of a TISO controller, as depicted in Figure 4, where  $y_1 = q$ ,  $y_2 = m$  and  $d = K_L$ . Such a formulation fits in the framework of single-input two-output (SITO) systems, whose properties have been extensively studied in recent years (see, e.g., [3], [11] and the references therein).

### 4 Some properties of SITO systems

In this section we recall some definitions and results introduced by Freudenberg and Middleton in [3] and discuss their application to the problem addressed in this paper. All the symbols and the quantities used in this section are referred to the block diagram shown in Figure 4. The SITO plant is indicated by  $P(s, \gamma) = [P_1(s, \gamma) \ P_{2,\gamma}(s)]^T$ , the transfer function between the disturbance input and the two outputs by  $P_d(s, \gamma) = [P_{d_1}(s, \gamma) \ P_{d_2}(s, \gamma)]^T$  and the TISO controller by  $C(s) = [C_1(s) \ C_2(s)]$ . First, recalling Definition 9 and Definition 4 of [3], we define the uncertain plant-disturbance and the uncertain plant-controller alignment angles for the problem addressed in this paper:

**Definition 1:**

The *uncertain plant-disturbance alignment angle* is defined as

$$\phi_{pd}(\omega, \gamma) \doteq \arccos \left( \frac{|P^H(j\omega, \gamma)P_d(j\omega, \gamma)|}{\|P(j\omega, \gamma)\| \|P_d(j\omega, \gamma)\|} \right) \quad (4)$$

**Definition 2:**

The *uncertain plant-controller alignment angle* is defined as

$$\phi_{pc}(\omega, \gamma) \doteq \arccos \left( \frac{|C(j\omega)P(j\omega, \gamma)|}{\|C(j\omega)\| \|P(j\omega, \gamma)\|} \right) \quad (5)$$

Next let us define the input and the output loop gain as  $L_I(s, \gamma) = C(s)P(s, \gamma)$  and  $L_O(s, \gamma) = P(s, \gamma)C(s)$  respectively. Thus the input and output sensitivity are  $S_I(s, \gamma) = 1/(1 + L_I(s, \gamma))$  and  $S_O(s, \gamma) = (I + L_O(s, \gamma))^{-1}$  while the input and output complementary sensitivity are  $T_I(s, \gamma) = L_I(s, \gamma)/(1 + L_I(s, \gamma))$  and  $T_O(s, \gamma) = L_O(s, \gamma)(I + L_O(s, \gamma))^{-1}$ , where  $I$  is the identity matrix. Then the closed loop transfer function of interest in the disturbance attenuation problem addressed in the paper is:

$$\frac{y(s)}{d(s)} = S_O(s, \gamma)P_d(s, \gamma) \quad (6)$$

where  $y(s) = [y_1(s) \ y_2(s)]^T = [q(s) \ m(s)]^T$  is the Laplace transform of the output and  $d(s) = K_L(s)$  is the Laplace transform of the disturbance input. The following proposition, obtained extending Proposition 9 of [3] to the problem addressed in this paper, states the dependence of transfer function (6) upon the uncertain plant-disturbance  $\phi_{pd}(\omega, \gamma)$  and the uncertain plant-controller  $\phi_{pc}(\omega, \gamma)$  alignment angles:

**Proposition 1:**

The transfer function  $S_O(s, \gamma)P_d(s, \gamma)$  that map  $d(s)$  to  $y(s)$  satisfy the following bounds:

$$S_{lw}(j\omega, \gamma) \leq \frac{\|S_O(j\omega, \gamma)P_d(j\omega, \gamma)\|}{\|P_d(j\omega, \gamma)\|} \leq S_{up}(j\omega, \gamma) \quad (7)$$

where:

$$S_{lw}(j\omega, \gamma) = \left\{ \sin^2 \phi_{pd}(j\omega, \gamma) + |\cos \phi_{pd}(j\omega, \gamma)| |S_I(j\omega, \gamma)| - \sin \phi_{pd}(j\omega, \gamma) |T_I(j\omega, \gamma)| \tan \phi_{pc}(j\omega, \gamma) \right\}^{\frac{1}{2}}$$

and

$$S_{up}(j\omega, \gamma) = \left\{ \sin^2 \phi_{pd}(j\omega, \gamma) + (\cos \phi_{pd}(j\omega, \gamma) |S_I(j\omega, \gamma)| + \sin \phi_{pd}(j\omega, \gamma) |T_I(j\omega, \gamma)| \tan \phi_{pc}(j\omega, \gamma))^2 \right\}^{\frac{1}{2}}$$

**Remark 1:** The ratio  $\alpha(\omega, \gamma) \doteq \frac{\|S_O(j\omega, \gamma)P_d(j\omega, \gamma)\|}{\|P_d(j\omega, \gamma)\|}$  is the disturbance attenuation coefficient provided by the closed loop. Large attenuation is obtained when  $\alpha(\omega, \gamma) \approx 0$  while no attenuation is provided when  $\alpha(\omega, \gamma) = 1$ . Then, the goal of the disturbance attenuation problem can be specified through the

assignment of desired values of  $\alpha(\omega, \gamma)$ .

**Result 1:**

The original SITO control problem formulated in section 3 can be simplified to an equivalent SISO disturbance attenuation one over the following set of frequencies:

$$\Omega = \{\omega \in \mathfrak{R} : \phi_{pd}(\omega, \gamma) = 0, \phi_{pc}(\omega, \gamma) \neq \pi/2, \gamma \in \Gamma\} \quad (8)$$

**Proof:**

From Proposition 1 it can be seen that  $\forall \omega$  such that  $\phi_{pd}(\omega, \gamma) = 0$  and  $\phi_{pc}(\omega, \gamma) \neq \pi/2$  we have  $S_{lw}(\omega, \gamma) = S_{up}(\omega, \gamma) = |S_I(\omega, \gamma)|$  which imply  $\alpha(\omega) = |S_I(\omega, \gamma)|$ . Therefore, if the two conditions  $\phi_{pd}(\omega, \gamma) = 0$  and  $\phi_{pc}(\omega, \gamma) \neq \pi/2$  are satisfied for all the systems belonging to  $\mathcal{M}_P$ , the SITO disturbance attenuation problem we are dealing with can be simplified to an equivalent SISO disturbance attenuation one that can be addressed through the frequency shaping of  $|S_I(\omega, \gamma)|$  over  $\Omega$ .

**Remark 2:** From Figure 5, which shows the plant-disturbance alignment angle of the uncertain system, obtained gridding the hypercube  $\Gamma$ , we can see that  $\phi_{pd}(\omega, \gamma) \approx 0$  in the frequency range  $[0, 1]$  rad/s. Moreover from equation (5) it is easy to show that the required condition  $\phi_{pc}(\omega, \gamma) \neq \pi/2$  is implied by  $\frac{C_2(j\omega)}{C_1(j\omega)} \neq -\frac{P_1(j\omega, \gamma)}{P_2(j\omega, \gamma)}$ , for which a sufficient condition is  $\angle \frac{C_2(j\omega)}{C_1(j\omega)} \neq \angle -\frac{P_1(j\omega, \gamma)}{P_2(j\omega, \gamma)}$ .

**Remark 3:** Note that  $\forall \omega \geq 1$  rad/s, i.e., over the frequency range where the problem cannot be simplified to an equivalent SISO one, the magnitude of the worst-case disturbance can be considered negligible being  $|K_L(\omega)| < -50$  dB, as shown by Figure 3.

## 5 Controller design

**Controller structure choice**

From Result 1 of Section 2 we see that the SITO control problem formulated in Section 3 can be solved by designing a TISO controller  $C(s) = [C_1(s)C_2(s)]$  through the shaping of  $S_I(j\omega, \gamma)$ , that is, by solving a SISO disturbance attenuation problem. Among the all possible structures of  $C(s)$  we focus on the choice presented in paper [2] based on the emulation of the common driver behaviour. In that paper we showed that such an approach leads to the design of a SISO controller  $\tilde{C}(s)$  of a feedback control system where the feedback signal is  $y_{fb} = q + mL$  which is the distance, measured at the look-ahead, between the longitudinal axis and the linear approximation of the centerline of the lane (see Figure 1). It can be easily shown that such a control strategy is equivalent to the choice  $C_1(s) = \tilde{C}(s)$  and  $C_2(s) = L\tilde{C}(s)$  for the internal structure of the TISO controller  $C(s)$ . Moreover, such a choice for the internal structure of  $C(s)$ , simplify the sufficient condition  $\angle \frac{C_2(j\omega)}{C_1(j\omega)} \neq \angle -\frac{P_1(j\omega, \gamma)}{P_2(j\omega, \gamma)}$ , formulated in Remark 2, to the following one:  $\angle -\frac{P_1(j\omega, \gamma)}{P_2(j\omega, \gamma)} \neq 0$ , being  $\frac{C_2(j\omega)}{C_1(j\omega)} = L$ . Now, from Figure 6, which shows the angle  $\angle -\frac{P_1(j\omega, \gamma)}{P_2(j\omega, \gamma)}$  for the uncertain system obtained gridding the hypercube  $\Gamma$ , we note that

$\angle - \frac{P_1(j\omega, \gamma)}{P_2(j\omega, \gamma)} \neq 0, \quad \forall \omega$ . Thus we can conclude that the proposed controller structure ensures the fulfillment of the condition  $\phi_{pc}(\omega, \gamma) \neq \frac{\pi}{2}$ , independently of the subsequent design of the SISO controller  $\tilde{C}$  as well as of the value of the lookahead distance  $L$  which can be chosen following the consideration made in [2].

#### Design technique

In order to meet the desired performance requirements formulated in Section 3 in the presence of the uncertainty described in Section 2, we formulate the problem in the  $H_\infty/\mu$  framework. More specifically the design of the controller  $\tilde{C}$  is performed solving the following  $\mu$ -synthesis problem:

$$\tilde{C}(s) = \arg \min_{\tilde{C}(s) \in \mathcal{RH}_{\infty\omega}} \sup \mu \quad (9)$$

where  $\mu = |W_2(s)T_I(s, \gamma_{nom})| + |W_1(s)S_I(s, \gamma_{nom})|$  and the frequency shaped weights  $W_1(s)$  and  $W_2(s)$  respectively embed the performance specification and the model uncertainty as described in the rest of the section.

#### Selection of weight $W_1$

In Remark 1 we have shown that the goal of disturbance attenuation problem we are dealing with can be specified assigning the shape of the attenuation coefficient  $\alpha(\omega, \gamma)$ . Then, in Result 1 we have shown that  $\alpha(\omega, \gamma) = |S_I(\omega, \gamma)| \forall \omega \in [0, 1]$  rad/s. Thus, the requirement on the attenuation of the disturbance in the frequency interval  $[0, 1]$  rad/s can be embedded in the design through the following sensitivity weighting function  $W_1(s) = \frac{0.075782(s+1.5)^2(s+1)}{(s+0.08)s^2}$ , whose inverse is shown in Figure 7. Finally we recall that, outside that frequency interval, the disturbance can be considered negligible.

#### Selection of weight $W_2$

In order to perform the design using  $H_\infty/\mu$  techniques we have to describe the uncertain system in terms of linear fractional transformations [8]. For this purpose a suitable, although conservative, description of the uncertainty generated by the considered parameters perturbation is provided by the following model set expressed in the input multiplicative form:

$$\mathcal{M}_{\tilde{P}} = \{\tilde{P} = (1 + \Delta_m)\tilde{P}_n : |\Delta_m(j\omega)| \leq W_2(j\omega), \forall \omega\} \quad (10)$$

where  $\tilde{P}_n(s, \gamma) = P_1(s, \gamma) + LP_2(s, \gamma)$  is the nominal plant,  $\Delta_m(s)$  is a complex function which represents the unknown modeling error and  $W_2(s)$  is a known function bounding the modeling error. A description of  $\Delta_m(j\omega)$  obtained gridding the hypercube  $\Gamma$  is shown in Figure 8 together with the upper bound  $W_2(s) = 0.6$ .

#### Optimization results

The  $\mu$ -synthesis was performed using the D-K iteration algorithm. Starting with initial scaling weighting matrix  $D$  set to identity, the procedure provides both robust stability and robust performance fulfillment (i.e.,  $\sup \mu < 1$ ) after the first K-iteration (i.e. solving a single  $H_\infty$  design problem). The obtained controller  $\tilde{C}(s)$  is  $\tilde{C}(s) = n(s)/d(s)$  where  $n(s) = [-3.5e6s^6 - 2.8e8s^5 - 6.9e9s^4 - 3.3e10s^3 - 1.3e11s^2 - 5.8e10s - 1.1e10]$  and  $d(s) = [s^7 + 4.2e2s^6 + 8.6e4s^5 + 9.6e6s^4 + 3.8e8s^3 + 1.1e9s^2 + 2.8e9s + 2.2e8]$ . In Figure 9

the upper and the lower bounds on the sensitivity function of the perturbed systems are depicted together with the inverse of the performance weight  $W_1^{-1}(j\omega)$ .

## 6 Experimental results

In this section we report the experimental results obtained testing the controlled vehicle. The test track is a straight section followed by a curve with radius  $R \approx 800$  m along an italian highway. The velocity was kept approximately constant at 110 km/h. From Figure 10, which shows the lateral offset  $q$ , we see that the designed SISO controller fulfills the specification about the position error with respect to the centerline of the lane.

## 7 Conclusions

In this paper we addressed the problem of vehicle lateral dynamics control in highway experimental conditions. The control system under consideration exhibits a single-input two-output (SITO) structure where the steering angle is the input while the two outputs are the lateral displacement and the angular orientation of the vehicle referred to the centerline of the lane. By exploiting some properties of SITO systems, the given control problem is simplified to the design of a single-input single-output (SISO) controller. The design of such a controller is performed through  $\mu$ -synthesis techniques in order to obtain robust performances in the face of model uncertainties. Experimental tests along highway paths using a FIAT Brava 1600 ELX provided by Centro Ricerche Fiat, showed the fulfillment of given specifications.

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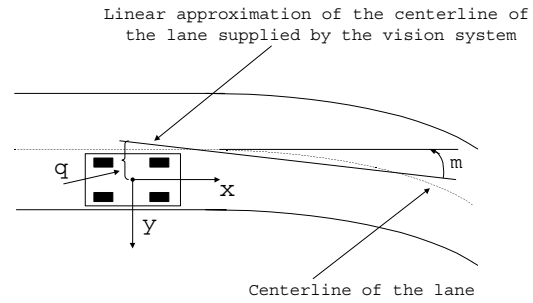


Figure 2: Centerline linear approximation supplied by the vision system.

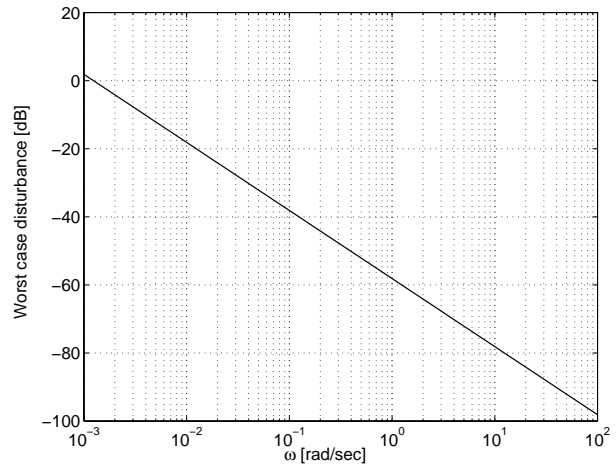


Figure 3: Magnitude of the worst case curvature disturbance  $|K_{L_{ws}}(\omega)|$ .

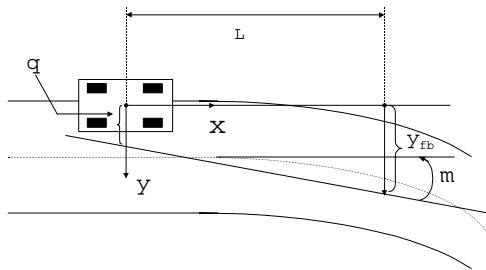


Figure 1: Feedback quantity  $y_{fb}$ .

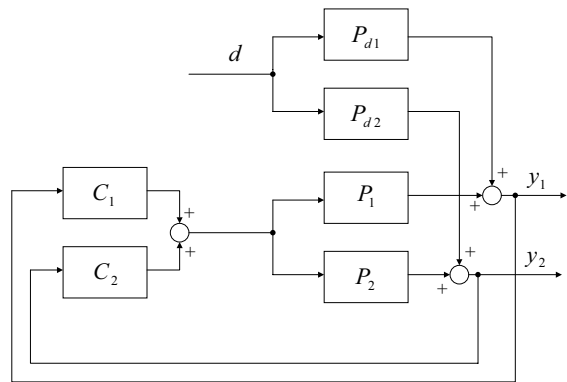


Figure 4: Block diagram of SITO feedback system.

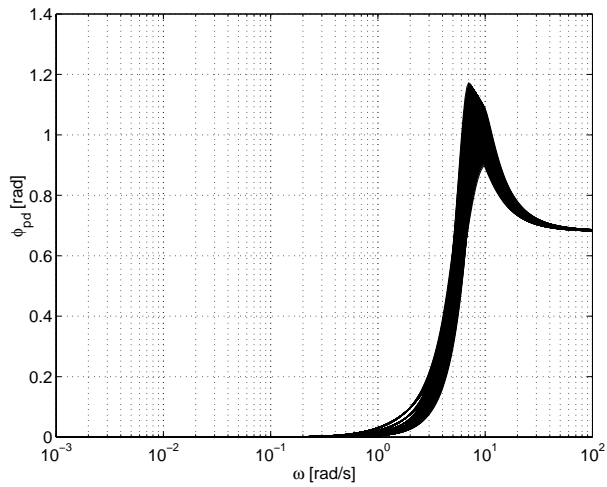


Figure 5: Alignment angle  $\phi_{pd}(\omega)$ .

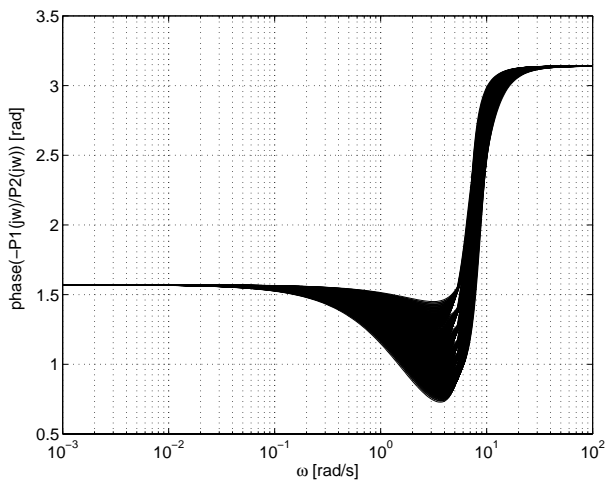


Figure 6: Angle  $\angle \frac{-P_1(j\omega)}{P_2(j\omega)}$  for the uncertain system.

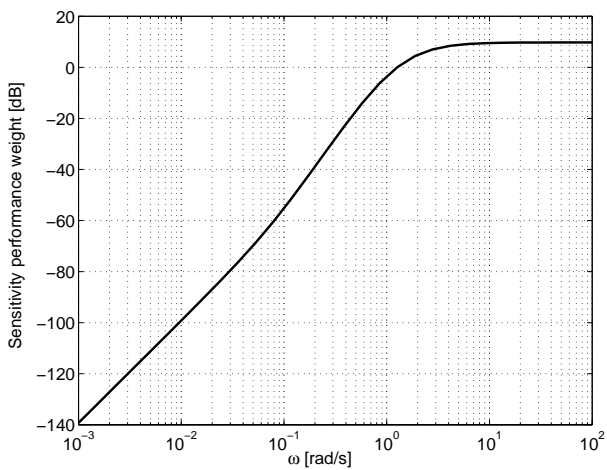


Figure 7: Sensitivity weight function  $W_1^{-1}(j\omega)$ .

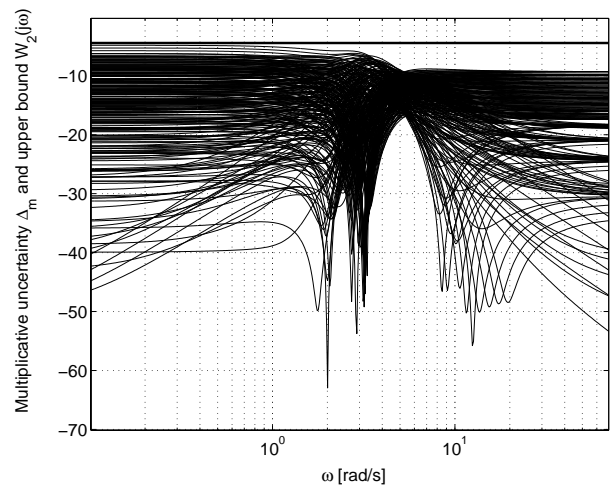


Figure 8: Uncertainty  $\Delta_m$  and weighting function  $W_2$  (solid).

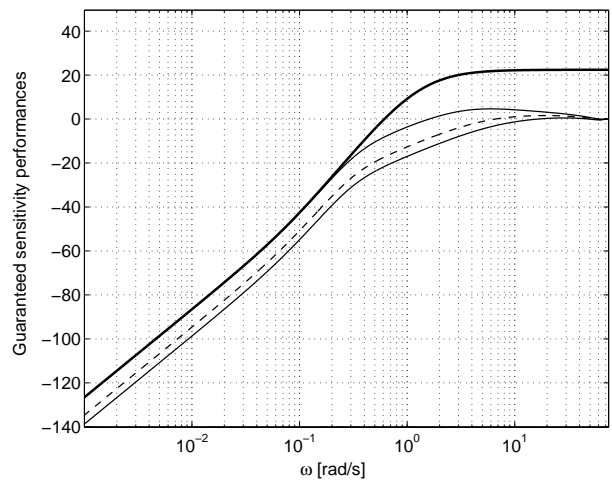


Figure 9: Guaranteed sensitivity bounds (thin), nominal sensitivity  $S_I$  (dashed) and performance weight  $W_1^{-1}$  (solid).

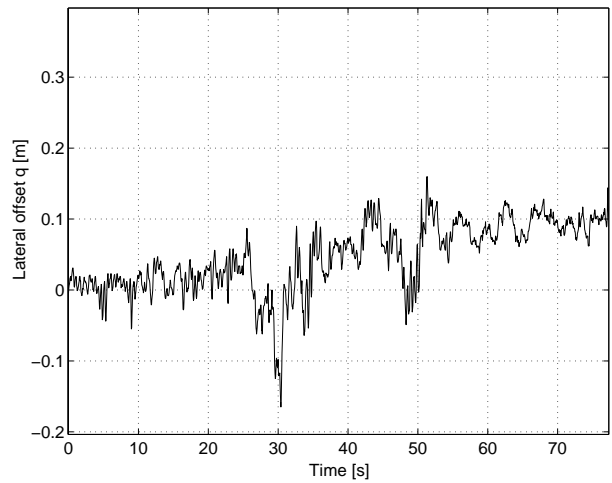


Figure 10: Experimental lateral offset  $q$ .