PARAMETER OPTIMISATION
IN ITERATIVE LEARNING CONTROL

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Abstract

In this paper parameter optimisation through a quadratic performance index is introduced as a method to establish a new iterative learning control law. With this new algorithm, monotonic convergence of the error to zero is guaranteed if the original system is a discrete-time LTI system and it satisfies a positivity condition. A high-order version of the algorithm is also derived and its convergence analysed. The theoretical findings in this paper are highlighted with simulations.

1. Introduction

Iterative learning control (ILC) can be used to overcome some of the design difficulties associated with conventional feedback control synthesis. In more precise terms, iterative learning control is a technique for improving the transient response and tracking performance of processes, machines, equipment, or systems that execute the same trajectory, motion, or operation repetitively. For example, in the general area of trajectory following in robotics, the specified task is regarded as the tracking of a given reference $r(t)$ or output trajectory on a specified time interval $t \in [0,T]$. Feedback control cannot, by its very nature, achieve this exactly as a non-zero error is required to activate the feedback mechanism. On the other hand, there exits several ILC algorithms that can achieve zero tracking error as the number of repetitions increases, e.g. Amann and Owens (1996).

Since the introduction of iterative learning control methodology by Arimoto et al (1984), a lot of progress has been made by several researchers in understanding the different theoretical and practical aspects of ILC, e.g. Oh (1988), Amann (1996), Longman (1997) Barton (2000) and Moore (2000).

To illustrate the type of result available, Togai and Yamano's paper (1985) proposed the following simple discrete-time ILC input-updating algorithm for a system with a relative degree one

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t+1) \tag{1}$$

Where the errors $e_k(t+1)$ and the control input signal $u_k(t)$ can be recorded during k^{th} repetition and this information can be used to compute a new input signal $u_{k+1}(t)$ that will be applied to the system during the next repetition or trial $k+1$. Undated law (1) showed that, with a LTI system defined over the time interval $t \in [0,T]$ and with a state-space description (A, B, C), if $CB \neq 0$ and the induced operator norm satisfies $\|I - \Gamma CB\|_i \leq \rho < 1$, then the output signal will track perfectly the given reference signal as $k \to \infty$.

Several other researchers have done further development on the algorithm with the equation (1), such as Lee and Bien (1998). Furthermore, Y. Q Chen et al (1998, 2000) have focused on the analysis of a high-order version of equation (1), given by

$$u_{k+1}(t) = u_k(t) + \sum_{i=1}^{M} \kappa_i(t) e_{k-i+1}(t+1) \tag{2}$$

The convergence conditions for this high-order learning control law have been established. Owens et al (2000) extended a class of ILC schemes from (2), which use information from the previous input vectors, the current trial error vector, and error vectors from the earlier trials. On trial $k+1$ the control input is calculated using the update-law

$$u_{k+1}(t) = \sum_{i=1}^{N} \alpha_i u_{k+1-i}(t) + \sum_{i=1}^{M} K_i(t) e_{k-i+1}(t+1) + K_0 e_{k+1}(t+1) \tag{3}$$

The guaranteed stability and convergence of the algorithm with the equation (3) have been fully analysed by Owens et al (2000).

Although the convergence properties of all these algorithms have been thoroughly analysed, it is not always clear how to select the free parameters of the algorithms in order to achieve fast or monotonic convergence. Hence in this paper, a performance index is suggested for the first-order ILC algorithm in (1). An optimal value Γ in (1) is calculated in each repetition using this performance index. Furthermore, the approach results in monotonic convergence in the l_2-topology if the original system satisfies a positivity condition. After that the optimisation approach is extended to the high-order algorithm similar to equation (2) and it is shown again that positivity guarantees monotonic convergence. Finally simulations are used to illustrate the theoretical findings in this paper.

2. Problem statement

Consider the following general multivariable, continuous and linear discrete system, which performs a given task over a time interval [0,T] repeatedly. Note that inputs and outputs are assumed to be sampled at intervals h

$$\begin{cases} x(t+1) = \phi x(t) + \Delta u(t) \\ y(t) = Cx(t) \end{cases} \quad (x(0)\text{ specified}),\ N \geq t \geq 0 \quad (4)$$

where ϕ, Δ, C are constant real $n \times n$, $n \times l$, $m \times n$ matrices respectively. Then the ILC synthesis objective can be specified as: (a) the tracking of a signal $r(t)$; (b) the intention that this tracking accuracy increases from iteration to iteration, and (c) that perfect tracking accuracy is ultimately achieved as the number of trials/iterations becomes infinite.

Assume that a SISO system is considered in this paper, thus it is useful in the analysis to replace this model by a matrix model relating a vector (time series) of inputs to a vector (time series) of outputs for each trial. Then the linear plant (4) can be described equivalently as

$$y = Gu + d \quad (5)$$

where G and d are the matrices

$$G = \begin{bmatrix} C\Delta & 0 & \cdots & 0 & 0 \\ C\phi\Delta & C\Delta & \ddots & 0 & 0 \\ C\phi^2\Delta & C\phi\Delta & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & C\Delta & 0 \\ C\phi^{N-1}\Delta & \cdots & \cdots & C\phi\Delta & C\Delta \end{bmatrix}, d = \begin{bmatrix} C\phi x_0 \\ C\phi^2 x_0 \\ C\phi^3 x_0 \\ \vdots \\ C\phi^N x_0 \end{bmatrix} \quad (6)$$

and G consists of the Markov parameters of the plant (4). In order to take into account the repetitive nature of the problem setting, the plant model (5) is written as $y_k = Gu_k + d$ where k is the trial number. Furthermore, the tracking error at trial k is defined as $e_k = r - Gu_k - d = (r-d) - Gu_k$ and hence, without lost of generality, it is possible to replace r by $r-d$ and therefore to assume that $d = 0$ in what follows. Equivalently, it is possible to assume that x_0=0.

The ILC problem statement can easily be seen to be equivalent to finding an iterative algorithm that converges to the minimizing input u_∞ for the optimisation problem

$$\min_u \left\{ \|e\|^2 : e = r - y, y = Gu \right\} \quad (7)$$

where the inner product is $< f, g >= f^T Qg$ and the norm is defined as $\| e \|^2 = e^T Qe$ and $Q = Q^T > 0$. A natural choice of Q is $Q = blockdiag\ \{Q_j\}$ where Q_j becomes a weighting factor of $e(j)$ in the norm. Note that the scaling of the output via the transformation $y_k \mapsto Q^{1/2} y_k$ and $G \mapsto Q^{1/2} G$ enables the analysis to proceed assuming $Q = I$. The optimal error $\|r - Gu_\infty\|^2$ is a measure for how well the iterative learning control procedure has solved the inversion problem. It also represents the best that the system can do in tracking the signal $r(t)$. The case of interest here is when the optimal error is exactly zero, that is when u_∞ is a solution of $r = Gu_\infty$ and hence solves the iterative learning control problem.

As an optimization problem in ILC, Furuta and Yamakita (1987) proposed the following gradient based algorithm $u_{k+1} = u_k + \varepsilon_k A^* e_k$, where A^* is the adjoint operator of the system and ε_k is a trial dependent gain. In this paper, we use this to modify the following simple feedforward control law (similar to the given in equation (1))

$$u_{k+1}(t) = u_k(t) + \beta_{k+1} e_k(t+1) \quad (8)$$

which is chosen as a starting point for further investigation where β_{k+1} a scalar gain parameter. Note that due to its very simple structure, equation (16) is very easy to implement in practise, whereas for example the causal implementation of the algorithm in Owens and Amann et al's paper (1993, 1996, 1998, 2000), which requires solving differential as difference equations numerically between trials. The important thing to observe here is that the parameter β_{k+1} is to be varied from each trial, which is different from (1). In order to calculate the control input on the $(k+1)^{th}$ trial based on (8), at the end k^{th} trial β_{k+1} is selected to be the solution of the quadratic optimisation problem

$$\beta_{k+1} = \arg\min_{u_{k+1}} \{ J_{k+1}(\beta_{k+1}) : e_{k+1} = r - y_{k+1}, y_{k+1} = Gu_{k+1} \} \quad (9)$$

where the performance index $J(\beta_{k+1})$ is defined as

$$J(\beta_{k+1}) = \|e_{k+1}\|^2 + w\ \beta_{k+1}{}^2\ ; \quad w \geq 0 \quad (10)$$

Using $e_k = r - Gu_k$ the tracking error update relation has the from

$$e_{k+1} = (I - \beta_{k+1} G) e_k,\ \forall k \geq 0 \quad (11)$$

The stationary condition $\dfrac{dJ}{d\beta_{k+1}} = 0$ (a necessary and a sufficient condition) gives the optimal β_{k+1} as

$$\beta_{k+1}^* = \frac{1}{2}\frac{e_k^T(G^T+G)e_k}{w+\|Ge_k\|^2} = \frac{<e_k, Ge_k>}{w+\|Ge_k\|^2}; \; w \ge 0 \quad (12)$$

This equation is a starting point for further analysis on the issue of convergence and convergence rates. Note that β_{k+1} can be computed from the known signal e_k and the signal Ge_k obtained either by using e_k as an input to a plant model with zero initial condition or by performing the equivalent experiment on the real plant.

3. Properties

The ILC algorithm defined in the previous section is conceptually simple, but possesses several useful properties as can be seen as follows.

Theorem 1: *For the algorithm defined by equation (8) and (10),*

a) The performance index satisfies the interlacing/ monotonicity condition $\|e_k\|^2 \ge J(\beta_{k+1}) \ge \|e_{k+1}\|^2$ *with equality holding if, and only if,* $\beta_{k+1}=0$.

b) The parameter sequence satisfies the condition $\sum_{k=0}^{\infty}\beta_{k+1}^2 < \infty$ *and hence* $\lim_{k\to\infty}\beta_{k+1}=0$

Note: (a) states that, despite its simplicity, the algorithm is a decent algorithm as the norm of the error is monotonically non-increasing in k*, and the "energy costs" from the first to the last trial are bounded, whilst (b) indicates that the learning rate becomes slower as the algorithm progresses to convergence (see theorem 2).*

Proof: (a) From optimality and the fact that the (non-optimal) choice of $\beta_{k+1}=0$ gives $J_{k+1}(0)=\|e_k\|^2$, the following estimate holds

$$J(0)=\|e_k\|^2 \ge optimal \;\; value = \|e_{k+1}\|^2 + w\,(\beta_{k+1}^*)^2 \ge \|e_{k+1}\|^2 \quad (13)$$

It shows that the norm of the error is monotonically non-increasing in k, if and only if $\beta_{k+1}=0$. For the second of the theorem (part (b)), it is necessary to use $\|e_k\|^2 \ge J(\beta_{k+1}) = \|e_{k+1}\|^2 + w\beta_{k+1}^2$. In the form $0 \le \|e_{k+1}\|^2 \le \|e_k\|^2 - w\beta_{k+1}^2$ and apply induction to give $0 \le \|e_{k+1}\|^2 \le \|e_0\|^2 - w\sum_{j=0}^{k}\beta_{j+1}^2$ for all *k*, which proves (b).

The algorithm has a number of other useful properties. The first is that the following limit exist:

$$\lim_{k\to\infty}\|e_k\|^2 = \lim_{k\to\infty} J_k(\beta_{k+1}) := J_\infty \ge 0 \quad (14)$$

The existence of the limit suggests that the algorithm has a form of convergence property. The details are developed below.

Theorem 2 *Under the assumptions of Theorem 1 and the additional assumption that* G *is positive in the sense that*

$$G+G^T > 0 \quad (15)$$

then following iterative learning convergence condition is obtained

$$\lim_{k\to\infty}\|e_k\|=0 \quad (16)$$

Proof: Without lost of generality, replace r by $r-d$ and therefore assume that $d=0$. It is clear from $\|Ge_k\|^2 \ge 0$ and the assumption that $G+G^T>0$, that there exists a real number $\sigma^2>0$ such that $(G+G^T)\ge\sigma^2 I$ and hence that

$$\beta_{k+1}^* = \frac{<e_k, Ge_k>}{w+\|Ge_k\|^2} \ge \frac{<e_k, Ge_k>}{w} \ge \frac{\sigma^2}{w}\|e_k\|^2 \ge 0 \quad (17)$$

Theorem 1 has shown that $\lim_{k\to\infty}\beta_{k+1}=0$ and hence the estimate (17) indicates that $\lim_{k\to\infty}<e_k, Ge_k> = 0$ and consequently $\lim_{k\to\infty}\|e_k\|=0$ which proves the theorem.

In summary, by using the simple updating rule (8) and the performance index (10), the positivity condition on G ensures that

a) The iterative learning control tracking error sequence $\{e_k\}$ converges in norm to zero, i.e. the iterative learning control algorithm has guaranteed convergence of learning

b) This convergence has the important property that the error norm sequence is monotonic.

So far these two theorems have proved the convergence of the ILC algorithm (8) under the assumption that the original system is positive. It is important to analyse the importance of the requirement that $G+G^T$ is positive definite. Its importance is underlined by the following result:

Proposition 1: *If* $G+G^T$ *is not positive definite but* G *is invertible, then* $\|e_k\|$ *does not necessarily converge to zero.*

Proof: Consider the expression for the optimal value β_{k+1} in (12). If $G+G^T$ is not positive definite, there exists a non-zero vector v such that $v^TGv=0$. Let v be such that $v=r-G_e z$ (such a z exists if G is invertible) and choose the initial control input sequence via $u_0=z$. This yields $e_0=v$ and hence $\beta_1=0$. By induction, β_k =0, $k\ge1$ and hence $e_k=e_0\ne0$, $k\ge1$. The lack of convergence is obvious.

In summary, despite its simplicity, the algorithm has very strong monotonic convergence property if the original plant is positive. However, some practical limitations induced by the positivity condition could occur. Thus, if the original plant does not satisfy the positivity condition, two different procedures were established in our previous report [Owens D

H & Feng K, 2002], which can be used to modify the original system so that it can become positive.

For the choice of the weighing parameter w in performance index (10), examination of the optimal value β_{k+1}^* in (12), suggest that β_{k+1}^* reduces as w increases, causing smaller change in control and hence, intuitively reducing convergent speed. However, the choice of $w=0$ will mean that, when e_k turns small, β_{k+1}^* is obtained by dividing a small number by a small number. This could be numerically unreliable so the choice of $w>0$ but small seems the practical way. The details how to choose a proper weighting parameter was fully analysed in our previous report [Owens D H & Feng K, 2002]. As a result adaptive weights $w = w_1 + w_2\|e_k\|^2$ substituted into (12) are introduced as a method to improve the convergence properties of the algorithm.

4. High order ILC algorithm

Intuitively, the inclusion of more degrees of freedom will improve algorithm performance. The purpose of this section is to show how more degrees of freedom can be introduced in an optimisation context. In this section the following high-order ILC algorithm is introduced

$$u_{k+1}(t) = u_k(t) + \sum_{i=1}^{M} \beta_{k+1}(i) e_{k-i+1}(t+1) \tag{18}$$

where $1 \le i \le M < k$, $e_i(t) = r(t) - y_i(t)$ is the tracking error; $\beta_{k+1}(i)$ are gains, M is the order of the ILC updating law and e_{k+1-i} are the errors from earlier trials. The parameter vector ($\beta_{k+1}(i)$) is selected as the solution of the following optimisation problem

$$J(\beta_{k+1}(1), \beta_{k+1}(2),\ldots, \beta_{k+1}(M)) = \|e_{k+1}\|^2 + \sum_{i=1}^{M} w_i \beta_{k+1}(i)^2 \tag{19}$$

where $w_i>0$ are weighting parameters of the performance index (19). Using $e_k = r - Gu_k$ and the updating law (18), the error evolution equation becomes

$$e_{k+1} = e_k - \sum_{i=1}^{M} \beta_{k+1}(i) G e_{k-i+1}, \forall k \ge 0, 1 < M < k \tag{20}$$

The stationary condition (necessary and sufficient) $\frac{\partial J}{\partial \beta_{k+1}(i)} = 0,$ $1 \le i \le M$, gives, after some calculation.

$$\begin{bmatrix} \|Ge_k\|^2 + w_1 & <Ge_k, Ge_{k-1}> & \cdots & <Ge_k, Ge_{k+1-M}> \\ <Ge_{k-1}, Ge_k> & \|Ge_{k-1}\|^2 + w_2 & \cdots & <Ge_{k-1}, Ge_{k+1-M}> \\ \vdots & \ddots & \ddots & \vdots \\ <Ge_{k+1-M}, Ge_k> & <Ge_{k+1-M}, Ge_{k-1}> & \cdots & \|Ge_{k+1-M}\|^2 + w_M \end{bmatrix} \begin{bmatrix} \beta_{k+1}^*(1) \\ \beta_{k+1}^*(2) \\ \vdots \\ \beta_{k+1}^*(M) \end{bmatrix} = \begin{bmatrix} <e_k, Ge_k> \\ <e_k, Ge_{k-1}> \\ \vdots \\ <e_k, Ge_{k+1-M}> \end{bmatrix} \tag{21}$$

or in more compact matrix form $D_k \beta_{k+1}^* = F_k$, where D_k and F_k are defined in an obvious way from (21). The matrix D_k can be rewritten as

$$D_k = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & w_M \end{bmatrix} + \begin{bmatrix} Ge_k \\ Ge_{k-1} \\ \vdots \\ Ge_{k+1-M} \end{bmatrix} \begin{bmatrix} Ge_k & Ge_{k-1} & \cdots & Ge_{k+1-M} \end{bmatrix} \tag{22}$$

It is clear that if either (i) $w_i > 0$, $i = 1, \cdots M$ or (ii) $Ge_k, Ge_{k-1}, \cdots, Ge_{k+1-M}$ are linearly independent then D_k^{-1} exists and β_{k+1}^* can be solved as $\beta_{k+1}^* = D_k^{-1} F_k$. Note that because of the assumption of SISO system, $<Ge_i, Ge_j>$ in (21) is equivalence to a scalar value. Thus the dimension of matrix D_k is only $M \times M$, and consequently D_k^{-1} is easy to be found.

Theorem 3: *For the high order algorithm defined by equation (18) and (19),*

a) The performance index satisfies the interlacing/ monotonicity condition

$$\|e_k\|^2 \ge J(\beta_{k+1}(1), \beta_{k+1}(2) \cdots \beta_{k+1}(M)) \ge \|e_{k+1}\|^2 \tag{23}$$

with equality holding if, and only if, $\beta_{k+1}(1) = \beta_{k+1}(2) = \cdots = \beta_{k+1}(M) = 0$.

b) The parameter sequence satisfies the condition

$$\sum_{j=0}^{\infty} \sum_{i=1}^{M} w_i \beta_{j+1}^2(i) < \infty \tag{24}$$

which immediately implies that

$$c) \lim_{k\to\infty} \beta_{k+1}(1) = \lim_{k\to\infty} \beta_{k+1}(2) = \cdots = \lim_{k\to\infty} \beta_{k+1}(M) = 0 \tag{25}$$

Note: (a) states that, the algorithm is a decent algorithm as the norm of the error is monotonically non-increasing in k, and the "energy costs" from the first to the last trial are bounded, whilst (b) and (c) indicates that the learning rate becomes slower as the algorithm progresses to convergence (see theorem 4).

Proof: The proof is similar to Theorem 1. (a) From optimality and the fact that the (non-optimal) choice of $\beta_{k+1}(1) = \beta_{k+1}(2) = \cdots = \beta_{k+1}(M) = 0$, $1 \le M \le k$ gives $J_{k+1}(0,0,\cdots 0) = \|e_k\|^2$, results in the following simple interlacing result

$$\begin{aligned} J(0,0\cdots,0) = \|e_k\|^2 &\ge optimal \quad value \\ &= \|e_{k+1}\|^2 + \sum_{i=1}^{M} w_i (\beta_{k+1}^*(i))^2 \ge \|e_{k+1}\|^2 \end{aligned} \tag{26}$$

This result shows that the norm of the error is monotonically non-increasing in k with equality, if and only if $\beta_{k+1}(1) = \beta_{k+1}(2) = \cdots = \beta_{k+1}(M) = 0$, $1 \le M \le k$. For the theorem (b) and (c) it is necessary to use the following estimate

$$\|e_k\|^2 \geq J(\beta_{k+1}(1), \beta_{k+1}(2), \cdots, \beta_{k+1}(M)) = \|e_{k+1}\|^2 + \sum_{i=1}^{M} w_i \beta_{k+1}^2(i) \quad (27)$$

where $1 \leq M \leq k$, in the form $0 \leq \|e_{k+1}\|^2 \leq \|e_k\|^2 - \sum_{i=1}^{M} w_i \beta_{k+1}^2(i)$ and apply induction to give $0 \leq \|e_{k+1}\|^2 \leq \|e_0\|^2 - \sum_{j=0}^{\infty}\sum_{i=1}^{M} w_i \beta_{j+1}^2(i)$ for all k which proves (b) and (c).

Convergence is guaranteed by the following theorem:

Theorem 4 *Under the assumptions of Theorem 3 and the additional assumption that G is positive in the sense that* $G + G^T > 0$ *, the optimal high order updating law (18) converges in the limit* $\lim_{k\to\infty}\|e_k\| = 0$.

Proof: From (21) and theorem 3, it follows that $\lim_{k\to\infty} < e_k, Ge_k >= 0$ and the result follows in a similar manner to theorem 2.

In summary, by using the high-order updating rule (18) and the performance index (19), the positivity condition on G ensures that iterative learning control algorithm has guaranteed convergence of learning and the error norm sequence is monotonic and decreasing.

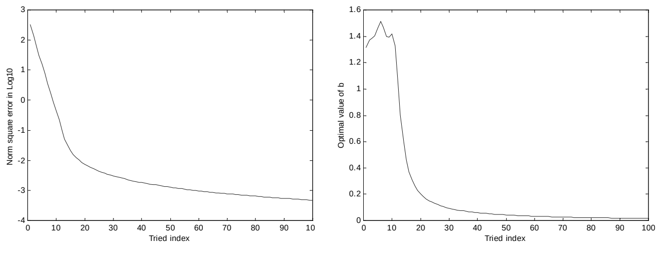
The theorems in this section have shown that if the original system (4) is positive the high-order ILC algorithm (18) converges to zero.

5. Numerical Example

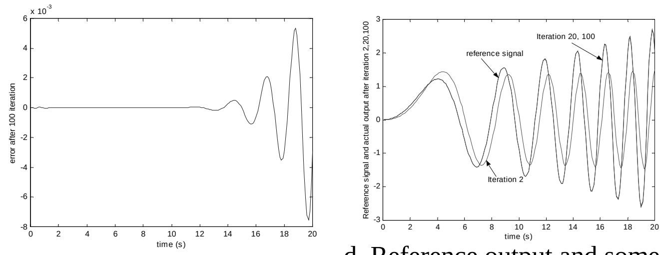
To demonstrate the effectiveness of the new parameter optimisation based ILC algorithm (8), consider a plant having the following transfer function

$$G(s) = \frac{2s+3}{s^2+4s+3} \quad (28)$$

Using sampling time $h = 0.1$ and the reference signal chosen is $r(t) = e^{t/20}\sin(\omega t/10)$ where $t \in [0,20]$. With these particular choices of sampling time and trial length, MATLAB indicates that the eigenvalues of $G + G^T$ lie between 0.1989 and 1.9786 and hence G is a positive matrix. The selection $w = 10^{-2}$ in (10) is chosen to provide numerical solutions of the evaluation of β_{k+1} at very small error values.



a. Norm square error in log10 b. Value of parameter β



c. Error after 100 iteration d. Reference output and some of actual output

Figure 1 Simulation results of parameter Norm-optimisation Iterative Learning Control.

The results confirm the theoretical prediction that norm of the error $\|e_k\|$ and β_{k+1} converge to zero as $k \to \infty$ and that the convergence of the error norm is monotonic. This is due to Theorem 1 and Theorem 2, which state that the positivity of the plant is a sufficient condition for monotonic convergence to zero.

Consider now the high-order algorithm in (18) where the order of the algorithm is selected to be $M = 2$. In order to make a comparison it with the first order algorithm possible, the weighting parameters of the algorithm are selected as $w_1 = w_2 = \frac{w}{2} = 0.5\times10^{-2}$, i.e. $w = w_1 + w_2 = 10^{-2}$. The plant model and the reference signal are the same as in the previous examples. Figure 2 shows that with this high-order algorithm the convergence increases relative to $M = 1$.

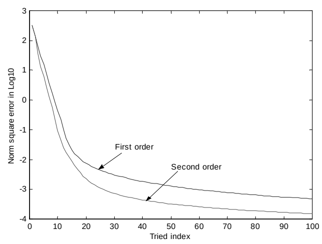


Figure 2 Simulation results of POILC – comparison with the first order and high order case.

Consider now the plant $G(s) = \frac{1}{(s+1)^2}$, and the reference signal $r(t) = e^{t/20}\sin(\omega t/10)$ where $t \in [0,20]$ and let the sampling interval h to be $h = 0.1$. In this case the eigenvalues of $G + G^T$ vary from -0.2086 to 1.8934, which shows that $G + G^T$ is not positive definite. Using the first-order algorithm in (30) with the same weighting parameter $w = 10^{-2}$ gives the behaviour shown in Figure 3, where the error converge to a non-zero limit, i.e. it fails to converge to zero!

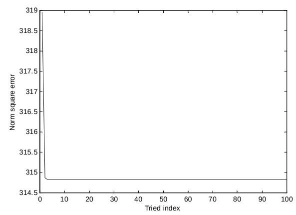


Figure 3 Simulation results of parameter Norm-optimisation Iterative Learning with non-positive system

6. Discussion and Conclusion

In this paper, parameter optimisation based iterative learning control (POILC) was introduced as a new method to solve the ILC problem when the original plant is a discrete-time LTI system. The resulting algorithm is of feed-forward type and it has guaranteed monotonic convergence to zero if the original system satisfies a positivity condition. Because of its computational simplicity, this new ILC algorithm is potentially straightforward to implement in real-time applications.

Based on the intuition that improved ILC should be obtainable if more design parameters are used, a high-order version of the algorithm was derived. This more complex algorithm uses the error data over the past M iterations as the basis of control updating and might be expected to improved algorithm performance markedly. A convergence theory was given for this algorithm with the surprising conclusion that positivity is again a sufficient condition on the plant for monotonic convergence of the error to zero. Simulation examples confirm the tendency for a high-order algorithm to give faster convergence than the first-order algorithm but it is still unclear how it works theoretically. This suggests that the use of higher order algorithms does have benefits in a parameter optimisation framework but that additional work is needed to improve on the results of this paper. Progress will be reported separately.

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