ACTIVE QUEUE MANAGEMENT FOR TCP-GOVERNED WIRELESS NETWORKS

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Abstract

A theoretical framework is proposed to analyze the performance of active queue management (AQM) protocols in TCP-governed wireless networks. It is assumed that the TCP-governed sources operate in the linear-increase-multiplicative-decrease mode. A bottleneck network topology involving a single router and multiple users is examined and it is shown that the problem of synthesizing stabilizing AQM controllers is equivalent to that of solving a class of bilinear matrix inequality (BMI) problems. A series of such BMI formulations for a class of AQM controllers is presented and it is observed that some of these formulations can be recast as linear matrix inequality (LMI) formulations, which can be solved efficiently. ■

Keywords: TCP, active queue management, wireless networks, LMI, and BMI ■

1 Introduction

1.1 Motivation

State-of-the-art Internet is a decentralized control system employing dynamic transmission control protocols at the sources and dynamic queue management protocols at the routers. The predominant transmission control protocol suites are variants of TCP [13] whereas the predominant queue management protocol is RED [8]. TCP is known to yield poor performance for streaming and real-time applications [3]. In addition, it scales poorly with the network bandwidth-delay product [18]. However, since it governs over 70% of the backbone data traffic, it is not possible to supplant TCP with an entirely different transmission protocol. A more realistic approach is to add a computationally inexpensive dynamic queue management functionality to the routers so that the performance is improved at the cost of relatively few changes to TCP. This functionality is referred to as *active queue management* (AQM); several examples of the AQM controllers are described in [8, 11, 14, 17], and [18].

Currently, the AQM controller design mostly concerns generating a packet dropping policy since, as the congestion mitigation action, TCP gives a drastic multiplicative decrease response to the event of multiple packet drops. The prevalent AQM controllers use a measure of the queue occupancy as the decision variable and generate a packet dropping policy as the output. So far, these controllers have addressed the need of wired infrastructure support only. Motivation of this paper is to explore whether the wireless setting offers any rich trade-offs which could be exploited in the AQM controller design.

1.2 Proposed Approach

We focus on a bottleneck topology in which a router on the path of multiple users and derive an analytical fluid flow model, defined by a set of ordinary differential equations, for the case in which some of the links are wireless erasure channels. It has been well observed that "... *in practice, increase in the controller complexity unnecessarily outpaces increase in the plant complexity, and the objective should be to minimize control system complexity subject to the achievement of accuracy specification in the face of uncertainty*" [25, Ch. 10]. In sympathy, this paper confines its attention to the class of AQM controllers having a *proportional-integral-derivative* (PID) structure. We show that the stability analysis problem can be formulated as a *bilinear matrix inequality* (BMI) problem, which in some cases gets reduced to solving a *linear matrix inequality* (LMI) problem, solution to which carries a lower computational complexity. This paper is primarily intended to introduce a framework and the underlying powerful robust control theoretic results can be used to incorporate more sophisticated controller structures.

1.3 Organization of the Paper

The paper is organized as follows. Notation is introduced and tabulated as and when necessary. Some standard stability concepts are noted down in Section 2.2. A summary of the prevalent AQM protocols is presented in Section 2.3. A linearized model of a bottleneck topology is derived in Section 3.1 and the stability analysis problem is formulated in Section 3.2. A list of BMI formulations is presented in Section 4 as a solution to this problem. Conservativeness of this approach and the computational complexity issues are commented upon in Section 5. The paper is concluded in Section 6.

2 Preliminaries

2.1 Notation

Capital letter symbols, such as F and G, denote operators whereas small letter symbols, such as x and y, denote real signals which may possibly be vector valued or matrix valued. The notation $\doteq$ stands for 'defined as'. The inner product

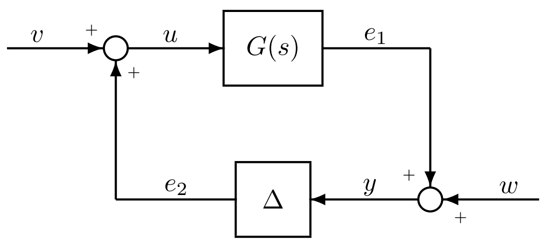

Figure 1: Block diagram representation of the system $\mathcal{S}$. The subsystem $G(s)$ is linear time invariant whereas the subsystem Δ is otherwise.

$\langle x, y \rangle \doteq \int_{-\infty}^{\infty} y(t)^T x(t)\, dt$. The norm $\|x\| \doteq \langle x, x \rangle$. The vector space of signals for which the norm exists is denoted $\mathcal{L}_2^n$. The vector space $\mathcal{L}_2^n$ is generally referred to as $\mathcal{L}_2$. Fourier transform of x is denoted $\widehat{x}(\cdot)$. The set of all integers is denoted $\mathbb{Z}$. Conjugate transpose of a vector or matrix $(\cdot)$ is denoted $(\cdot)^*$; its transpose is denoted $(\cdot)^T$. A matrix $D \in \mathbb{R}^{n \times n}$ is said to be *Hurwitz* if each of its eigenvalues has a strictly negative real part. A block diagonal matrix $D \in \mathbb{R}^{n \times n}$ having entries D_{ii} on its diagonal is denoted $\text{diag}(D_{11}, D_{22}, \ldots, D_{nn})$. An identity matrix is denoted I. Other terms not defined here may be found in [20] and [24].

2.2 Stability and IQCs

In stability analysis, a given system $\mathcal{S}$ is often decomposed into two interconnected subsystems — a linear time invariant subsystem G in the feedforward path and an otherwise subsystem Δ in the feedback path (see Fig. 1).

Definition 1 [Stability of a System]
The system $\mathcal{S}$ is said to be stable if there exists a positive constant C such that

$$\int_0^T \left(|e_1|^2 + |e_2|^2\right) dt \leq C \int_0^T \left(|v|^2 + |w|^2\right) dt$$

and if, in addition, the map $(e_1, e_2) \to (v, w)$ has a causal inverse on $\mathcal{L}_2$. □

Definition 2 [IQC]
The pair of signals w and v in the space $\mathcal{L}_2$ is said to satisfy the integral quadratic condition (IQC) defined by *any measurable Hermitian valued function Π if*

$$\int_{-\infty}^{\infty} \begin{bmatrix} \widehat{v}(j\omega) \\ \widehat{w}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \widehat{v}(j\omega) \\ \widehat{w}(j\omega) \end{bmatrix} d\omega \geq 0. \quad (1)$$

A bounded operator $G : \mathcal{L}_2 \to \mathcal{L}_2$ is said to satisfy the IQC defined by Π *if (1) holds for all pairs of signals w and v in $\mathcal{L}_2$ with $w \doteq G(v)$.* □

The following well known theorem characterizes stability of the system $\mathcal{S}$ in terms of IQC's.

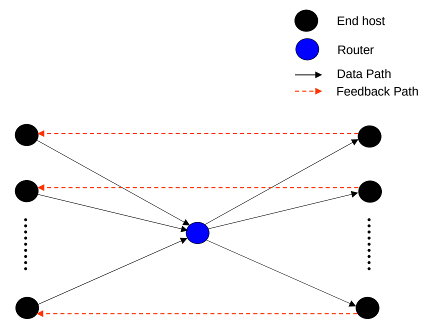

Figure 2: The bottleneck network topology shows a single router on the data paths from multiple sources to multiple destinations. The feedback path is drawn in dotted lines to indicate that the feedback path may not necessarily be the same as the feedforward data path.

Theorem 1 [IQC Stability Theorem, [20]]
Given $G(s)$ with no poles in the closed right half s-plane and a bounded causal operator Δ. Suppose the following conditions hold:

1. the system $\mathcal{S}$, with Δ replaced by $\tau\Delta$, is well posed for all $\tau \in [0, 1]$;
2. the operator $\tau\Delta$ the satisfies IQC defined by Π for all $\tau \in [0, 1]$;
3. it holds that

$$\int_{-\infty}^{\infty} \begin{bmatrix} \widehat{G}(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \widehat{G}(j\omega) \\ I \end{bmatrix} d\omega < 0.$$

Then the system $\mathcal{S}$ is stable. □

A comprehensive introduction to robust stability and IQC's is given in [12] and [20].

2.3 Prevalent AQM Mechanisms

The simplest AQM controller is the bang-bang controller under which a router chooses to drop an incoming data packet if its buffer is full and chooses to serve it otherwise; this primitive controller monopolized the market until the first improvements were put forth by [8]. It is well known that bang-bang controllers lead to large oscillations [1]; indeed, chaotic behavior has been reported in [26]. To alleviate this problem, [8] proposes RED which a preemptive randomized packet drop protocol. This protocol monitors the averaged queue-length and marks packets with a probability if it exceeds a threshold; the marking probability is a static and monotonically increasing function of the average queue-length.

Tuning of the thresholds and the slope of increase parameters remained a heuristic exercise, see e.g. [5] and [6], until a rigorous control theoretic interpretation of RED was established by [11]; the main results therein are presented more informally in [10]. It shows that RED amounts to using a *proportional* (P) controller along with a low pass filter. A fuller analysis of RED is given by [18] which effectively shows that the stability margin of a TCP/RED governed system reduces as the delay-bandwidth product increases; a more specific characterization is given in [14]. Removal of the low pass filter from the RED design yields the classical P controller which has a higher closed loop bandwidth at the cost of reduced robustness (see [7] and [11]). A standard improvement over a P controller is a *proportional-integral*, i.e. PI, controller [2] and [11] has proposed a PI controller. A *tracking* controller has been proposed by [17]. Necessity of the derivative action in an AQM controller was first identified by [16]. Other important works include [9] and [19].

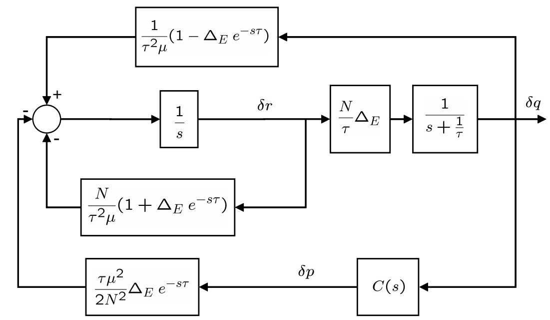

Figure 3: A linearized TCP model for the bottleneck topology shown in Fig. 2. Note the dependency on the round trip time τ. The AQM controller is represented by $C(s)$. In the wired setting, Δ_E is the identity operator.

3 Model Description and Problem Formulation

3.1 Model Description

We consider the bottleneck network topology described in Fig. 2; the mathematical notation is described in the Table 1. The end hosts are assumed to operate in a linear increase multiplicative decrease mode. A link from a source to the router can be either wired or wireless. Linearizing on the same lines as [11], the linearized model on the operating point (r_0, q_0, p_0) is described by the following equations.

$$\begin{aligned}\dot{\delta r}(t) &= -\frac{N}{\tau^2\mu}\left(\delta r(t) + \delta\widetilde{r}(t-\tau)\right) \\ &\quad - \frac{1}{\tau^2\mu}\left(\delta q(t) - \delta\widetilde{q}(t-\tau)\right) \\ &\quad - \frac{\tau\mu^2}{2N^2}\delta\widetilde{p}(t-\tau) \qquad (2) \\ \dot{\delta q}(t) &= \frac{N}{\tau}\delta\widetilde{r}(t) - \frac{1}{\tau}\delta q(t) \qquad (3)\end{aligned}$$

$$\delta r \doteq r - r_0, \quad \delta q \doteq q - q_0, \quad \delta p \doteq p - p_0, \quad \widetilde{(\cdot)} \doteq \Delta_E(\cdot),$$

where Δ_E is the erasure channel operator with $\|\Delta_E\| \le 1$. The block diagram representation is shown in Figure 3.

3.2 Problem Formulation

Problem 1 Given the round trip time and the controller structure, derive the analytical conditions which must be satisfied by a stabilizing AQM controller. □

We restrict the controller to be of PID form. The problem formulation assumes that the round-trip time τ is known at the router. The assumption is realistic because the round trip time information is available in the packet headers.

4 Main Result

4.1 Proportional Control Synthesis

When a P controller is used as an AQM controller, $C(s) = k_p$ for some $k_p \in \mathbb{R}$. The controller synthesis problem is to find a k_p that stabilizes the system described by (2) and (3). Now, the controller output, viz. δp, can be seen to be

$$\delta p(t) = k_p \delta q(t) = \begin{bmatrix} 0 & k_p \end{bmatrix} \begin{bmatrix} \delta r(t) \\ \delta q(t) \end{bmatrix}.$$

Define the state of the system $x \doteq \begin{bmatrix} \delta r & \delta q \end{bmatrix}^T$. Then, (2) and (3) can be rewritten as

$$\dot{x}(t) = A_0 x(t) + (A_d + BK)\, x(t-\tau) \qquad (4)$$

where

$$A_0 \doteq \begin{bmatrix} -\frac{N}{\tau^2\mu} & -\frac{1}{\tau^2\mu} \\ \frac{N}{\tau} & -\frac{1}{\tau} \end{bmatrix}, \quad A_d \doteq \begin{bmatrix} -\frac{N}{\tau^2\mu} & \frac{1}{\tau^2\mu} \\ 0 & 0 \end{bmatrix},$$
$$B \doteq \begin{bmatrix} -\frac{\tau\mu^2}{2N^2} & 0 \end{bmatrix}^T, \quad K \doteq \begin{bmatrix} 0 & k_p \end{bmatrix}$$

It is well known that the time-delay system (4) is stable if

$$\begin{bmatrix} A_0^T P + P A_0 + P_1 & P(A_d + BK) \\ (A_d + BK)^T P & -P_1 \end{bmatrix} < 0,$$

$P > 0$ and $P_1 > 0$ [4]. This is a BMI with respect to the variables P, P_1, and K. A wide range of controller synthesis problems are formulated as BMI problems [23]. In general, the BMI problems are NP-hard and several heuristic methods, such as the DK-iteration, have been proposed to solve them. This BMI, however, can be converted into an LMI. Multiply every block entry of Eq (5) on the left and on the right by P^{-1} and set $Q = P^{-1}$, $Q_1 = P^{-1}P_1P^{-1}$ and $Y = KP^{-1}$, then we obtain the condition

$$\begin{bmatrix} A_0 Q + Q A_0^T + Q_1 & A_d Q + BY \\ Q A_d^T + Y^T B^T & -Q_1 \end{bmatrix} < 0,$$

$Q > 0$ and $Q_1 > 0$. Furthermore, the matrix Q should be diagonal and the matrix Y should have the form $Y = \begin{bmatrix} 0 & y \end{bmatrix}$ due to the structure of the matrix K. This is an LMI with respect to the variables Q, Q_1 and Y. A proportional controller k_p can be readily obtained by solving the above LMI.

Table 1: Table of Notation for the Network Topology

Symbol	Meaning
r	average transmission rate over an interval
q	average queuelength over an interval
τ	nominal round-trip time
μ	link capacity
τ_p	propogation delay
N	number of active connections
p	probability of a packet mark
Δ_E	erasure channel operator

4.2 PID Controller Synthesis

When a PID controller is used as a AQM controller, $C(s) = k_p + k_i/s + k_d\ s$ with $k_p, k_i, k_d \in \mathbb{R}$. The controller synthesis problem is to find values of parameters k_p, k_i and k_d that stabilize the the system given by (2) and (3). Now, the controller output, viz. δp can be seen to be

$$\delta p(t) = k_p\ \delta q(t) + k_i \int_0^t \delta q(\xi) d\xi + k_d\ \dot{\delta q}(t).$$

Define a new state $x_3(t)$ to be $\dot{x_3}(t) = \delta q(t)$. Then, $\delta p(t)$ can be expressed as

$$\begin{aligned}\delta p(t) &= k_p\ \delta q(t) + k_i\ x_3(t) + k_d\ \dot{\delta q}(t)\\ &= k_p\ \delta q(t) + k_d\ \left(\frac{N}{\tau}\delta r(t) - \frac{1}{\tau}\delta q(t)\right) + k_i\ x_3(t)\\ &= \begin{bmatrix} k_d & k_p & k_i\end{bmatrix}\begin{bmatrix}\frac{N}{\tau} & -\frac{1}{\tau} & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\delta r(t)\\ \delta q(t)\\ x_3(t)\end{bmatrix}.\end{aligned}$$

if we assume zero initial condition, that is, $x_3(0) = 0$. If we define $x(t) = \begin{bmatrix}\delta r(t) & \delta q(t) & x_3(t)\end{bmatrix}^T$, the system (2) and (3) can be rewritten as

$$\dot{x}(t) = A_0 x(t) + (A_d + BKC)x(t-\tau) \tag{5}$$

where

$$A_0 = \begin{bmatrix}-\frac{N}{\tau^2\mu} & -\frac{1}{\tau^2\mu} & 0\\ \frac{N}{\tau} & -\frac{1}{\tau} & 0\\ 0 & 1 & 0\end{bmatrix},\quad A_d = \begin{bmatrix}-\frac{N}{\tau^2\mu} & \frac{1}{\tau^2\mu} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$B = \begin{bmatrix}-\frac{\tau\mu^2}{2N^2} & 0 & 0\end{bmatrix}^T,\quad K = \begin{bmatrix}k_d & k_p & k_i\end{bmatrix},$$

$$C = \begin{bmatrix}\frac{N}{\tau} & -\frac{1}{\tau} & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

The system (5) is stable if

$$\begin{bmatrix}A_0^T P + P A_0 + P_1 & P(A_d + BK)\\ (A_d + BK)^T P & -P_1\end{bmatrix} < 0,$$

$P > 0$ and $P_1 > 0$. This is a BMI with respect to the variables P, P_1, and K. With the same procedure as in the previous section, we get the condition

$$\begin{bmatrix}A_0 Q + Q A_0^T + Q_1 & A_d Q + BY\\ Q A_d^T + Y^T B^T & -Q_1\end{bmatrix} < 0,$$

$Q > 0$ and $Q_1 > 0$ where $Q = P^{-1}$, $Q_1 = P^{-1}P_1P^{-1}$ and $Y = KCP^{-1}$.

4.3 PID Controller Synthesis for Erasure Channels

When a PID controller is used as a AQM controller, $C(s) = k_p + k_i/s + k_d\ s$ and controller synthesis problem is to find values of parameters k_p, k_i and k_d which make the system (2) and (3) stable. The control input $\delta p(t)$ for PID AQM controller can be said

$$\delta p(t) = k_p\ \delta q(t) + k_i \int_0^t \delta q(\xi) d\xi + k_d\ \dot{\delta q}(t).$$

Define a new state $x_3(t)$ to be $\dot{x_3}(t) = \delta q(t)$. Then, $\delta p(t)$ can be expressed as

$$\begin{aligned}\delta p(t) &= k_p\ \delta q(t) + k_i\ x_3(t) + k_d\ \dot{\delta q}(t)\\ &= k_p\ \delta q(t) + k_d\ \left(\frac{N}{\tau}\Delta_E\ \delta r(t) - \frac{1}{\tau}\delta q(t)\right) + k_i\ x_3(t)\\ &= \begin{bmatrix}\frac{k_d N}{\tau}\Delta_E & k_p - \frac{k_d}{\tau} & k_i\end{bmatrix}\begin{bmatrix}\delta r(t)\\ \delta q(t)\\ x_3(t)\end{bmatrix}\\ &= \begin{bmatrix}k_d & k_p & k_i\end{bmatrix}\begin{bmatrix}\frac{N}{\tau}\Delta_E & -\frac{1}{\tau} & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\delta r(t)\\ \delta q(t)\\ x_3(t)\end{bmatrix}.\end{aligned}$$

if we assume zero initial condition, that is, $x_3(0) = 0$. If we define $x(t) = \begin{bmatrix}\delta r(t) & \delta q(t) & x_3(t)\end{bmatrix}^T$, the system (2) and (3) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= (A_0 + H_0\Delta_E E_0)x(t) +\\ &\quad \Delta_E\left(A_d + BK(C + H_1\Delta_E E_1)\right)x(t-\tau)\end{aligned} \tag{6}$$

where

$$A_0 = \begin{bmatrix}-\frac{N}{\tau^2\mu} & -\frac{1}{\tau^2\mu} & 0\\ 0 & -\frac{1}{\tau} & 0\\ 0 & 1 & 0\end{bmatrix},\quad A_d = \begin{bmatrix}-\frac{N}{\tau^2\mu} & \frac{1}{\tau^2\mu} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$H_0 = \begin{bmatrix}0\\1\\0\end{bmatrix},\quad E_0 = \begin{bmatrix}\frac{N}{\tau} & 0 & 0\end{bmatrix},\quad B = \begin{bmatrix}-\frac{\tau\mu^2}{2N^2}\\ 0\\ 0\end{bmatrix},$$

$$K = \begin{bmatrix}k_d & k_p & k_i\end{bmatrix},\quad C = \begin{bmatrix}\frac{N}{\tau}\Delta_E & -\frac{1}{\tau} & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},$$

$$H_1 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T,\quad E_1 = \begin{bmatrix}\frac{N}{\tau} & 0 & 0\end{bmatrix}.$$

In PID case, we have additional uncertainty block Δ_E^2. By defining $\Delta_4 \doteq \Delta_E^2$, $\Delta(s) \doteq \mathrm{diag}((e^{-s\tau}I,\ \Delta_E\ I,\ \Delta_E)$ satisfies the IQC defined by

$$\Pi = \left[\begin{array}{c|c}\Pi_{11} & \Pi_{12}\\ \hline \Pi_{21} & \Pi_{22}\end{array}\right], \tag{7}$$

$\Pi_{11} \doteq \mathrm{diag}(Q, X, x_1, x_2)$, $\Pi_{12} \doteq \mathrm{diag}(0, Y, 0, 0)$, $\Pi_{21} \doteq \Pi_{12}^T$, $\Pi_{22} \doteq -\Pi_{11}$. Using some algebraic operations, it can be verified that the system is stable if

$$\begin{bmatrix}A_g & B_g\\ I & 0\end{bmatrix}^*\begin{bmatrix}0 & P\\ P & 0\end{bmatrix}\begin{bmatrix}A_g & B_g\\ I & 0\end{bmatrix} + \begin{bmatrix}C_g & D_g\\ 0 & I\end{bmatrix}^*\Pi\begin{bmatrix}C_g & D_g\\ 0 & I\end{bmatrix}$$

is strictly negative, where

$$A_g \doteq A_0, \quad B_g \doteq \begin{bmatrix} 0 & I & H_0 & BKH_1 \end{bmatrix},$$

$$C_g \doteq \begin{bmatrix} I \\ 0 \\ E_0 \\ 0 \end{bmatrix}, \quad D_g \doteq \begin{bmatrix} 0 & 0 & 0 & 0 \\ A_d + BKC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ E_1 & 0 & 0 & 0 \end{bmatrix}.$$

This is a BMI with respect to P, Q, X, Y, x_1, x_2 and K.

5 Discussion

It may be observed that the delay-bandwidth product term and the round trip time term enter the system matrices inversely. This fact can be cleverly used in deducing stability margins. A strength of IQC based approach is that it gives an elegant decomposition of the robustness analysis problems for complex systems, as is the case with the networks problems — the non-LTI subsystems can be *pulled out* and their IQCs can be combined in a relatively straightforward matter to form a composite IQC for the overall system. For example, consider the case where the overall uncertainty Δ comprises time-delay and saturation nonlinearity. Then, $\Delta = \operatorname{diag}(\Delta_1, \Delta_2)$ where $\Delta_1(s) = e^{-s\tau}$ and $\Delta_2(s) = \text{sat}(\cdot)$ and that each subsystem, Δ_i, satisfies the IQC defined by

$$\Pi_i = \begin{bmatrix} \Pi_{i(11)} & \Pi_{i(12)} \\ \Pi^*_{i(12)} & \Pi_{i(22)} \end{bmatrix},$$

where the block structures are consistent with the size of the subsystem Δ_i. Then the overall system Δ satisfies the IQC defined by

$$\Pi = \left[\begin{array}{cc|cc} \Pi_{1(11)} & 0 & \Pi_{1(12)} & 0 \\ 0 & \Pi_{2(11)} & 0 & \Pi_{2(12)} \\ \hline \Pi^*_{1(12)} & 0 & \Pi_{1(22)} & 0 \\ 0 & \Pi^*_{2(12)} & 0 & \Pi_{2(22)} \end{array}\right].$$

Now let us transform this frequency dependent condition to a non-frequency dependent condition. If we define state space realizations of $H(s)$ and $G(s)$ to be

$$H(s) \overset{s}{=} \left[\begin{array}{c|c} A_h & B_h \\ \hline C_h & D_h \end{array}\right]$$

$$G(s) \overset{s}{=} \left[\begin{array}{c|c} A_g & B_g \\ \hline C_g & 0 \end{array}\right] = \left[\begin{array}{c|c} A_0 + A_d + BK & A_d + BK \\ \hline I & 0 \end{array}\right],$$

then we have a state space realization of the serial connection of $H(s)$ and $G(s)$ expressed as

$$H(s)G(s) \overset{s}{=} \left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & 0 \end{array}\right]$$

where

$$\bar{A} \doteq \begin{bmatrix} A_g & 0 \\ B_h C_g & A_h \end{bmatrix} = \begin{bmatrix} A_0 + A_d & 0 \\ B_h & A_h \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} K & 0 \end{bmatrix},$$

$$\bar{B} \doteq \begin{bmatrix} B_g \\ 0 \end{bmatrix} = \begin{bmatrix} A_d \\ 0 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} K,$$

$$\bar{C} \doteq \begin{bmatrix} D_h C_g & C_h \end{bmatrix} = \begin{bmatrix} D_h & C_h \end{bmatrix}.$$

With this state space realization, the frequency dependent LMI can be transformed to a frequency independent LMI

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \bar{C}^T Q \bar{C} & P\bar{B} \\ \bar{B}^T P & -Q \end{bmatrix} < 0$$

by Kalman-Yakubovich-Popov lemma and setting $\widehat{g}(j\omega) = Q > 0$. This is a BMI with respect to P, Q and K.

Remark 1 In steady-state, a TCP governed source probes the available network bandwidth via a linear-increase-multiple-decrease (LIMD) protocol: in the linear increase mode, the transmission rate increases linearly with time whereas in the multiple decrease mode, it decreases by a proportionate amount in response to a congestion notification event, such as a series of packet drops [15]. The model used in this paper aims to represent the system behavior near the steady state regime and has completely ignored the slow start and time out phenomena. □

Remark 2 Designing a P controller is a static output feedback problem. Whether the static output feedback problem can be decided in time polynomial in the size of the problem data is still an open problem. Hence, the LMI in (5) is *not* equivalent to the BMI in (5). Note that in deriving the LMI in (5), we restrict matrix Q to be a diagonal matrix due to the structure of the matrix K. By doing so, we do not search the set of all pairs of control Lyapunov functions and stabilizing controllers. Therefore, the LMI condition (5) for P controller synthesis is conservative in the sense that no solution for LMI (5) does not imply infeasibility of LMI (5), and there could be stabilizing P controllers even if (5) is infeasible. The LMI condition (5) can be numerically solved very efficiently using interior point methods. □

Remark 3 Packet losses occur due to buffer overflows in the wired setting. In the wireless setting, however, a packet can be lost either due to a buffer overflow or due to channel fading. This indicates that the wireless channel states should possibly be a factor in the AQM controller design. Such a controller is multi input multi output. Its inputs comprise the channel states and a measure of the queue occupancy. Its outputs comprise the packet dropping policy and, possibly, recommended transmission parameters. In contrast, in the wired setting, the controller is single input single output. Per se, the problem of synthesizing transmit power allocation policies (see, e.g. [21]) can be viewed as a MIMO AQM controller synthesis problem; however, its approach is based on queueing theory concepts and not on feedback control theoretic concepts. The approach used in this paper may serve to compliment the Lyapunov drift based approach of [21] in deriving the transmit power allocation policies for mobile networks. □

6 Conclusion

We have proposed a theoretical framework to analyze the performance of AQM protocols in TCP-governed wireless networks. We assumed that the TCP-governed sources operate in the linear-increase-multiplicative-decrease mode. A bottleneck network topology involving a single router and multiple users is examined and it is shown that the problem of synthesizing stabilizing AQM controllers is equivalent to that of solving a class of BMI problems. A series of such BMI formulations for a class of AQM controllers is presented and it is observed that some of these formulations can be recast as LMI formulations, which can be solved efficiently. We have illustrated the use of integral quadratic constraints in addressing the model uncertainties. Future directions are outlined. ■

7 Acknowledgment

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