THE INFLUENCE OF MEASUREMENT NOISE ON THE PARAMETER ESTIMATION OF MAX–PLUS–LINEAR SYSTEMS

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Abstract

The present paper investigates the influence of noise on the estimation results for max–plus–linear systems. These systems are a suitable description for many processes modelled by discrete event systems. It is shown that measurements corrupted by gaussian noise lead to a bias in the estimated values. In addition a correction factor is determined that attempts to compensate this bias.

1 Introduction

The solution of automation tasks is typically based on a mathematical model that describes the process behaviour in an adequate way. When considering processes in manufacturing or chemical industry, it is observed that their behaviour can be described by a sequence of discrete events that mark the transition from one processing step to another. Thus, a discrete event model is appropriate to describe such processes. The system behaviour is then represented by a Markov– or a Semi–Markov chain, by timed automata or by timed Petri nets [2]. The focus of this contribution is on a particular class of timed Petri nets where synchronization but no concurrency occurs, namely the timed event graphs. In recent years, this particular Petri net class has gained significant attention [1]. This is due to the fact that the sequences of event times for such processes can be described by equations which are linear in a particular algebra, the so called Max–Plus algebra. The resulting equations exhibit a structural equivalence to system descriptions from conventional control engineering as transfer functions or state space models. Thus, a system theory for these max–plus–linear systems has been developed [1] and various concepts well known from control engineering have been adapted to this system class in control design [7, 11] and diagnosis [9]. The application of such methods requires a process model which can be obtained by theoretical modelling or identification algorithms.

Identification by parameter estimation for max–plus–linear systems has been considered in several publications using different approaches. The principle of estimating the system parameters has been adapted to discrete event systems in [5, 7] and [10], where the prediction error of a model given by the impulse response or in state space, respectively is minimized. In contrast, state space models have been determined using the system's Markov parameters in [4]. In all these publications it is assumed that the measurements are not corrupted by noise. However, it is a known fact from identification of continuous system models, that measurement noise may produce biases in the estimates, leading to a discrepancy between estimated system parameters and their corresponding true values [6]. This paper investigates the influence of noise on the estimated values for max–plus–linear systems.

The following section briefly reviews the basic notions of max–plus–linear systems. The estimation algorithm is then briefly discussed, followed by the investigation on the influence of noise. Finally, the results are illustrated in an example.

2 Max–Plus–linear systems

We consider in the sequel discrete event systems where the evolution of the events is governed by synchronization effects and no concurrency. The behaviour of these systems is completely specified if the event occurrence times of each event and the initial conditions are known. Thus, the time instant when event e_i occurs for the k–th time is denoted by the dater $x_i(k)$. Similarly, the input event times are given by $u_j(k)$. A relation between the event times $x(k+1), x(k) \in \mathbf{R}^n_{\max}$ and the input event times $u(k+1) \in \mathbf{R}^p_{\max}$, where $\mathbf{R}_{\max} = \mathbf{R} \cup \{-\infty\}$, can then be described by the following equation [1]:

$$x(k+1) = A_0 \otimes x(k+1) \oplus A_1 \otimes x(k) \oplus B_0 \otimes u(k+1)\ , \quad (1)$$

where $A_0, A_1 \in \mathbf{R}_{\max}^{n\times n}$, $B_0 \in \mathbf{R}_{\max}^{n\times p}$. The operators $\oplus$ and $\otimes$ are the addition and multiplication operators of the max–plus algebra and are defined by

$$x \oplus y = \max(x,y)\ , \qquad x \otimes y = x + y$$
$$\forall\, x, y \in \mathbf{R}_{\max} = \mathbf{R} \cup \{-\infty\}$$

The neutral elements of the max–plus addition and the max–plus multiplication are $-\infty = \varepsilon$ and 0, respectively. Note that ε is absorbing with respect to $\otimes$. Matrix addition and multiplication are defined similar to the conventional algebra:

$$\begin{aligned} P\,,\ Q \in \mathbf{R}_{\max}^{n\times p}\ ,\ (P \oplus Q)_{ij} &= P_{ij} \oplus Q_{ij}\ , \\ \begin{matrix} P \in \mathbf{R}_{\max}^{n\times p} \\ Q \in \mathbf{R}_{\max}^{p\times q} \end{matrix}\ ,\ (P \otimes Q)_{ij} &= \bigoplus_{k=1}^{p} (P_{ik} \otimes Q_{kj})\ . \end{aligned}$$

Equation (1) can be used for parameter identification as will be shown in the subsequent section. However, it is not suited for system analysis or simulation. If $(A_0^n)_{ij} = \varepsilon,\ \forall\ i,j = 1,\dots,n$, as will be assumed in the following considerations, (1) can be transformed into a recursive evolution equation

$$x(k+1) \ = \ A \otimes x(k) \oplus B \otimes u(k+1)\ , \qquad (2)$$

where

$$A = A_0^* \otimes A_1\ , \quad B = A_0^* \otimes B_0 \quad \text{and} \quad A_0^* = \bigoplus_{k=0}^{n-1} A_0^k\ .$$

The structural equivalence between (2) and the discrete time state space equation makes it possible to adapt well known concepts from system theory to this particular system class, provided the model and its parameter can be determined. The following section describes an identification procedure wich allows the determination of the model parameters using event time measurements. The model structure is assumed to be known.

3 Parameter estimation by minimization of an equation error

The following parameter estimation problem shall be considered:

Given the system model

$$x(k+1) = A_0 \otimes x(k+1) \oplus A_1 \otimes x(k) \oplus B_0 \otimes u(k+1)\ ,$$

the input event times $u(k),\ k = 1,\dots,N$, and the measurements $x(k),\ k = 0,\dots,N$, corrupted by noise, determine estimates $\hat{A}_{0,ij}$, $\hat{A}_{1,ij}$ and $\hat{B}_{0,ij}$ for those elements $A_{0,ij}$, $A_{1,ij}$ and $B_{0,ij}$ of the system matrices that are different from ε such that the equation error

$$\begin{aligned} \xi(k+1) &= x(k+1) - \Big(\hat{A}_0 \otimes x(k+1) \oplus \hat{A}_1 \otimes x(k) \oplus \\ &\qquad \oplus \hat{B}_0 \otimes u(k+1)\Big) \qquad (3) \\ &= x(k+1) - \underbrace{\begin{bmatrix} \hat{A}_0 & \hat{A}_1 & \hat{B}_0 \end{bmatrix}}_{=:\,\widehat{\Theta}} \otimes \underbrace{\begin{bmatrix} x_(k+1) \\ x(k) \\ u(k+1) \end{bmatrix}}_{=:\,m(k+1)} \\ &= x(k+1) - \widehat{\Theta} \otimes m(k+1) \end{aligned}$$

is minimized and the estimated parameters $\hat{A}_{0,ij}$, $\hat{A}_{1,ij}$ and $\hat{B}_{0,ij}$ are as close as possible to the true system parameters given by $A_{0,ij}$, $A_{1,ij}$ and $B_{0,ij}$. It is assumed, that it is known which entries of the system matrices are equal to ε and which are not.

To solve this problem, first an estimate for the system parameters is determined under the assumption that no noise is present. Considering the given measurements of $x(k+1)$ and $m(k+1)$, the equation error matrix results in

$$\begin{aligned} &\begin{bmatrix} \xi(N) & \dots & \xi(1) \end{bmatrix} = \\ &= \begin{bmatrix} x(N) & \dots & x(1) \end{bmatrix} - \widehat{\Theta} \otimes \begin{bmatrix} m(N) & \dots & m(1) \end{bmatrix} \\ &= X - \widehat{\Theta} \otimes M\ . \qquad (4) \end{aligned}$$

The data matrices X and M contain the event times, whereas $\widehat{\Theta} = [\hat{A}_0\ \ \hat{A}_1\ \ \hat{B}_0]$ denotes the matrix of estimated parameters based on the given measurements of x and m.

The result is obtained [1, 3, 5, 7] by computing the greatest solution of the inequality

$$X \ \geq \ \Theta \otimes M \qquad (5)$$

which is given by

$$\begin{aligned} \widehat{\Theta} &= X \otimes' \left(-M^T\right)\ , \\ \widehat{\Theta}_{ij} &= \bigoplus_{k=1}^{N}{}'\big(\underbrace{X_{ik}}_{x_i(k)} \otimes' \underbrace{-M_{jk})}_{-m_j(k)}\ \big) \\ &= \min_{k=1}^{N} \left(x_i(k) - m_j(k)\right) \qquad (6) \end{aligned}$$

where the operators "$\oplus'$" and "$\otimes'$" of the min–plus algebra [3] correspond to conventional minimization and addition, respectively. As shown in [7], this solution, determined in the absence of noise by (6), has two particular properties

$$X \ = \ \widehat{\Theta} \otimes M\ , \qquad (7)$$
$$\widehat{\Theta} \ \geq \ \Theta\ . \qquad (8)$$

From (7) immediately follows that the equation error $\xi(k) = 0$, $\forall\ k = 1,\dots,N$. However, the property (8) shows that an estimated parameter value may in general differ from the true parameter value even if no noise is present. This issue has been addressed in [10]. It was shown that the true parameter values can be achieved by applying certain excitation signals to the system. In the sequel it will be assumed that such signals can be found and applied to the system, such that the estimation would result in the true system parameters, that is, $\widehat{\Theta} = \Theta$ if the measurements were not corrupted by noise. The influence of measurement noise on the estimated values is now investigated.

4 Influence of noise

Assume now that the measurements x_i and m_j are corrupted by noise

$$x_i = x_{u,i} + v_i \ , \qquad m_j = m_{u,j} + w_j \ ,$$

where the variables x_u and m_u denote the undisturbed values of the measurements and v_i and w_j are white gaussian noise with mean μ and variance σ^2. The cross–correlation between two different time measurements shall be zero. It should be noted here that x_u and m_u correspond to time instants and not to the value of one of the system's physical states. Noise in time measurements could result for example from measuring whether a fluid's level in a tank has exceeded a certain threshold – if the surface of the fluid is in motion, the measured time instant is not the correct one. Noise can also occur due to quantization of the measured event times.

Using the above assumptions, the estimate is obtained from (6)

$$\begin{aligned}
\widehat{\Theta}_{ij} &= \min_{k=1}^{N} \big(x_i(k) - m_j(k)\big) \\
&= \min_{k=1}^{N} \big(x_{u,i}(k) + v_i(k) - m_{u,j}(k) - w_j(k)\big) \\
&= \min_{k=1}^{N} \Big(\big(x_{u,i}(k) - m_{u,j}(k)\big) + \big(v_i(k) - w_j(k)\big)\Big) \\
&\geq \min_{k=1}^{N} \big(x_{u,i}(k) - m_{u,j}(k)\big) + \min_{k=1}^{N} \big(v_i(k) - w_j(k)\big) \ .
\end{aligned}$$

Due to the assumption that the true system parameters are obtained in the absence of noise, one obtains from the above considerations

$$\widehat{\Theta}_{ij} \quad \geq \quad \Theta_{ij} + \min_{k=1}^{N} \big(v_i(k) - w_j(k)\big) \ .$$

The expected estimated value then results from

$$\mathrm{E}\{\widehat{\Theta}_{ij}\} \quad \geq \quad \Theta_{ij} + \mathrm{E}\{\min_{k=1}^{N} \big(v_i(k) - w_j(k)\big)\} \qquad (9)$$

since Θ_{ij} is deterministic. Thus, it is concluded that the noise may lead to biased estimated values. The bias depends on the expected value $\mathrm{E}\{\min_{k=1}^{N} \big(v_i(k) - w_j(k)\big)\}$ and will be analyzed in more detail in the sequel.

For the computation of this expectation value, first the probability density function (pdf) of $\delta(k) = v_i(k) - w_j(k)$ is required. If $j = 1, \ldots, n$, or $j = n+1, \ldots, 2n$, then $m_j(k)$ corresponds to the measurement $x_j(k)$ or $x_j(k-1)$, respectively, corrupted by the noise $w_j(k)$. Since both variables v_i and w_j are assumed gaussian with mean μ and variance σ^2, and since v_i and w_j are independent by assumption, the pdf of $\delta(k)$ is also gaussian with mean $\mu_\delta = 0$ and variance $\sigma_\delta^2 = 2\sigma^2$:

$$f_\delta(\delta) \quad = \quad \frac{1}{\sigma_\delta\sqrt{2\pi}}\,\mathrm{e}^{-\dfrac{(\delta-\mu_\delta)^2}{2\sigma_\delta^2}} = \frac{1}{2\sigma\sqrt{\pi}}\,\mathrm{e}^{-\dfrac{\delta^2}{4\sigma^2}} \ .$$

If otherwise $j = 2n+1, \ldots, 2n+p$, then $m_j(k)$ corresponds to the input signal u_{j-2n} which is assumed to be uncorrupted by the noise such that $w_j(k) = 0$. Then, $\mu_\delta = \mu$ and $\sigma_\delta^2 = \sigma^2$.

The bias in equation (9) is the expectation value of a stochastic variable obtained from the minimum of N values of δ. The following theorem illustrates the dependence of this bias on the variance σ^2 and the mean μ of the noise and the number of measurements N included in the estimation.

Theorem 4.1 *Given the sequence of independent gaussian random variables $\delta(k)$ with mean μ_δ and variance σ_δ^2. Then, there exists a $K(N)$ such that*

$$\begin{aligned}
E\{\min_{k=1}^{N} \big(\delta(k)\big)\} &= \mu_\delta + \sigma_\delta K(N) \\
K(N) &= \sqrt{2}\int_{-\infty}^{\infty} \beta f_{\min_N}(\beta)d\beta
\end{aligned}$$

where β is a random variable that results from the minimum of N independent gaussian random variables with zero mean and variance $\sigma_\beta^2 = \frac{1}{2}$ and the pdf of β is denoted by $f_{\min_N}(\beta)$.

Proof: The expected value $\mathrm{E}\{\min_{k=1}^{N} \big(\delta(k)\big)\}$ is given by [8]

$$\mathrm{E}\{\alpha = \min_{k=1}^{N} \big(\delta(k)\big)\} \quad = \quad \int_{-\infty}^{\infty} \alpha f_{min_N}(\alpha)d\alpha \ . \qquad (10)$$

Thus, in the first part of the proof the pdf $f_{min_N}(\alpha)$ is determined. First the distribution function $F_{min_N}(\alpha)$ is considered. For $N = 2$ the distribution is given by [8]

$$\begin{aligned}
F_{min_2}(\alpha) &= F(\alpha) + F(\alpha) - F(\alpha)F(\alpha) \\
&= 2F(\alpha) - F^2(\alpha) = 1 - (1 - F(\alpha))^2 \ ,
\end{aligned}$$

where $F(\alpha) = F(\delta)$. Assume that

$$F_{min_N}(\alpha) \quad = \quad 1 - (1 - F(\alpha))^N \qquad (11)$$

holds for some N. Then,

$$\begin{aligned}
F_{min_{N+1}}(\alpha) &= F(\alpha) + F_{min_N}(\alpha) - F(\alpha)F_{min_N}(\alpha) \\
&= F(\alpha) + 1 - (1 - F(\alpha))^N - \\
&\quad\; F(\alpha)\big(1 - (1 - F(\alpha))^N\big) \\
&= F(\alpha) + (1 - F(\alpha))\big(1 - (1 - F(\alpha))^N\big) \\
&= 1 - (1 - F(\alpha))^{N+1} \ .
\end{aligned}$$

Thus, (11) holds for any $N \geq 2$. The desired *pdf* can then be determined from

$$f_{min_N}(\alpha) = \frac{\partial F_{min_N}(\alpha)}{\partial \alpha} = N(1 - F(\alpha))^{N-1} f(\alpha) \qquad (12)$$

Thus,

$$\int_{-\infty}^{\infty} \alpha f_{min_N}(\alpha) d\alpha = \int_{-\infty}^{\infty} \alpha N(1 - F(\alpha))^{N-1} f(\alpha) d\alpha \; .$$

Substituting $\beta = \frac{\alpha - \mu_\delta}{\sqrt{2}\sigma_\delta}$ one obtains

$$E\{\alpha\} = \mu_\delta + \sigma_\delta \sqrt{2} \int_{-\infty}^{\infty} \beta N(1 - F(\beta))^{N-1} f(\beta) d\beta \; .$$
$$= \mu_\delta + \sigma_\delta \sqrt{2} \int_{-\infty}^{\infty} \beta f_{min_N}(\beta) d\beta \; .$$

□

From the result of theorem 4.1 and (9) one concludes that

$$\mathrm{E}\{\widehat{\Theta}_{ij}\} \geq \Theta_{ij} + \mu_\delta + \sigma_\delta K(N)$$
$$\geq \Theta_{ij} + \sqrt{2}\,\sigma K(N) \; , j = 1, \ldots, 2n \; , \qquad (13)$$
$$\mathrm{E}\{\widehat{\Theta}_{ij}\} \geq \Theta_{ij} + \mu + \sigma K(N) \; , \qquad (14)$$
$$j = 2n + 1, \ldots, 2n + p \; ,$$

where μ and σ^2 are the mean and the variance of the measurement noise, respectively. Note that the first n columns of Θ correspond to A_0 whereas the columns $n + 1, \ldots, 2n$ and $2n + 1, \ldots, 2n + p$ correspond to A_1 and B_0, respectively.

The above equations hold if the system matrix A_1 in (1) contains no diagonal elements other then ε[1], since only in this case all $\delta(k)$ are independent. For the estimation of diagonal elements in A_1, consecutive values $\delta(k) = v_i(k) - v_i(k-1)$ and $\delta(k-1) = v_i(k-1) - v_i(k-2)$ are obviously not independent such that the distribution $F_{\min_N}$ cannot be computed as in theorem 4.1. In contrast, the computation of the distribution $F_{\min_N}$ and the corresponding *pdf* requires the consideration of the correlation coefficient of consecutive values $\delta(k)$ and $\delta(k-1)$.

The factor $K(N)$ can be computed in advance, depending only on the number of data sets used for the estimation of a particular parameter. It is given for the $N = 2, \ldots, 9$, in table 1 and for larger values of N in figure 1.

From table 1 and figure 1 it is seen, that $K(N)$ is negative. The absolute value of $K(N)$ increases with increasing N. However, the influence of new measurements added to the data set decreases as N increases. Since $K(N)$ is negative, the noise

[1]Note that all diagonal elements of A_0 are equal to ε. Otherwise there would exist some index j such $(A_0^n)_{jj} \neq \varepsilon$ thus contradicting the assumption from section 2.

N	2	3	4	5
$K(N)$	-0.5642	-0.8463	-1.0294	-1.1630
N	6	7	8	9
$K(N)$	-1.2672	-1.3522	-1.4236	-1.4850

Table 1: $K(N)$ for $N = 2, \ldots, 9$.

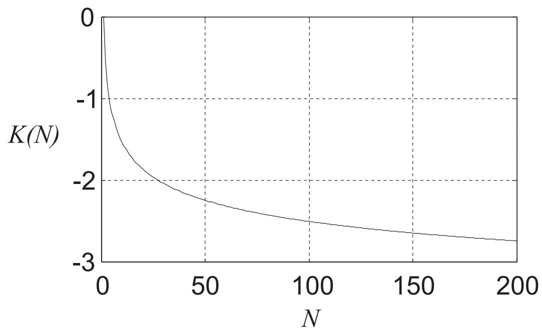

Figure 1: $K(N)$ for $N = 2, \ldots, 200$.

may lead to an underestimation of the system parameters. This is an issue if the estimated model is used to design controllers that compute the greatest input that must produce output signals below a certain threshold, thus attempting to solve the zero–lateness problem [7]. An underestimation of a parameter may lead to control signals that are issued too late to achieve the given output time. To avoid underestimation, (13) and (14) can be used to compute an upper bound for the system parameters:

$$A_{0,ij} \leq \mathrm{E}\{\hat{A}_{0,ij}\} - \sqrt{2}\,\sigma K(N) \; , \qquad (15)$$
$$A_{1,ij} \leq \mathrm{E}\{\hat{A}_{1,ij}\} - \sqrt{2}\,\sigma K(N) \; , \qquad (16)$$
$$B_{0,ij} \leq \mathrm{E}\{\hat{B}_{0,ij}\} - \mu - \sigma K(N) \; . \qquad (17)$$

Using the right hand side of (15), (16) and (17) as estimation result, an underestimation can be avoided. In addition, equation (17) shows that for particular values of μ, σ and $K(N)$ the correction term in (17) is equal to zero. Thus, the correction of B_{ij} can also be made during the estimation, by adding an appropriate bias to the measurements that shifts the noise mean to the desired value and in addition using an appropriate number of measurements N in the estimation procedure.

5 Example

Consider now a manufacturing cell shown in figure 2, where parts are delivered to the machine by a conveyor, machined and released to an additional conveyor. The capacity of each conveyor is limited to one part. The machine can process one part at the same time. Figure 3 gives a discrete event model that describes the system behaviour as a Petri net.

Let $x_1(k)$ be the time instant when the k–th part is loaded onto the conveyor 1. After τ_{21} time units, this part is ready to enter the machine. The dater $x_2(k)$ denotes the time when the k–th part enters the machine. After the machining operation which

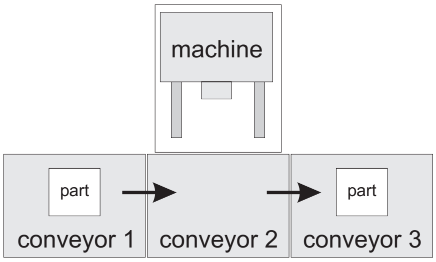

Figure 2: Manufacturing cell.

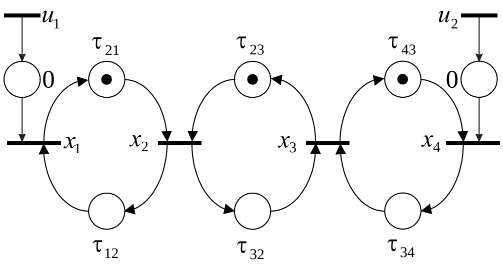

Figure 3: Petri net model of the manufacturing cell.

takes τ_{32} time units, the part is released to conveyor 2 at time $x_3(k)$ reaching a final position on conveyor 3 after τ_{43} time units. From this final position, the part is picked up at time $x_4(k)$. Conveyor 1 and 3 can receive a new part after τ_{12} and τ_{34} time units, respectively, whereas the machine must be prepared for a new operation for τ_{23} time units. The input event times $u_1(k)$ and $u_2(k)$ correspond to the time instants when a new part is available to be delivered for conveyor 1 or removed from conveyor 3, respectively.

Using the above reasoning and the initial state of the system described by the initial marking in the Petri net from figure 3, the following max–plus–linear equations hold for $x(k)$:

$$\begin{aligned} x_1(k+1) &= \tau_{12} \otimes x_2(k+1) \oplus u_1(k+1) \\ x_2(k+1) &= \tau_{21} \otimes x_1(k) \oplus \tau_{23} \otimes x_3(k) \\ x_3(k+1) &= \tau_{32} \otimes x_2(k+1) \oplus \tau_{34} \otimes x_4(k+1) \\ x_4(k+1) &= \tau_{43} \otimes x_3(k) \oplus u_2(k+1) \end{aligned}$$

These equations are of the structure given by (1)

$$x(k+1) = A_0 \otimes x(k+1) \oplus A_1 \otimes x(k) \oplus B_0 \otimes u(k+1)$$

$$A_0 = \begin{bmatrix} \varepsilon & \tau_{12} & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \tau_{32} & \varepsilon & \tau_{34} \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}, \; A_1 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \tau_{21} & \varepsilon & \tau_{23} & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \tau_{43} & \varepsilon \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 0 \end{bmatrix}^T .$$

The entries of A_0, A_1 and B_0 that are different from ε are unknown and must be determined from measured event times x and u.

First an appropriate persistent excitation was designed using the principles given in [10]. In order to illustrate the effect of the noise, the parameters of the original system given in the appendix were used in this design process to make sure, that deviations in the estimated parameters are not due to a design that results from overestimated parameters but only to the measurement noise. The number of required measurements was set to $N = 8$. In addition, it is assumed that the entries equal to ε in the matrices A_0, A_1 and B_0 are known. Using the designed input signals, the parameters estimated based on the uncorrupted measurements are equal to the true system parameters.

Now, the measurements $x(k)$ corrupted by gaussian noise with $\sigma^2 = 1$ are considered. A sequence of estimates $\hat{A}_0^{(r)}$, $\hat{A}_1^{(r)}$ and $\hat{B}_0^{(r)}$ for the matrices A_0, A_1 and B_0 is determined from

$$\begin{aligned} \hat{A}_{0,ij}^{(r)} &= \min_{k=r+1}^{r+8} \big(x_i(k+1) - x_j(k+1) \big) , \\ \hat{A}_{1,ij}^{(r)} &= \min_{k=r+1}^{r+8} \big(x_i(k+1) - x_j(k) \big) , \\ \hat{B}_{0,ij}^{(r)} &= \min_{k=r+1}^{r+8} \big(x_i(k+1) - u_j(k+1) \big) , \\ & \quad r = 2, \ldots, 1592 . \end{aligned}$$

The matrices $\hat{A}_0$, $\hat{A}_1$ and $\hat{B}_0$ are then computed as the average from $\hat{A}_0^{(r)}$, $\hat{A}_1^{(r)}$ and $\hat{B}_0^{(r)}$, respectively:

$$\hat{A}_0 = \begin{bmatrix} \varepsilon & 1.2116 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1.2337 & \varepsilon & -0.1130 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix},$$

$$\hat{A}_1 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -0.9086 & \varepsilon & 0.1225 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 0.2007 & \varepsilon \end{bmatrix},$$

$$\hat{B}_0 = \begin{bmatrix} -1.4049 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & -1.4229 \end{bmatrix}^T .$$

As the comparison with the original system parameters shows, the noise leads to an underestimation in all estimated parameters. Using the given noise variance $\sigma^2 = 1$ and the parameter $K(8)$ from table 1 one obtains with (15), (16) and (17) upper bounds for the system parameters:

$$\hat{A}_{0c} = \begin{bmatrix} \varepsilon & 3.2249 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 3.2470 & \varepsilon & 1.9002 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix},$$

$$\hat{A}_{1c} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 1.1047 & \varepsilon & 2.1358 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 2.2139 & \varepsilon \end{bmatrix},$$

$$\hat{B}_{0c} = \begin{bmatrix} 0.0187 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 0.0007 \end{bmatrix}^T .$$

Figure 4 shows the simulation results obtained for $x_3(k)$ after transforming the implicit system equations (1) the into the

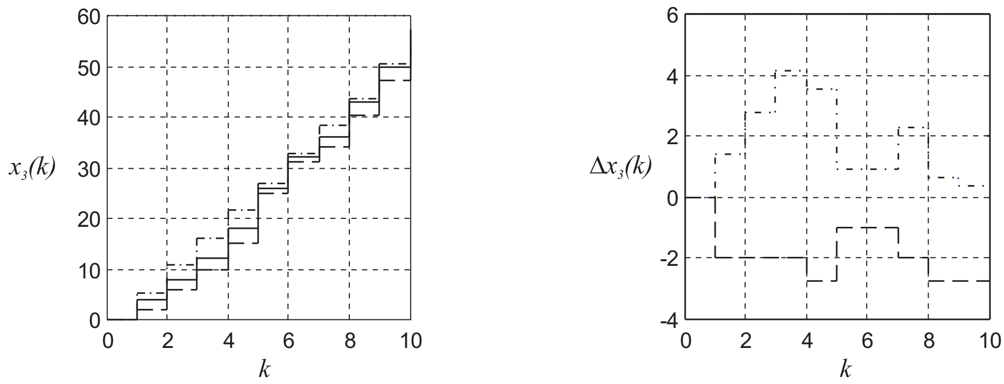

Figure 4: Left: Simulation results obtained for the system (A, B) $(-)$, $(\hat{A}, \hat{B})$ $(--)$, and $(\hat{A}_c, \hat{B}_c)$ $(-\cdot)$. Right: Deviations to the uncorrupted case without correction $(--)$ and with correction $(-\cdot)$.

evolution equation (2) using the parameters estimated based on the uncorrupted data, the parameters estimated using the noisy data and the parameters obtained from (15), (16) and (17). Note that the negative entries different from ε in $\hat{A}_0$, $\hat{A}_1$ and $\hat{B}_0$ have been set to $zero$ to ensure their interpretability. In addition to the event times, the right hand side of figure 4 displays the deviations to the uncorrupted case. Obviously, the estimation based on the noisy data leads to a system model which is ”faster” than the original system, whereas the correction by (15), (16) and (17) yields a ”slower” model.

6 Conclusions

The present paper addressed the issue of estimating the parameters of max–plus–linear systems when the measurements are corrupted by gaussian noise. It was shown, that noise may lead to biases in the estimated parameters resulting in an underestimation of system parameters which may be an issue when computing controllers that solve the zero–lateness problem. This issue can be overcome by using a correction factor depending on the noise variance and the number of measurements used for identification to determine upper bounds for the system parameters.

A Original system parameters

$$\begin{array}{lll} \tau_{12} = 3 & \tau_{21} = 1 & \tau_{23} = 1 \\ \tau_{32} = 2 & \tau_{34} = 1 & \tau_{43} = 2 \end{array}$$

$$x(0) = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array}\right]^T$$

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