ROBUST SERVO CONTROL DESIGN FOR MECHANICAL SYSTEMS USING MIXED UNCERTAINTY MODELLING

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Abstract

In this paper, a servo control synthesis based on the μ method is applied. With this method, a robust compensator that achieves performance specifications can be designed, which provides the track of the predefined reference signal, rejects the effects of the disturbances and takes structured uncertainty into consideration. In the mixed μ synthesis, both the real parametric and the complex uncertainties are handled together, which usually yields a less conservative compensator than the traditional robust control design methods. The design strategy is illustrated for an inverted pendulum device, which involves real parametric uncertainties.

1 Introduction and motivation

Recently, the complex μ synthesis has become widespread because this method yields a compensator that achieves nominal performance and robust stability and takes structured uncertainties into consideration. In this method, the structure of uncertainties is represented by a diagonal structure with full or scalar complex blocks. For real parametric uncertainties the representation of the complex μ method can be arbitrarily conservative. In practice, there are several components whose parameters change around their operational points. If the parametric uncertainties are taken into account, the magnitude of the unmodelled dynamics could be decreased. In this case, the structure of uncertainties is represented by repeated real blocks. Applying the mixed μ method parametric uncertainties can be taken into consideration, which is more realistic than the traditional approaches, and the design process yields a less conservative compensator than other robust control design methods, [1, 5, 8, 10, 14].

In the last decade, the two degree-of-freedom compensator for servo problems has been widely used in practice. They comprise two components, a pre-filter and a feedback component. Several methods for designing servo controllers exist, [9, 11, 18], etc. Keviczky (1996) followed a different path to design a servo system through an iterative scheme. In this scheme the model identification and the controller design steps are performed in a sequential way to improve the performance properties of the controlled system, [12, 13]. In an earlier work of our project, a servo control design methodology based on $\mathcal{H}_\infty$ and complex μ was presented, [6].

The aim of this paper is to apply the mixed μ synthesis to an inverted pendulum device. In a number of papers the inverted pendulum is considered as a good demonstration tool for the illustration of control design. Different control strategies using traditional methods or modern methods have been shown. In this paper, more information of the mechanical system is assumed to be known and these data can be taken into account in the mixed μ method. The design process yields a controlled system with better performance behavior and the conservatism is also decreased.

The organization of the paper is as follows. Section 2 presents the problem setup, i.e. the specifications of the servo design for an inverted pendulum. Section 3 discusses the robust servo control design based on the mixed μ synthesis. Section 4 demonstrates the application of the μ synthesis in both complex and mixed μ methods, and gives some comparison results.

2 Servo control specifications for uncertain systems

The simplified structure of the inverted pendulum that is installed in our laboratory is shown in Figure 1. The cart is propelled by a DC servomotor supported by a power amplifier, the cart position and the rod angle is measured by potentiometers. Direct digital control can be realized by means of a computer complemented with analog to digital and digital to analog converters. The objective of the experiment is to design a controller which stabilizes the rod and keeps the cart in a desired position. Let $\bar{m}_1$ be the mass of the rod, $\bar{l}$ the length of the rod, m_2 the mass of the cart, R_m the armature resistance, K_m the motor torque constant, K_g the gear-ratio of gearbox, and r the radius of the gear. The state space form of the nominal model is as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \hline y_x \\ y_\theta \end{bmatrix} = \left[\begin{array}{cccc|c} c_2 & \frac{1}{l}g(\frac{\bar{m}_1}{m_2}+1) & -g\frac{1}{l}c_2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & c_1 & 0 & -g\frac{1}{l}c_1 & 0 \\ 0 & -\frac{1}{l}c_1 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \hline u \end{bmatrix} \tag{1}$$

where x_i's are the state variables in the controllability state space representation form, u is the input voltage, y_x is the car displacement and y_θ is the rod angle, [16]. In Eq. (1), the $c_1 = \frac{K_g K_m A_m}{R_m m_2 r}$ and $c_2 = -\frac{K_g^2 K_m^2}{R_m m_2 r^2}$ are constants.

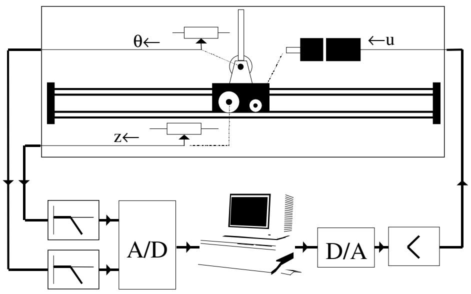


Figure 1: Schematic diagram of the experiment

The controller must be designed in such a way that the following criteria are met. The closed-loop system must be stable. The control voltage must not exceed 10 V. Let the reference signal for the displacement be a square signal with 0.2 magnitude, as well as a 0.2 amplitude sine signal.

The output signals, i.e. displacement y_x (tracking) and rod angle y_θ (interaction), must satisfy the following specifications:

Specification 1: The settling time must be less than 10 sec: $|y_x(t) - \bar{y}_x| < 0.02$, for all $t \leq 10$.

Specification 2: The overshoot must not exceed 10%: $y_x(t) < 0.22$ for all t.

Specification 3: The steady-state error must be below 1%: $|y_x(t) - \bar{y}_x| < 0.002$.

Specification 4: The interaction must be minimal: $y_\theta(t) < 0.1$ for all t.

Specification 5: Applying the disturbance signal the angle must be minimal: $y_\theta(t) < 0.1$ for all t.

The difficulties of the control design is that the model contains uncertainties, which are caused the parametric uncertainties. The parametric uncertainties are generated in a laboratory environment by varying the length of the rod l and its mass m_1.

The parameters are assumed to be uncertain, with a nominal value and a range of possible variation:

$$m_1 = \bar{m}_1(1 + d_m\delta_m), \quad l = \bar{l}(1 + d_l\delta_l) \tag{2}$$

with d_m, d_l scalars, in which $-1 \leq \delta_m, \delta_l \leq 1$. The d scalar indicates the percentage of variation that is allowed for a given parameter around its nominal value. The changing of δ parameters in the interval $\begin{bmatrix} -1 & 1 \end{bmatrix}$ determines the actual parameter deviation. The l and m_1 parameters occur in the differential equation so their LFT representation can be drawn up in the following way:

$$\frac{1}{l} = \frac{1}{\bar{l}(1 + d_l\delta_l)} = \mathcal{F}_l\left(\begin{bmatrix} \frac{1}{l} & -\frac{d_l}{l} \\ 1 & -d_l \end{bmatrix}, \delta_l\right) \tag{3}$$

$$m_1 = \bar{m}_1(1 + d_m\delta_m) = \mathcal{F}_l\left(\begin{bmatrix} \bar{m}_1 & 1 \\ d_m\bar{m}_1 & 0 \end{bmatrix}, \delta_m\right) \tag{4}$$

In Eq. (3) and Eq. (4), let $M_m = \begin{bmatrix} \bar{m}_1 & 1 \\ d_m\bar{m}_1 & 0 \end{bmatrix}$ and $M_l = \begin{bmatrix} \frac{1}{l} & -\frac{d_l}{l} \\ 1 & -d_l \end{bmatrix}$ be the uncertain blocks.

The δ uncertainty blocks from the motion equations must be pulled out. Let the input and output of δ_m be y_{m_1} and u_{m_1}, and δ_l be y_l and u_l, respectively. In the differential equations of the nominal plant the length of the rod l occurs in several times. In general such parameters can only be treated as a repeated scalar block. It means that different uncertain parameters must be handled by the same uncertain coefficients (d, δ). Thus, l can be modelled as a three times repeated parameter. The u_l^i and y_l^i ($i = 1, 2, 3$) represent the input and output signals of the length uncertainty, and u_m^i, y_m^i represent the signals of the mass uncertainty.

Applying equations (3) and (4), the state space form containing uncertain parameters between $\left[\begin{array}{cccc|cccc|cc} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4 & y_l^1 & y_l^2 & y_l^3 & y_m^1 & y_x & y_\theta \end{array}\right]$ and $\left[\begin{array}{cccc|cccc|c} x_1 & x_2 & x_3 & x_4 & u_l^1 & u_l^2 & u_l^3 & u_m^1 & u \end{array}\right]$ can be formulated. The uncertain state space model and its illustration are shown in the following.

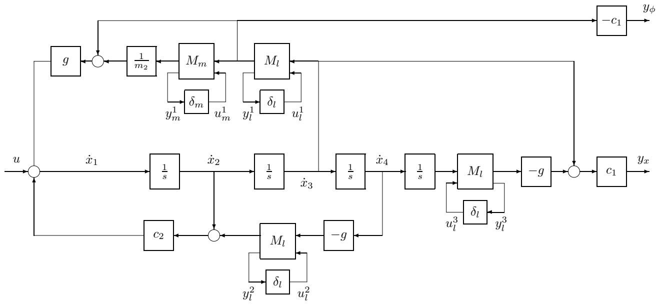


Figure 2: Block structure of the uncertain model

3 Robust servo control design using the mixed μ synthesis

Consider the closed-loop system which includes the feedback structure of the model G and controller K, and elements associated with the uncertainty models and performance objectives (Figure 3). In the diagram, r is the reference, u is the control input, y is the output, n is the measurement noise, and z_e is the deviation of the output from the required one. The structure of the controller K may be partitioned into two parts: $K = \begin{bmatrix} K_r & K_y \end{bmatrix}$, where K_y is the feedback part of the controller and K_r is the pre-filter part.

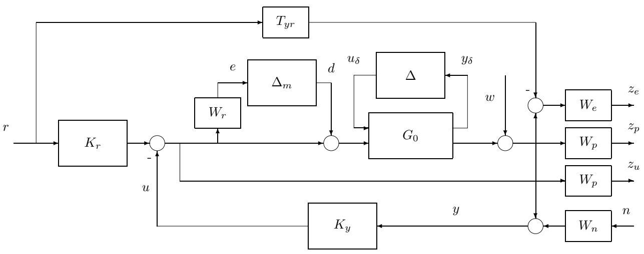


Figure 3: Closed loop interconnection structure

The required transfer function T_{yr} from r to y is defined by the designer. In our application, T_{yr} is used to introduce time domain specification into the design process. Inside the dashed box there are two blocks to represent both the unmodelled dynamic and the parametric uncertainty. The uncertainties of the rod length and the rod mass are represented by the Δ_r block, whose input and output are denoted by u_δ and y_δ. The transfer function Δ_r contains the $\delta_l I_{3\times3} < 1$ and $\delta_m < 1$ components in diagonal form. The unmodelled dynamics is represented by W_r and Δ_m. It is assumed that the transfer function W_r is known, and it reflects the uncertainty in the model. The transfer function Δ_m is assumed to be stable and unknown with the norm condition, $\|\Delta_m\|_\infty < 1$. In the diagram, e is the input of the perturbation, d is its output.

The weighting function W_e reflects the relative importance of the different frequency domains in terms of tracking error. The weighting function W_n represents the impact of the different frequency domains in terms of sensor noise n. The weighting function W_p represents the performance of the rod angle. The role of the weighting function W_u represents the different frequency domains of the input effort. Necessary and sufficient conditions for robust stability and robust performance can be formulated in terms of the structured singular value denoted as μ, [2].

By applying the weighting functions and the compensator, the augmented plant P can be formalized between the outputs $\left[\begin{array}{c|cccc|ccc|ccc} e & y_l^1 & y_l^2 & y_l^3 & y_m^1 & z_e & z_p & z_u & r & y_x & y_\theta \end{array}\right]$ and the inputs $\left[\begin{array}{c|cccc|ccc|c} d & u_l^1 & u_l^2 & u_l^3 & u_m^1 & r & w & n & u \end{array}\right]$:

$$\left[\begin{array}{c|cccc|ccc|c}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_r \\
\hline
0 & -d_l & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -d_l & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -d_l & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{d_l}{l} d_m \bar{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
W_e G_{yu}^x & 0 & 0 & W_e \frac{d_l}{l} g c_1 & 0 & -W_e T_{yr} & W_e & 0 & W_e G_{yu}^x \\
W_p G_{yu}^\theta & W_p \frac{d_l}{l} c_1 & 0 & 0 & 0 & 0 & W_p & 0 & W_p G_{yu}^\theta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_u \\
\hline
0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\
G_{yu}^x & 0 & 0 & \frac{d_l}{l} g c_1 & 0 & 0 & I & W_n & G_{yu}^x \\
G_{yu}^\theta & \frac{d_l}{l} c_1 & 0 & 0 & 0 & 0 & I & W_n & G_{yu}^\theta
\end{array}\right] \tag{5}$$

The mixed real and complex μ involves three types of blocks: repeated real scalar, repeated complex scalar and full blocks. Three nonnegative integers, S_r, S_c, and F represent the number of repeated real scalar blocks, the number of repeated complex blocks, and the number of full blocks. The admissible set of uncertainties $\tilde{\Delta}$ is defined as

$$\tilde{\Delta} = \begin{bmatrix} \Delta_r & 0 & 0 \\ 0 & \Delta_m & 0 \\ 0 & 0 & \Delta_p \end{bmatrix}, \tag{6}$$

where $\Delta_r \in \mathbb{R}^{4\times4}$, $\Delta_m \in \mathbb{C}^{1\times1}$, $\Delta_p \in \mathbb{C}^{4\times3}$. The first block, Δ_r is a repeated real scalar block which represents the parametric uncertainties. The second block of this structured set corresponds to the scalar-block uncertainty Δ_m which is used to describe the unmodelled dynamics. The Δ_p is a fictitious uncertainty block, which is used to incorporate the $\mathcal{H}_\infty$ nominal performance objective into the μ framework. Given a matrix M, the mixed $\mu_{\tilde{\Delta}}$ function is then defined by:

$$\mu_{\tilde{\Delta}}(M) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \tilde{\Delta},\ \det(I - M\Delta) = 0\}} \tag{7}$$

unless no $\Delta \in \tilde{\Delta}$ makes $I - M\Delta$ singular, in which case $\mu_\Delta(M) = 0$. Thus $1/\mu_{\tilde{\Delta}}(M)$ is the "size" of the smallest perturbation Δ, measured by its maximum singular value, which makes $\det(I - M\Delta) = 0$. Unfortunately equation (7) is not suitable for computing μ since the implied optimization problem may have multiple local maxima. However tight upper and lower bounds for μ may be effectively computed for both complex and mixed perturbation sets. Algorithms for computing these bounds have been documented in several papers, see e.g. [4, 19].

Let us define the following expressions:

$$\mathcal{Q} = \left\{\Delta \in \tilde{\Delta} :\ \phi_i \in [-1, 1],\ |\delta_i| = 1,\ \Delta_i \Delta_i^* = I_{m_i}\right\} \tag{8}$$

$$\mathcal{D} = \left\{\begin{array}{c} \operatorname{diag}\left[\tilde{D}_1,\ \tilde{D}_2,\ \tilde{D}_3,\ \tilde{D}_4,\ d_1,\ I_2\right] : \\ \tilde{D}_1 \in \mathbb{C}^{1\times1},\ \tilde{D}_2 \in \mathbb{C}^{1\times1},\ \tilde{D}_3 \in \mathbb{C}^{1\times1},\ \tilde{D}_4 \in \mathbb{C}^{1\times1} \\ d_1 \in \mathbb{R},\ I_2 = \mathcal{I}^{4\times3} \end{array}\right\} \tag{9}$$

$$\mathcal{G} = \left\{\begin{array}{c} \operatorname{diag}\left[G_1,\ G_2,\ G_3,\ G_4,\ 0,\ 0\right] : \\ G_1 \in \mathbb{C}^{1\times1},\ G_2 \in \mathbb{C}^{1\times1},\ G_3 \in \mathbb{C}^{1\times1},\ G_4 \in \mathbb{C}^{1\times1} \end{array}\right\} \tag{10}$$

The upper bound can be formulated as a convex optimization problem, so the global minimum can be found. For a constant matrix M and both complex and mixed uncertainty structure $\tilde{\Delta}$, an upper bound for $\mu_{\tilde{\Delta}}(M)$ that take the phase information of the real parameters into account can be formulated into an optimization problem:

$$\inf_{D\in\mathcal{D},\, G\in\mathcal{G}} \min_{\beta} \left\{\beta \mid M^*DM + j(GM - M^*G) - \beta^2 D \le 0\right\} \tag{11}$$

The goal of the mixed μ synthesis is to minimize overall stabilizing controllers K, the peak value $\mu_\Delta(\cdot)$ of the closed loop transfer function $\mathcal{F}_l(P,K)$. The formula is as follows:

$$\min_K \sup_\omega \mu_{\bar{\Delta}}[\mathcal{F}_l(P,K)(j\omega)] \tag{12}$$

Using this upper bound, the optimization is reformulated as

$$\min_K \sup_\omega \inf_{D\in\mathcal{D},\, G\in\mathcal{G}} \min_\beta \{\beta \mid \quad \bar{\sigma}(\Gamma(\omega)) \le 1)\} \tag{13}$$

where

$$\Gamma(\omega) = \left(\frac{D_\omega F_l(P,K)(j\omega)D_\omega^{-1}}{\beta} - jG_\omega\right)(I + G_\omega^2)^{-\frac{1}{2}} \tag{14}$$

where D_ω, G_ω are selected from the set of scaling $\mathcal{D}$, $\mathcal{G}$ independently of every ω.

The scaling G allows one to exploit the phase information about the real parameters so that a better upper bound can be obtained. The optimization problem can be solved in an iterative way using for D, G and K, similarly to $D-K$ iteration. For fixed $K(s)$ the problem of finding $D(\omega)$, $G(\omega)$ and β is just the mixed upper bound problem. Having found these scalings we may fix $\beta^* = \max\beta$ and fit transfer function matrices $D(s)$ and $G(s)$ to $D(\omega)$ and $jG(\omega)$. It can then be shown, that using spectral factorization, a stable interconnection $P_{DG}(s)$ can be formed, which approximates $\Gamma(\omega)$ across frequency ω. For given β^*, $D(s)$ and $G(s)$ the problem of finding the controller $K(s)$ will be reduced to a standard $\mathcal{H}_\infty$ problem. The optimization algorithm is called $D,G-K$ iteration, see [3, 17, 20].

4 The μ synthesis for an inverted pendulum

The control design based on the μ synthesis is performed in two ways. The first approach is based on the complex μ synthesis, in which the model uncertainties are represented by complex frequency dependent Δ blocks and a-priory information about the real parametric uncertainties is not used in the design process. The second approach is based on the mixed μ synthesis, in which the real parametric uncertainties are taken into consideration, i.e. both the complex and the real frequency independent uncertainties are handled in Δ blocks. The nominal parameters of the inverted pendulum are shown in Table 1.

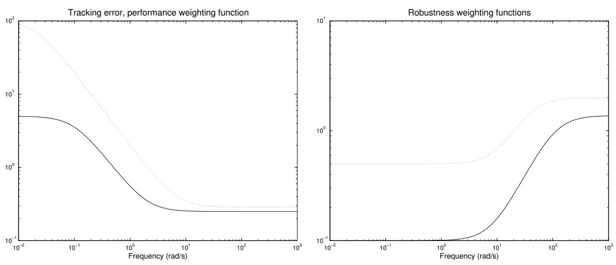
Let the required transfer function from the reference to the displacement of the cart be the following simple first-order system: $T_{yr} = \frac{1}{s+1}$. The reference tracking should ideally be decoupled at the output channels and must fulfil the requirements determined in the time domain. In order to

Table 1: Parameters of the pendulum

Parameters (symbols)	Value
mass of the rod (m_1)	0.210 kg
length of the rod (l)	0.305 m
mass of the cart (m_2)	0,455 kg
armature resistance (R_m)	2.6 ω
motor torque constant (K_m)	0.00767 Nm
gear-ratio of gearbox (K_g)	3.7
radius of the gear (r)	0.00635 m

meet our requirements for the tracking error, apply a W_e weighting function, which reduces the steady state error below 1%: $W_e = 100\frac{s/7+1}{s/0.02+1}$. It follows from the condition that the transfer function from the reference signal to the cart position must be less than $1/W_e$ in the $\mathcal{H}_\infty$ norm sense i.e. less than $\frac{1}{100}$ in steady state.

Let the frequency weighting function of the control input be $W_u = \frac{1}{20}$. The fact that the magnitude of the reference signal is 0.2 m entails that the effect of the reference signal on the control input does not exceed 26 dB. It is assumed that the sensor noise is 5 mm in the cart position and 0.01 rad in the rod angle in the entire frequency domain, thus the weighting function of the sensor noise is represented by $W_n = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.01 \end{bmatrix}$. It is assumed that in the low frequency domain disturbances at the angle should be rejected by a factor of 5 by using $W_p = 5\frac{s/2+1}{s/0.1+1}$. The weighting functions for the performance and the tracking error are illustrated in the left hand side of Figure 4.



(a) Performance (b) Robustness

Figure 4: The weighting functions for performance and uncertainties

In the first approach, uncertainty is modelled as a complex scalar block with multiplicative uncertainty at the plant input. Let the frequency weighting function of the unmodelled dynamics be as follows: $W_r^1 = 0.5\frac{s/10+1}{s/40+1}$. It means that in the low frequency domain, the uncertainties are about 50% and, in the upper frequency domain they are up to 100%. The upper bound of the unmodelled dynamics is illustrated by the dashed line in the right hand side of Figure 4. This estimation is analyzed in both simulation and real examinations. If a smaller upper bound is applied e.g. $W_r^{12} = 0.25\frac{s/10+1}{s/40+1}$ in the control design then the robust performance cannot be guaranteed. It means that the weighting function W_r^{12} does not cover the entire model uncertainty, which comes from the parametric uncertainty and the neglected dynamics.

In the second approach, in which mixed uncertainty is applied, information about the model uncertainties between the model and the plant must be used in the control design, and the magnitude of the unmodelled dynamics is reduced. Thus the uncertainties are selected significantly smaller than in the previous case: $W_r^2 = 0.1\frac{s/8+1}{s/110+1}$. It means that in the low frequency domain the modelling error is about 10% and, in the upper frequency domain it is up to 100%.

The complex μ synthesis is performed by using the $D-K$ iteration. The compensator order is selected 18, and all the nominal performance, robust stability, and robust performance are achieved. Using a simulation procedure, the step responses of the cart position and the rod angle with the control input are shown in Figure 5. The tracking of the square reference signal meets the requirements both in the transient time domain and in steady state. The interaction between signals is also eliminated according to the specifications. In the weighting function W_r^{12} case, the robust stability requirement is not met and the oscillation of the angle is relatively high.

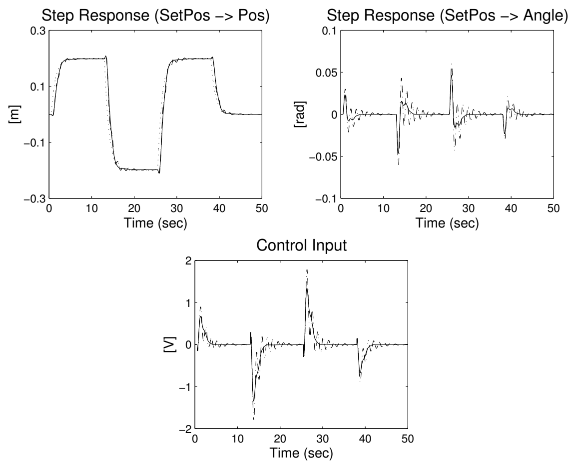


Figure 5: Step responses of the controlled system designed by complex μ synthesis

The mixed μ synthesis is performed by using the $D, G-K$ iteration. The compensator order is selected 44, and all the nominal performance, the robust stability, and the robust performance are achieved. The price of the mixed μ synthesis is usually a controller with rather large order, which can be usually reduced. The controller reduction method is based on the balanced realization and optimal Hankel norm approximation [7]. The order of the controller reduced is selected 12. The result of the robustness

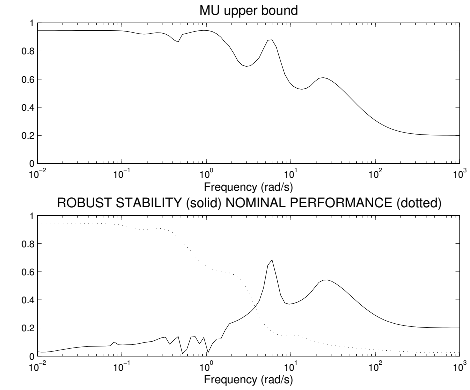


Figure 6: Performance and robustness analysis of the mixed μ controller

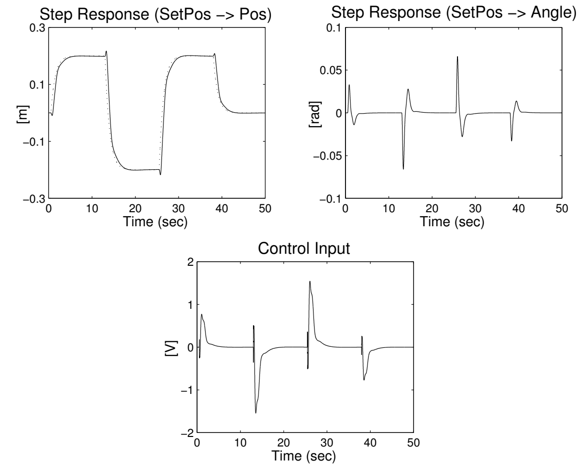


Figure 7: Step responses of the controlled system designed by mixed μ synthesis

and performance analysis is shown in Figure 6. Using a simulation procedure, the step responses of the cart position and the rod angle with the control input are shown in Figure 7.

Finally, the compensators are used for the real inverted pendulum, and they are analyzed for impulse disturbance. The impulse responses of the controlled system using complex μ controller are shown in Figure 8. The impulse responses of the mixed μ case are in Figure 9.

5 Conclusions

In this paper, the mixed μ synthesis has been presented through the application of an inverted pendulum. As a result of the control design the following conclusions can be drawn. The magnitude of the unmodelled dynamics between the model and the plant should be reduced if real parametric uncertainties can be taken into consideration. It means that information about the parametric uncertainties must be used in the control design. In the case of complex μ synthesis, in which the model uncertainties

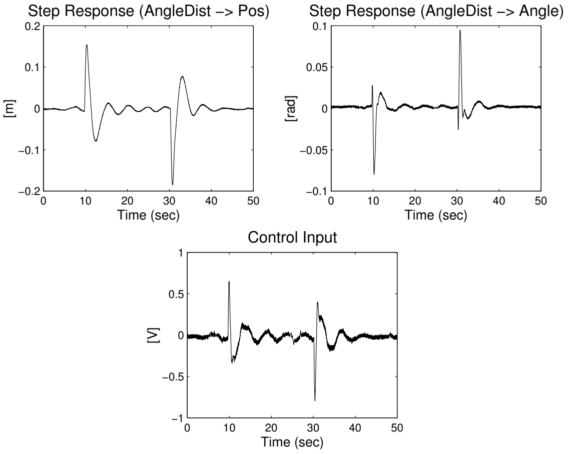


Figure 8: Measured signals using the complex μ compensator

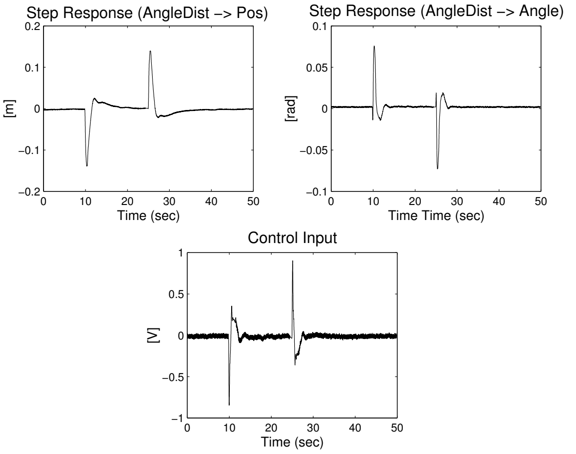


Figure 9: Measured signals using the mixed μ compensator

are handled by full or scalar complex blocks, the magnitude of the uncertainty must be assumed larger than in the mixed μ synthesis because of the worst case principle, or the designed compensator might not be robust against uncertainties. As a consequence the bandwidth of the controlled system can be increased in case of the mixed μ. The price of the mixed μ synthesis is usually a controller with a large order, however, it can be effectively reduced by using a controller reduction method.

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