EXTENDED GEOMETRIC CONDITIONS FOR NON-INTERACTING CONTROLS IN LINEAR SYSTEMS AND CONSEQUENCES ON RELATED ISSUES

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Abstract

An extension of the structural conditions for non-interacting controls is considered for linear discrete time-invariant systems. The approach is strictly geometric and exploits the basic properties of the minimum conditioned invariant containing a given subspace. The investigation is motivated by the impact of this extension on the dual setting of fault detection and identification.

1 Introduction

The non-interacting control problem solved by means of feedforward dynamic units as in the pioneering work by Basile and Marro [1] is considered, and an extension of the structural conditions first proven therein — and widely used in the literature up to now [3, 10] — is presented, which is valid whenever non-interaction is required for a finite short time, rather than for an infinite time. Although the interest of non-interaction restricted to a short time concerns very specific situations, the study of extended conditions for non-interacting controls is motivated by the impact they have on the theoretical aspects of the dual problem. Since the early papers by Massoumnia, Verghese and Willsky [5, 6], some duality has been recognized between non-interaction and fault detection and identification (FDI). Indeed, the scheme adopted in [5, 6] is the dual counterpart of that proposed in [1]. Leaving aside the great number of issues arising in practical implementation of FDI strategies, this work simply aims at providing a relaxed version of the structural conditions for non-interacting controls, which, in the dual setting, allows the theoretical aspects of the FDI problem to be solved by means of an elementary decision logic cascaded to the observers (residual generators). The approach is strictly geometric [2, 9] and relies on a peculiar property of the minimum conditioned invariant containing a given subspace recently pointed out in [4]. The discussion refers to discrete-time systems, since connections between geometric properties of the subspaces involved and synthesis procedures are more intuitive than in the continuous-time case [8, 7].

For the sake of simplicity, the main idea will be expressed with respect to the so-called fundamental problems, i.e. the non-interacting control problem where the output partition consists of two blocks only, and, by duality, the FDI problem where the failure mode inputs are partitioned into two groups. As already stated, the control scheme for non-interaction is the one based on feedforward dynamic compensation shown in Fig.1, while the corresponding layout for FDI is sketched in Fig.2. In the non-interaction problem stated for the controlled system (A, B, C) with the output partition (y_1, y_2) corresponding to the matrices C_1 and C_2, the compensation unit Σ_{c_1} is such that an impulse at input h_1 generates a control action which steers the state of the controlled system along a trajectory belonging to the reachable subspace constrained to the maximum controlled invariant contained in $\mathcal{C}_2$, the kernel of C_2. Thus, an impulse at h_1 will never affect y_2, while it spans the space of y_1 (Fig.3). In geometric terms, feasibility of such a compensation unit is expressed by the structural condition $C_1\,(\mathcal{S}_2^* \cap \mathcal{V}_2^*) = \operatorname{im} C_1$, where $\mathcal{S}_2^*$ is the minimum $(A, \mathcal{C}_2)$-conditioned invariant containing $\mathcal{B}$, the image of B, and $\mathcal{V}_2^*$ is the maximum $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{C}_2$. By duality, in the fundamental FDI problem, the corresponding condition guarantees that observer output o_1 is affected by a fault occurring at system input m_1, but will never be affected by a fault occurring at system input m_2. The relaxed condition corresponding to one previously considered is $C_1\,(\mathcal{S}_2^* \cap \mathcal{C}_2) = \operatorname{im} C_1$, i.e. the subspace where the system state trajectory is steered is enlarged from $\mathcal{S}_2^* \cap \mathcal{V}_2^*$ to $\mathcal{S}_2^* \cap \mathcal{C}_2$. This means that the space of y_1 is spanned while the state trajectory lies on $\mathcal{S}_2^* \cap \mathcal{C}_2$. Afterwards, the state leaves $\mathcal{C}_2$, thus affecting also y_2. In other terms, non-interaction is restricted to the time when the state trajectory belongs to $\mathcal{S}_2^* \cap \mathcal{C}_2$ (Fig.4). With obvious meaning of the symbols, the extended condition for output y_2 is $C_2\,(\mathcal{S}_1^* \cap \mathcal{C}_1) = \operatorname{im} C_2$. In the dual setting, the extended conditions allow the FDI problem to be solved even though the standard conditions are not satisfied, provided that a simple decision logic is added to the observer. In fact, if the extended conditions referred to the primal problem are satisfied, then observer units can be designed such that a fault occurring at system input m_1 affects output o_2 only after output o_1, and vice versa for a fault occurring at system input m_2. The finite time interval when only one observer output is affected, enables a simple logic to detect and isolate the fault.

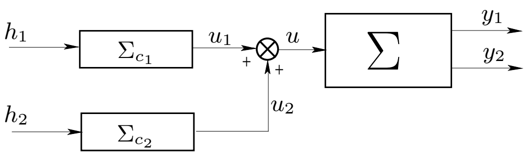

Figure 1: Non-interaction block diagram.

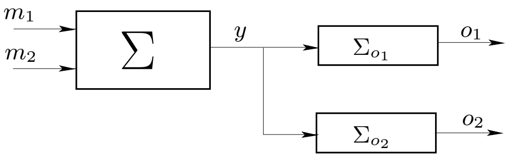

Figure 2: FDI block diagram.

2 Notation and preliminaries

Throughout this paper, $\mathbb{R}$ stands for the field of real numbers. Sets, vector spaces and subspaces are denoted by script capitals like $\mathcal{V}$, matrices and linear maps by slanted capitals like A. The notation for the image and the kernel of A is $\operatorname{im} A$ and $\ker A$, respectively. The symbols for the transpose, the pseudoinverse, and the spectrum of A are respectively A^{T}, $A^{\#}$, and $\sigma(A)$. The discrete time-invariant linear system

$$\begin{aligned} x(t+1) &= A\,x(t) + B\,u(t), \\ y(t) &= C\,x(t), \end{aligned} \tag{1}$$

is considered, with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^p$, controlled output $y \in \mathbb{R}^q$. The matrices B and C are assumed to be of full rank. For the sake of brevity, the following symbols will also be used: $\mathcal{B}$ for $\operatorname{im} B$, $\mathcal{C}$ for $\ker C$, $\max \mathcal{V}(A,\mathcal{B},\mathcal{C})$ or $\mathcal{V}^*$ for the maximum $(A,\mathcal{B})$-controlled invariant contained in $\mathcal{C}$, $\min \mathcal{S}(A,\mathcal{C},\mathcal{B})$ or $\mathcal{S}^*$ for the minimum $(A,\mathcal{C})$-conditioned invariant containing $\mathcal{B}$, and $\mathcal{R}_{\mathcal{V}^*}$ for the reachable subspace on $\mathcal{V}^*$. The symbol F will denote any state feedback matrix such that $(A+BF)\,\mathcal{V}^* \subseteq \mathcal{V}^*$. With the above notation, $\mathcal{R}_{\mathcal{V}^*}$ can also be expressed as the minimum $(A+BF)$-invariant containing $\mathcal{V}^* \cap \mathcal{B}$, i.e. $\mathcal{R}_{\mathcal{V}^*} = \min \mathcal{J}\,(A+BF, \mathcal{V}^* \cap \mathcal{B})$. The invariant zeros of the triple (A,B,C) are defined as the internal unassignable eigenvalues of $\mathcal{V}^*$, i.e. $\mathcal{Z} := \sigma(A+BF)_{\mathcal{V}^*/\mathcal{R}_{\mathcal{V}^*}}$, where $\mathcal{V}^*/\mathcal{R}_{\mathcal{V}^*}$ is the quotient space of $\mathcal{V}^*$ with respect to $\mathcal{R}_{\mathcal{V}^*}$. The triple (A,B,C) is left-invertible if and only if $B^{-1}\mathcal{V}^* = \{0\} \;\Leftrightarrow\; \mathcal{V}^* \cap \mathcal{B} = \{0\} \;\Leftrightarrow\; \mathcal{V}^* \cap \mathcal{S}^* = \{0\}$, is right-invertible if and only if $C\mathcal{S}^* = \mathbb{R}^q \Leftrightarrow \mathcal{S}^* + \mathcal{C} = \mathbb{R}^n \Leftrightarrow \mathcal{S}^* + \mathcal{V}^* = \mathbb{R}^n$. Hence, (A,B,C) is both right- and left-invertible if and only if $\mathcal{S}^* \oplus \mathcal{V}^* = \mathbb{R}^n$. The subspace $\mathcal{S}^*$ is computable as the last term of the sequence

$$\begin{aligned} &\mathcal{S}^{(1)} := \mathcal{B}, \\ &\mathcal{S}^{(i)} := A\left(\mathcal{S}^{(i-1)} \cap \mathcal{C}\right) + \mathcal{B}, \;\; i = 2, \dots k, \end{aligned} \tag{2}$$

where the value of $k\,(\leq n)$ is determined by the condition $\mathcal{S}^{(k+1)} = \mathcal{S}^{(k)}$. The symbol A_F will also be used in place of $A+BF$. The subspace $\mathcal{R}_{\mathcal{V}^*}$ is computable as the last term of

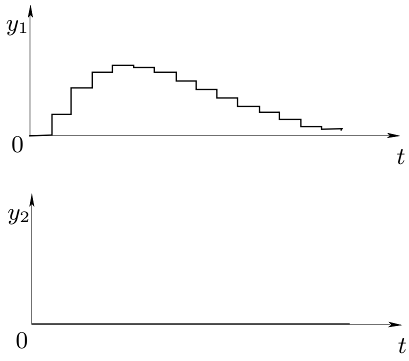

Figure 3: System outputs for an impulse occurring at input h_1, when the standard structural condition is satisfied.

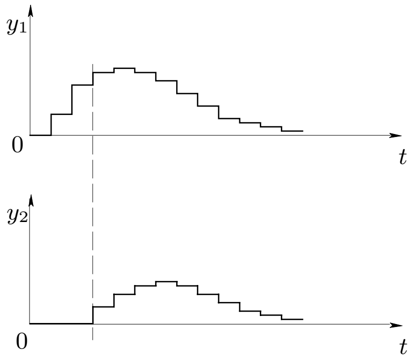

Figure 4: System outputs for an impulse occurring at input h_1, when the extended structural condition only is satisfied.

the sequence

$$\begin{aligned} &\mathcal{R}_{\mathcal{V}^*}^{(1)} := \mathcal{B} \cap \mathcal{V}^*, \\ &\mathcal{R}_{\mathcal{V}^*}^{(i)} := \left(A_F \mathcal{R}_{\mathcal{V}^*}^{(i-1)} + \mathcal{B}\right) \cap \mathcal{V}^*, \;\; i = 2, \dots k, \end{aligned} \tag{3}$$

with k such that $\mathcal{R}_{\mathcal{V}^*}^{(k+1)} = \mathcal{R}_{\mathcal{V}^*}^{(k)}$. The relation $\mathcal{R}_{\mathcal{V}^*} = \mathcal{V}^* \cap \mathcal{S}^*$ is also well-known. The sequence derived from (2) by intersecting each subspace with $\mathcal{C}$ converges to the subspace, herein denoted by $\mathcal{R}_{\mathcal{C}}^*$, of the states reachable from the origin along trajectories belonging to $\mathcal{C}$, i.e. $\mathcal{R}_{\mathcal{C}}^* = \mathcal{S}^* \cap \mathcal{C}$. This sequence can also be expressed by means of the algorithm

$$\begin{aligned} &\mathcal{R}_{\mathcal{C}}^{(1)} := \mathcal{B} \cap \mathcal{C}, \\ &\mathcal{R}_{\mathcal{C}}^{(i)} := \left(A\,\mathcal{R}_{\mathcal{C}}^{(i-1)} + \mathcal{B}\right) \cap \mathcal{C}, \;\; i = 2, \dots k, \end{aligned} \tag{4}$$

with k such that $\mathcal{R}_{\mathcal{C}}^{(k+1)} = \mathcal{R}_{\mathcal{C}}^{(k)}$. The standard non-interacting control problem and the geometric conditions for its solution will be briefly recalled herein. Let system (1) be the controlled system. The matrix A is assumed to be stable[1]. A block partition $(y_1, \dots, y_k)$, with $k \leq q$, of the controlled output y is considered. The feedforward non-interacting control problem with respect to the partition $(y_1, \dots, y_k)$ of the controlled output y is the problem of finding a precompensator Σ_c, ruled by

$$\begin{aligned} z(t+1) &= A_c\,z(t) + B_c\,h(t), \\ u(t) &= C_c\,z(t) + D_c\,h(t), \end{aligned} \tag{5}$$

[1]This assumption can be relaxed to stabilizability of the pair (A,B) and detectability of the pair (A,C) — see Section 5.

whose input h is partitioned into $(h_1,\ldots,h_k)$, with the property that if the system is initially in the zero state, any action on the single block input h_i causes only the corresponding block output y_i to be changed, while all the other outputs remain identically equal to zero. The symbols $C_1,\ldots,C_k$ will denote the submatrices of C corresponding to the partition $(y_1,\ldots,y_k)$. The symbols $\mathcal{C}_1,\ldots,\mathcal{C}_k$ will denote the kernels of the matrices $C_1,\ldots,C_k$, respectively. Also the following notation will be used $\bar{\mathcal{C}}_i := \bigcap_{j\neq i}\mathcal{C}_j$, for $i=1,\ldots,k$. The corresponding subspaces are denoted by $\bar{\mathcal{V}}_i^* := \max\mathcal{V}(A,\mathcal{B},\bar{\mathcal{C}}_i)$, $\bar{\mathcal{S}}_i^* := \min\mathcal{S}(A,\bar{\mathcal{C}}_i,\mathcal{B})$, and $\mathcal{R}_{\bar{\mathcal{V}}_i^*} = \bar{\mathcal{V}}_i^* \cap \bar{\mathcal{S}}_i^*$, for $i=1,\ldots,k$. As is well-known, a non-interacting controller exists if and only if the structural conditions $C_i\mathcal{R}_{\bar{\mathcal{V}}_i^*} = \operatorname{im} C_i$, $i=1,\ldots,k$, are satisfied. Clearly, the conditions for the existence of a non-interacting controller are closely related to the meaning of the generic i-th reachable subspace $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ on $\bar{\mathcal{V}}_i^*$ as the maximum subspace reachable from the origin along trajectories which can be indefinitely maintained in $\bar{\mathcal{V}}_i^*$, hence in $\bar{\mathcal{C}}_i$. Therefore, these trajectories affect the output y_i, but are invisible at any other output y_j, $j\neq i$.

3 Main results

As mentioned above, if non-interaction is not required for infinite time, the geometric conditions for non-interacting controls previously recalled can be relaxed by exploiting the basic property of the subspaces $\mathcal{R}_{\bar{\mathcal{C}}_i} = \bar{\mathcal{S}}_i^* \cap \bar{\mathcal{C}}_i$, $i=1,\ldots,k$. In fact, they are the maximum subspaces respectively reachable from the origin along trajectories belonging to $\bar{\mathcal{C}}_i$. For the sake of simplicity, the main results will be stated referring to the fundamental problem, where $k=2$. The extension to a generic number of blocks k is straightforward, although it implies the use of heavier notation.

Definition 1 *A controller Σ_c ruled by (5) with the input partition (h_1,h_2) such that, starting at the zero state, by acting on a single input h_i, $i=1,2$, with the other input identically zero, the space of the corresponding system output y_i is spanned within the time when the other output is zero will be called a conditioned non-interacting controller with respect to the system output partition (y_1,y_2).*

Theorem 1 *A conditioned non-interacting controller with respect to the system output partition (y_1,y_2) exists if and only if*

$$C_i\left(\mathcal{S}_j^* \cap \mathcal{C}_j\right) = \operatorname{im} C_i,\ i=1,2,\ j=1,2,\ j\neq i. \qquad (6)$$

Proof: Only if. This part of the proof is directly implied by the maximality of the subspace $\mathcal{S}_j^*\cap\mathcal{C}_j$, $j=1,2$, as the set of the states reachable from the origin in a finite number of steps, along trajectories belonging to $\mathcal{C}_j$, hence invisible at the j-th output. In fact, if conditions (6) do not hold for some i, it means that the image in C_i of the maximum subspace reachable from the origin in a finite number of steps, along trajectories lying on $\mathcal{C}_j$, is not the whole image of C_i. Then, the maximality of $\mathcal{S}_j^*\cap\mathcal{C}_j$ implies that a controller, steering the state along a trajectory which, starting at the origin, is such that the whole space of the i-th output is spanned, does not exist.

If. This part of the proof is constructive, since it yields the design of the precompensator Σ_c. The condition $C_1(\mathcal{S}_2^*\cap\mathcal{C}_2) = \operatorname{im} C_1$ is considered first, and it is shown how this allows a precompensation unit Σ_{c_1} to be designed, which satisfies the requirements. The precompensator Σ_{c_1} should be ruled by

$$\begin{aligned} z_1(t+1) &= A_{c_1}\, z_1(t) + B_{c_1}\, h_1(t), \\ u_1(t) &= C_{c_1}\, z_1(t) + D_{c_1}\, h_1(t), \end{aligned}$$

and should be such that, in the presence of a non-zero signal at its input h_1, it produces a control action u_1 forcing the system state, initially in the origin, to reach the subsequent subspaces generated by Algorithm (4) for $\mathcal{R}_{\mathcal{C}_2}^* = \mathcal{S}_2^*\cap\mathcal{C}_2$. By pursuing this trajectory, the state remains in $\mathcal{C}_2$ for the first ρ_2 steps, where ρ_2 represents the number of steps for the algorithm of $\mathcal{S}_2^*\cap\mathcal{C}_2$ to converge. Moreover, at step ρ_2 at most, the output y_1 have changed. Let $M_2^{(i)}$, $i=1,\ldots,\rho_2$, denote the respective bases of the subsequent subspaces generated by Algorithm (4) for $\mathcal{S}_2^*\cap\mathcal{C}_2$, i.e.

$$\begin{aligned} &\operatorname{im} M_2^{(1)} = \mathcal{B}\cap\mathcal{C}_2 = \mathcal{R}_{\mathcal{C}_2}^{(1)}, \\ &\operatorname{im} M_2^{(i)} = (A\mathcal{R}_{\mathcal{C}_2}^{(i-1)} + \mathcal{B})\cap\mathcal{C}_2 = \mathcal{R}_{\mathcal{C}_2}^{(i)},\ \ i=2,\ldots,\rho_2, \end{aligned}$$

with $\operatorname{im} M_2^{(\rho_2)} = \mathcal{S}_2^*\cap\mathcal{C}_2 = \mathcal{R}_{\mathcal{C}_2}^*$. Let the matrix X_f be defined as $X_f := M_2^{(\rho_2)}$. For any column of X_f, which represents a state belonging to $\mathcal{S}_2^*\cap\mathcal{C}_2$, a control sequence of ρ_2 steps, steering the state from the origin to the state represented by the given column, along a trajectory whose intermediate states belong to the subsequent subspaces generated by the algorithm of $\mathcal{S}_2^*\cap\mathcal{C}_2$, exists. Let $X(i) = M_2^{(i)}\beta(i)$, $i=1,\ldots,\rho_2-1$, denote the matrices of the intermediate states at the i-th step, and let $U(i)$, $i=1,\ldots,\rho_2-1$, denote the matrices of the corresponding controls. The unknowns $\beta(i)$ and $U(i)$, $i=1,\ldots,\rho_2-1$, are obtainable by the relations

$$\begin{aligned} &\begin{bmatrix} \beta(i) \\ U(i) \end{bmatrix} = \begin{bmatrix} AM_2^{(i)}\ B \end{bmatrix}^{\#} X(i+1),\ \ i=1,\ldots,\rho_2-1, \\ &X(\rho_2) = X_f,\ \ U(0) = B^{\#}X(1). \end{aligned}$$

Then, the precompensation unit Σ_{c_1} can be implemented as a finite impulse response system defined by

$$u_1(t) = \sum_{\ell=0}^{\rho_2-1} \Phi(\ell)h_1(t-\ell), \quad t=0,1,\ldots,$$

with $\Phi(\ell) = U(\ell)$, $\ell=0,1,\ldots,\rho_2-1$. Hence, the quadruple $(A_{c_1},B_{c_1},C_{c_1},D_{c_1})$ is

$$A_{c_1} = \begin{bmatrix} 0 & I & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & I \\ 0 & \ldots & \ldots & \ldots & 0 \end{bmatrix},\quad B_{c_1} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ I \end{bmatrix}, \qquad (7)$$

$$C_{c_1} = \begin{bmatrix} \Phi(\rho_2-1) & \ldots & \Phi(1) \end{bmatrix},\quad D_{c_1} = \Phi(0).$$

The design of the precompensation unit Σ_{c_2}, defined by the quadruple $(A_{c_2}, B_{c_2}, C_{c_2}, D_{c_2})$, can be carried out in a similar way. Hence, the quadruple (A_c, B_c, C_c, D_c) defining the compensator Σ_c, ruled by (5) with $z(t) = \left[z_1(t)^{\mathrm{T}}\ z_2(t)^{\mathrm{T}}\right]^{\mathrm{T}}$, $h(t) = \left[h_1(t)^{\mathrm{T}}\ h_2(t)^{\mathrm{T}}\right]^{\mathrm{T}}$ and $u(t) = u_1(t) + u_2(t)$, is

$$A_c = \begin{bmatrix} A_{c_1} & 0 \\ 0 & A_{c_2} \end{bmatrix}, \quad B_c = \begin{bmatrix} B_{c_1} & 0 \\ 0 & B_{c_2} \end{bmatrix},$$
$$C_c = \begin{bmatrix} C_{c_1} & C_{c_2} \end{bmatrix}, \quad D_c = \begin{bmatrix} D_{c_1} & D_{c_2} \end{bmatrix}. \qquad \blacksquare$$

The FDI problem will be stated referring to a discrete time-invariant linear system ruled by

$$\begin{aligned} x(t+1) &= A_d\, x(t) + B_d\, m(t), \\ y(t) &= C_d\, x(t), \end{aligned}$$

with state $x \in \mathbb{R}^n$, failure mode input $m \in \mathbb{R}^q$, output $y \in \mathbb{R}^p$. The matrices B_d and C_d are assumed to be of full rank. The matrix A_d is assumed to be stable. As in the previous discussion on non-interacting controls, also in the fault detection and identification setting only the fundamental problem will be considered. Therefore, the partition of the failure mode inputs consists of two blocks, (m_1, m_2). The corresponding submatrices of B_d will be denoted by B_{d_1} and B_{d_2}.

Definition 2 *An observer Σ_o, ruled by*

$$\begin{aligned} z(t+1) &= A_o\, z(t) + B_o\, y(t), \\ o(t) &= C_o\, z(t) + D_o\, y(t), \end{aligned}$$

with the output partition (o_1, o_2), such that, starting at the zero state, a non-zero signal at a single system input m_i, $i = 1, 2$, with the other input identically zero, causes the corresponding output o_i to change within the time when the other output is zero will be called a conditioned observer with respect to the system input partition (m_1, m_2).

The extended conditions for the solution of the FDI problem will be proven by exploiting duality with non-interaction. To this aim, a system ruled by (1) is introduced, with $A = A_d^{\mathrm{T}}$, $B = C_d^{\mathrm{T}}$, $C = B_d^{\mathrm{T}}$, and the output partition (y_1, y_2) corresponding to $C_1 = B_{d_1}^{\mathrm{T}}$, $C_2 = B_{d_2}^{\mathrm{T}}$. System (1) with matrices satisfying the above conditions will be also referred to as the primal system.

Theorem 2 *A conditioned fault observer with respect to the system input partition (m_1, m_2) exists if the primal system satisfies conditions (6).*

Proof: Assume that the primal system satisfies conditions (6), then a non-interacting controller Σ_c can be designed according to the procedure illustrated in the “if” part of the proof of Theorem 1. Let r_{11} and r_{22} respectively denote the number of steps for computing $\mathcal{S}_2^* \cap \mathcal{C}_2$ and $\mathcal{S}_1^* \cap \mathcal{C}_1$ by means of Algorithm (4). Let r_{12} and r_{21} respectively denote the minimum number of steps for input h_1 to affect output y_2 and input h_2 to affect output y_1. The compensator Σ_c guarantees that

$$r_{11} < r_{12}, \quad r_{22} < r_{21}. \tag{8}$$

Consider a fault observer Σ_o, defined by duality with respect to Σ_c, i.e. $A_o = A_c^{\mathrm{T}}$, $B_o = C_c^{\mathrm{T}}$, $C_o = B_c^{\mathrm{T}}$, and $D_o = D_c^{\mathrm{T}}$, so that the corresponding observer units are defined by $A_{o_i} = A_{c_i}^{\mathrm{T}}$, $B_{o_i} = C_{c_i}^{\mathrm{T}}$, $C_{o_i} = B_{c_i}^{\mathrm{T}}$, and $D_{o_i} = D_{c_i}^{\mathrm{T}}$, for $i = 1, 2$. In the dual setting, conditions (8) respectively mean that input m_2 does not affect output o_1 before m_1 and that input m_1 does not affect output o_2 before m_2. However, in order to solve the conditioned FDI problem by means of a simple logic comparing the outputs of the observer units within the time when they are significant, the conditions

$$r_{11} < r_{21}, \quad r_{22} < r_{12}, \tag{9}$$

must be satisfied. In fact, conditions (9) respectively mean that input m_1 does not affect output o_2 before o_1 and that input m_2 does not affect output o_1 before o_2. Under these conditions, starting from an initial situation where both the observer unit outputs are zero, a simple inspection of the observer unit outputs within a receding time window whose length is $r = \max\{r_{11}, r_{22}\}$ enables the fault to be detected and isolated. Conditions of the type in (9) can always be derived from (8) by eventually modifying one of the precompensation units by inserting a cascade of a suitable number of unit delays at its input. For instance, assume that (8) hold with $r_{11} = r_{21}$. Then, by inserting a cascade of $\delta = r_{12} - r_{21}$ unit delays at input h_2, the new coefficients $r'_{22} = r_{22} + \delta$ and $r'_{21} = r_{21} + \delta = r_{12}$ satisfy the conditions $r_{11} < r_{12} = r'_{21}$ and $r'_{22} < r'_{21} = r_{12}$. Consequently, the observer obtained by duality with the precompensator including the cascade of delays solves the conditioned FDI problem. $\blacksquare$

Clearly, the proposed solution does not deal with simultaneous faults.

4 Mixed standard and extended conditions

As far as the non-interacting control problem is concerned, a common situation is that where the standard conditions are satisfied for some block outputs, while the extended conditions only are satisfied for the others. Clearly, whenever the standard conditions hold, exploiting the property of the generic i-th reachable set $\mathcal{R}_{\mathcal{V}_i^*}$ on $\mathcal{V}_i^*$ is the convenient choice. In fact, $\mathcal{R}_{\mathcal{V}_i^*}$ is the maximum set reachable from the origin along trajectories which belong to $\mathcal{V}_i^*$ and can be maintained on $\mathcal{R}_{\mathcal{V}_i^*}$ by means of a suitable control action. The procedure herein presented for designing the precompensation unit when the standard condition holds is different from both the one presented by Basile and Marro [1] and the one proposed by Massoumnia in the dual setting [5, 6], while it is consistent with the one presented in the previous section. In fact, when the standard condition holds, the procedure herein presented yields a precompensation unit consisting of a finite impulse response system and a dynamic unit connected as shown in Fig.5, while the precompensation unit reduces to the finite impulse response system only, when the sole extended condition holds.

The design of precompensation unit Σ_{c_1} is now illustrated under assumption $C_1\,(\mathcal{S}_2^* \cap \mathcal{V}_2^*) = \operatorname{im} C_1$. The precompensator Σ_{c_1}, ruled by (5), should be such that, in the presence of a

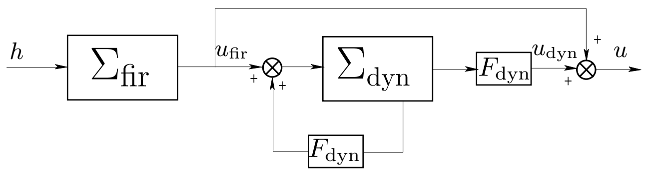

Figure 5: Block diagram of the precompensation unit when the standard condition holds.

non-zero signal at its input h_1, it produces a control action u_1 forcing the system state, initially in the origin, to reach the subsequent subspaces generated by Algorithm (3) for $\mathcal{S}_2^* \cap \mathcal{V}_2^*$, within the first ρ_2 steps, where ρ_2 is equal to the number of steps for the algorithm of $\mathcal{S}_2^* \cap \mathcal{V}_2^*$ to converge, while from step ρ_2+1 on it should simply generate the control action maintaining the state on $\mathcal{R}_{\mathcal{V}_2^*}$. Thus, at step ρ_2 at most, the output y_1 has changed, while output y_2 will remain identically equal to zero. Let F be any matrix such that $(A+BF)\,\mathcal{V}_2^* \subseteq \mathcal{V}_2^*$ and $\sigma(A+BF)|_{\mathcal{R}_{\mathcal{V}_2^*}}$ is stable, and set $A_F := A+BF$. Let $M_2^{(i)}$, $i=1,\ldots,\rho_2$, now denote the respective bases of the subsequent subspaces generated by Algorithm (3) for $\mathcal{S}_2^* \cap \mathcal{V}_2^*$, i.e.

$$\begin{aligned} &\operatorname{im} M_2^{(1)} = \mathcal{B} \cap \mathcal{V}_2^* = \mathcal{R}_{\mathcal{V}_2^*}^{(1)}, \\ &\operatorname{im} M_2^{(i)} = (A_F \mathcal{R}_{\mathcal{V}_2^*}^{(i-1)} + \mathcal{B}) \cap \mathcal{V}_2^* = \mathcal{R}_{\mathcal{V}_2^*}^{(i)},\ i=2,..,\rho_2, \end{aligned}$$

with $\operatorname{im} M_2^{(\rho_2)} = \mathcal{S}_2^* \cap \mathcal{V}_2^* = \mathcal{R}_{\mathcal{V}_2^*}^*$. Let X_f be defined as $X_f := M_2^{(\rho_2)}$. The synthesis procedure is similar to that presented in Section 3. For any column of X_f, which represents a state belonging to $\mathcal{S}_2^* \cap \mathcal{V}_2^*$, a control sequence of ρ_2 steps, steering the state from the origin to the state represented by the given column, along a trajectory whose intermediate states belong to the subsequent subspaces generated by the algorithm of $\mathcal{S}_2^* \cap \mathcal{V}_2^*$, exists. Let $X(i) = M_2^{(i)}\,\beta(i)$, $i=1,2,\ldots,\rho_2-1$, denote the matrices of the intermediate states at the i-th step, and let $U(i)$, $i=1,2,\ldots,\rho_2-1$, denote the matrices of the corresponding controls. The unknowns $\beta(i)$ and $U(i)$, $i=1,2,\ldots,\rho_2-1$, can be derived by the relations

$$\begin{aligned} &\begin{bmatrix} \beta(i) \\ U(i) \end{bmatrix} = \left[A_F M_2^{(i)}\ B\right]^{\#} X(i+1),\ \ i=1,..,\rho_2-1, \\ &X(\rho_2) = X_f,\ \ U(0) = B^{\#} X(1). \end{aligned}$$

From step ρ_2 on, the precompensator Σ_{c_1} should only generate the control action maintaining the state on $\mathcal{R}_{\mathcal{V}_2^*}$, so it should include also a dynamic unit reproducing the dynamics (arbitrarily assignable) of $\mathcal{R}_{\mathcal{V}_2^*}$. Then, the precompensation unit Σ_{c_1} should be implemented as a finite impulse response system $\Sigma_{\rm fir}$ and another dynamic unit $\Sigma_{\rm dyn}$ connected as shown in Fig.5. The finite impulse response system is defined by

$$u_{\rm fir}(t) = \sum_{\ell=0}^{\rho_2-1} \Phi(\ell) h_1(t-\ell), \quad t=0,1,\ldots,$$

with $\Phi(\ell) = U(\ell)$, $\ell=0,1,\ldots,\rho_2-1$, hence, by the quadruple $(A_{\rm fir}, B_{\rm fir}, C_{\rm fir}, D_{\rm fir})$, whose matrices are related to the gain matrices $\Phi(\ell)$ respectively as $(A_{c_1}, B_{c_1}, C_{c_1}, D_{c_1})$ in (7). The dynamic unit $\Sigma_{\rm dyn}$ should be ruled by

$$\begin{aligned} \eta(t+1) &= A_{\rm dyn}\,\eta(t) + B_{\rm dyn}\,v(t), \\ u_{\rm dyn}(t) &= F_{\rm dyn}\,\eta(t), \end{aligned}$$

where the matrices $(A_{\rm dyn}, B_{\rm dyn}, F_{\rm dyn})$ are derived as described below. Consider the state space basis transformation represented by the matrix $T=[T_1\ T_2]$ with T_1 and T_2 such that $\operatorname{im} T_1 = \mathcal{R}_{\mathcal{V}_2^*}$ and $\operatorname{im} T_2 = \mathcal{R}_{\mathcal{V}_2^*}^{\perp}$, respectively. Set $A_F' := T^{-1} A_F T$, $B' := T^{-1} B$, $F' := FT$, and $\nu := \dim \mathcal{R}_{\mathcal{V}_2^*}$. Thus, in order to reproduce the dynamics of $\mathcal{R}_{\mathcal{V}_2^*}$, the matrices of $\Sigma_{\rm dyn}$ should be defined as $A_{\rm dyn} := A_F'(1:\nu, 1:\nu)$, $B_{\rm dyn} := B'(1:\nu,:)$, and $F_{\rm dyn} := F'(:,1:\nu)$. Finally, the precompensation unit Σ_{c_1} is defined by the quadruple $(A_{c_1}, B_{c_1}, C_{c_1}, D_{c_1})$, with

$$\begin{aligned} &A_{c_1} = \begin{bmatrix} A_{\rm fir} & 0 \\ B_{\rm dyn} C_{\rm fir} & A_{\rm dyn} \end{bmatrix}, \quad B_{c_1} = \begin{bmatrix} B_{\rm fir} \\ B_{\rm dyn} D_{\rm fir} \end{bmatrix}, \\ &C_{c_1} = \begin{bmatrix} C_{\rm fir} & F_{\rm dyn} \end{bmatrix}, \qquad D_{c_1} = D_{\rm fir}. \end{aligned}$$

5 Extension to stabilizable systems by output dynamic feedback

Consider the controlled system (1) and the fundamental non-interacting control problem introduced in Section 3. In this Section, it is shown how the assumption that the matrix A is stable can be removed. If the pair (A,B) is stabilizable and the pair (A,C) is detectable, a dynamic unit Σ_F exists, ruled by

$$\begin{aligned} &z(t+1) = (A+GC)\,z(t) + B\,u(t) - G\,y(t) + v_2(t), \\ &u_F(t) = F\,z(t), \quad y_F(t) = z(t), \end{aligned}$$

which, inserted in the feedback connection shown in Fig.6, guarantees that the corresponding extended system is stable. Let the extended state, input and output be respectively defined as $\hat{x}(t) = \left[x(t)^{\rm T}\ z(t)^{\rm T}\right]^{\rm T}$, $\hat{v}(t) = \left[v_1(t)^{\rm T}\ v_2(t)^{\rm T}\right]^{\rm T}$, and $\hat{y}(t) = \left[y(t)^{\rm T}\ z(t)^{\rm T}\right]^{\rm T}$. Then, the extended system is

$$\begin{aligned} \hat{x}(t+1) &= \hat{A}\,\hat{x}(t) + \hat{B}\,\hat{v}(t), \\ \hat{y}(t) &= \hat{C}\,\hat{x}(t), \end{aligned}$$

with

$$\begin{aligned} &\hat{A} = \begin{bmatrix} A & BF \\ -GC & A+BF+GC \end{bmatrix}, \hat{B} = \begin{bmatrix} B & 0 \\ B & I \end{bmatrix}, \\ &\hat{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}. \end{aligned} \tag{10}$$

Let the matrices $\hat{C}_i$, $i=1,2$, be defined as $\hat{C}_i = \begin{bmatrix} C_i & 0 \\ 0 & I \end{bmatrix}$. Their respective kernels have the same dimensions as those of the matrices C_i, $i=1,2$, in the nonextended state space. More precisely, they can be expressed as $\hat{\mathcal{C}}_i = \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{C}_i \right\}$, $i=1,2$. Then, the subspaces involved in the non-interacting control synthesis procedures have the following properties.

Property 1 *In the extended state space:*

i)
$$\hat{\mathcal{V}}_i^* := \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{V}_i^* \right\}, \quad i = 1, 2, \tag{11}$$

is the maximum $(\hat{A}, \hat{B})$-controlled invariant contained in $\hat{\mathcal{C}}_i$.

ii)
$$\hat{\mathcal{R}}_{\hat{\mathcal{V}}_i^*} := \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{R}_{\mathcal{V}_i^*} \right\}, \quad i = 1, 2, \tag{12}$$

is the reachable set on the corresponding $\hat{\mathcal{V}}_i^*$, i.e. $\hat{\mathcal{R}}_{\hat{\mathcal{V}}_i^*} = \hat{\mathcal{S}}_i^* \cap \hat{\mathcal{V}}_i^*$ with $\hat{\mathcal{S}}_i^* = \min \mathcal{S}(\hat{A}, \hat{\mathcal{C}}_i, \hat{\mathcal{B}})$.

iii)
$$\hat{\mathcal{R}}_{\hat{\mathcal{C}}_i} := \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{R}_{\mathcal{C}_i} \right\}, \quad i = 1, 2, \tag{13}$$

is the maximum set reachable from the origin in a finite number of steps along trajectories belonging to the corresponding $\hat{\mathcal{C}}_i$, i.e. $\hat{\mathcal{R}}_{\hat{\mathcal{C}}_i} = \hat{\mathcal{S}}_i^* \cap \hat{\mathcal{C}}_i$.

Proof: Let W be a basis matrix of the generic i-th subspace $\mathcal{C}_i$, so that $\hat{W} := \begin{bmatrix} W \\ 0 \end{bmatrix}$ is a basis matrix of $\hat{\mathcal{C}}_i$, and let V be a basis matrix of the corresponding i-th subspace $\mathcal{V}_i^*$, so that $\hat{V} := \begin{bmatrix} V \\ 0 \end{bmatrix}$ is a basis matrix of the subspace $\hat{\mathcal{V}}_i^*$ defined by (11), i.e.

$$\hat{\mathcal{C}}_i = \operatorname{im} \hat{W} = \operatorname{im} \begin{bmatrix} W \\ 0 \end{bmatrix},\ \hat{\mathcal{V}}_i^* = \operatorname{im} \hat{V} = \operatorname{im} \begin{bmatrix} V \\ 0 \end{bmatrix}. \tag{14}$$

It will be shown that matrices $\hat{X}$, $\hat{U}$ exist such that

$$\hat{A}\hat{V} = \hat{V}\hat{X} + \hat{B}\hat{U}. \tag{15}$$

In fact, condition (15) is equivalent to $(\hat{A}, \hat{\mathcal{B}})$-controlled invariance of $\hat{\mathcal{V}}_i^*$. Since $\mathcal{V}_i^*$ is an $(A, \mathcal{B})$-controlled invariant, matrices X, U exist such that $AV = VX + BU$. Then, by taking into account (10), it can be easily shown that (15) holds with $\hat{X} = X$ and $\hat{U} = \begin{bmatrix} U \\ -GCV - BU \end{bmatrix}$. Moreover, since $\mathcal{V}_i^*$ is the maximum $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{C}_i$, relations (14) imply that $\hat{\mathcal{V}}_i^*$ is the maximum $(\hat{A}, \hat{\mathcal{B}})$-controlled invariant contained in $\hat{\mathcal{C}}_i$. In order to prove propositions i) and ii), the subspaces $\hat{\mathcal{S}}_i := \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{S}_i^* \right\}$, with $i = 1, 2$, are introduced. Relations (10) and (14) imply $\hat{\mathcal{S}}_i^* \supseteq \hat{\mathcal{S}}_i$. Hence, the second one in (14) implies $\hat{\mathcal{S}}_i^* \cap \hat{\mathcal{V}}_i^* = \hat{\mathcal{S}}_i \cap \hat{\mathcal{V}}_i^* = \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{S}_i^* \cap \mathcal{V}_i^* \right\} = \hat{\mathcal{R}}_{\hat{\mathcal{V}}_i^*}$, with $i = 1, 2$, while the first one in (14) implies $\hat{\mathcal{S}}_i^* \cap \hat{\mathcal{C}}_i = \hat{\mathcal{S}}_i \cap \hat{\mathcal{C}}_i = = \left\{ \begin{bmatrix} x \\ 0 \end{bmatrix},\ x \in \mathcal{S}_i^* \cap \mathcal{C}_i \right\} = \hat{\mathcal{R}}_{\hat{\mathcal{C}}_i}$, with $i = 1, 2$. ■

As a consequence of Property 1, the procedures described in Sections 3 and 4 can be applied to the extended system without modifying the respective orders of the precompensation units. Output feedback can also be used to improve system robustness.

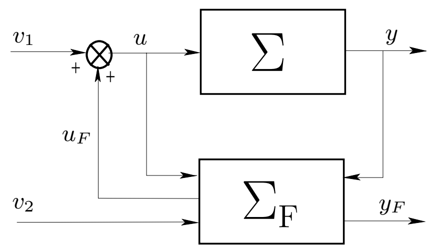

Figure 6: Stabilizing dynamic output feedback.

6 Concluding remarks

A result of a certain relevance in the framework of the geometric approach to FDI has been presented: extended geometric conditions for non-interacting controls have been shown and their impact on the dual setting has been analysed. Discrete-time systems have been considered, since the geometric properties of the subspaces involved are more intuitive and more directly connected with the synthesis procedure than in the continuous-time case. However, their extension to continuous-time systems, although non-trivial, is not prevented and is worth investigation.

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