SLIDING MODE CONTROL WITH ADAPTIVE FUZZY APPROXIMATOR FOR MIMO UNCERTAIN SYSTEMS

N. Manamanni, A. Hamzaoui and N. Essounbouli
Université de Reims Champagne Ardenne
Laboratoire d'Automatique et de Micro-électronique
LAM, Faculté des Sciences - Moulin de la housse BP 1039 51687 Reims Cedex 2.
Tel : + 33326918386, Fax : + 33326913106, noureddine.manamanni@univ-reims.fr ,

Keywords: Sliding mode control, fuzzy logic systems, uncertain systems, MIMO systems, adaptive control

Abstract

This paper deals with the combination of sliding mode control (SMC) and fuzzy logic system (FLS) for a class of non-linear multi-input multi-output (MIMO) uncertain systems. An adaptive scheme for the fuzzy control is developed to approximate the unknown system functions. A SMC is then applied to reduce the effects of both approximation errors and external disturbances. Thanks to Lyapunov's theory, the stability of the closed loop system is demonstrated. Finally, simulation results, concerning the case of a MIMO uncertain system with external disturbances is considered in order to show the efficiency of the combination of the used controllers.

1 Introduction

The tracking trajectory problem, subjected to external perturbations, can be solved essentially by an H_∞ approach or a SMC one. In the case of time variant uncertain systems, an adaptive control is required. Hence, handling this problem by an H_∞ control with fuzzy approximation [3,6] consists in synthesizing an adaptation law that approximate the system, and then adding a control law (deduced from Riccati equation) in view of attenuating the effect of both the approximation error and external disturbances. A drawback of this kind of approach is that it requires the determination of the weighting matrix by the designer, our mean by drawback here, is to find a trade off between the initial values of the control signal and the attenuation level. To avoid this drawback, one can choose the method using SMC. Obviously, the sliding mode control [19,20,2], and the fuzzy logic control [8,18,21] have been widely used and already proved their efficiency in modern control theory for non-linear dynamic systems. Moreover, the combination of sliding mode control (SMC) and fuzzy logic control (FLC) has been recently applied for robust control of non-linear systems [11,1,15]. This is certainly due to the fact that this combination allows to avoid the drawbacks of each controller. Indeed, one of the problems can be the chattering in SMC, which is quite undesirable in some dynamic systems. The other is the difficulty to prove the stability of FLC, whereas the stability of SMC is inherent. Hence, one can take benefit of the two controllers, in one hand, the ability of SMC to decouple high dimensional systems and, therefore, reduce the rule base size of FLC [19] and in other hand, the benefit of a FLC to approximate the uncertain and perturbed model [11,1] and also, the robust property given by the two controllers. It's known that in general, there is a direct trade-off between chattering and robustness.
In the other hand, these two controllers are very similar and complementary [16]. They both use rules, logic rules for FLC and decision rules for the SMC. Note that, for the SMC, the rule called switching function uses the state measure as an input to produce as an output the implemented feedback control.
In [23] Wong used this combination by adding a PI controller, which uses a FLC to eliminate the steady state error. Lin and Chen [10] use Genetic algorithms to mix the SMC and FLC in order to reduce chattering in the system. In Ha et al [4,5], a combination of the two controllers for a linearised system [4] and a class of non linear system [5] is considered. The system robustness is improved by reducing the influence of unmodelled uncertainties and fuzzy tuning is used to move the sliding surfaces. Other works [9,24] have also dealt with the concept of combination of SMC using adaptive law for the FLS. These applications consider simple-input simple-output (SISO) systems, or decoupled MIMO systems [9].
This paper deals with a class of uncertain non-linear MIMO systems whose accurate mathematical model is difficult to formulate or not available and where external disturbances are considered.
The contribution of this work is to use the combination of SMC and adaptive FLC for a particular class of systems, the non decoupled MIMO systems. Hence, one uses, in one hand, the robustness of the SMC, and in the other hand the "intelligence" of the FLC. This method is particularly attractive for non-linear systems since it can result in many cases in invariant control systems, i.e. systems completely insensitive to parametric uncertainties and external disturbances. Thus, this approach improves the tracking performance in the sense that the FLC approximates as closely as possible the model plant and the SMC attenuates the effect due to both the approximation error and the external disturbances.

2 Tracking by Sliding Mode control

Let us consider a MIMO plant described by the following state equations:

$$\begin{cases} X^{(n)} &= F(x)+G(x)\,u+d \\ x &= (X,^T\ \dot{X}^T,\ldots,\left(X^{(n-1)}\right)^T)^T \\ y &= X \end{cases} \tag{1}$$

Where $F \in \Re^n$ and $G \in \Re^{n\times n}$ are supposed to be unknown (uncertain) but with a limited bound, $u \in \Re^n$ are the inputs, $y \in \Re^n$ are the outputs of the system, x is the state vector, which is assumed to be available for measurement and $d \in \Re^n$ represents the unknown but bounded and smooth external disturbances (load, white noise….). In order for the system to be controllable, we require that $det(G(x)) \neq 0$ for x in the operational field of the system. Thus, the above n-degree system is in the normal form.

The control objective is to let y track a reference signal y_r; i.e. in the presence of unknown but bounded perturbations, the tracking error, $e = y - y_r$, should be as small as possible under the constraints that the closed-loop system must be globally stable and robust in the sense that all variables are uniformly bounded. When the system is well known, a sliding mode control can be applied.

The sliding hyperplanes are selected as Hurwitz polynomials of the tracking errors of the associated state. Thus, the sliding surface s (switching line for second order system) is defined in the state space as:

$s(t)=0$ where $s = \Lambda e$ and $\Lambda=[\Lambda_1,\Lambda_2,\ldots\Lambda_{n-1},1]$ guarantees the stability of the systems dynamics on the sliding surface if $\Lambda_i>0$ $I=1,..,n-1$.

Given the initial condition, the problem of tracking, $y=y_r$ is equivalent to that of remaining on the surface $s(t)=0$ for all $t>0$. Then, a sufficient condition of this behavior is to satisfy the attractivity condition $s^T\dot{s} \leq -\eta|s| < 0$, where η is a positive constant given by the designer [17].

A possible choice of the SMC, satisfying the condition above, can be given by [17,13,14] which leads to:

$$u = G_0^{-1}(x)(-F_0(x)+y_r^{(n)}-Ps-\Omega-K\,\mathbf{sgn(s)}) \tag{2}$$

$$sgn(s)=[sgn(s_1),\ldots,sgn(s_m)]^T$$

$$sgn(s_i)=signum(s_i)$$

Where $\Omega=\sum_{i=1}^{n-1}\Lambda_i e^{(i)}$, P is a diagonal positive definite matrix and $K=diag(k_i)$, with $k_i>0$. The k_i express the switching gains which are used to guarantee a sliding regime on the switching surface $s(t)$, see [17] for further details. These gains (k_i) are selected, in the presence of perturbations, in order to maintain the attractivity condition, so in the worst case, given by [7]:

$min(k_i) \geq \left\|F-F_0+\left(G-G_0\right)u\right\|+\|d\|+\eta$, F_0 and G_0 represent the nominal value of F and G respectively. In this section, the system is assumed well known and the perturbations bounded, so by choosing a Lyapunov function as $V_1=\frac{1}{2}s^Ts$, one can easily prove that the system remains stable $(\dot{V}_1<0)$ when the condition $k_i \geq |d|+\eta>0$ is satisfied. This condition guarantees a finite time convergence to the sliding surface.

The controller given in (2) will have chattering phenomena near the sliding surface due to the *sgn* function involved. This can be avoided [16] by introducing a boundary layer with width Φ. Thus replacing $sgn(s)$ by $sat(s/\Phi)$, gives:

$$u = G^{-1}(x)(-F(x)+y_r^{(n)}-Ps-\Omega-Ksat(s/\phi)) \tag{3}$$

where,

$$sat(\varphi)=\begin{cases} sgn(\varphi), & if\ |\varphi|>1 \\ \varphi, & if\ |\varphi|<1 \end{cases}$$

Moreover, the system may be subject to parameter variations and external disturbances. Consequently the control given in (2) may be sensitive to uncertainties during the reaching phase, since F and G are not known. Therefore, F and G should be approximated, to maintain the same structure of the controller. To solve this problem, we propose an adaptive scheme using fuzzy logic system.

3 Fuzzy adaptive law and stability analysis

3.1 Fuzzy logic system (FLS)

An FLS consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine manipulating fuzzy rules, and the defuzzifier [8]. The knowledge base for the FLS comprises a collection of fuzzy IF-THEN rules. The fuzzifier maps a real point in the input space (measurement of the system's state) to a fuzzy set. In general there are two possible choices of this mapping, namely singleton or non-singleton. In this paper we use the singleton fuzzifier mapping. The fuzzy inference engine performs a mapping from fuzzy sets of the input to fuzzy sets in the output space, based on the fuzzy IF-THEN rules (in the fuzzy rule base) and the compositional rule of inference. The defuzzifier maps fuzzy sets in the output space to a crisp point in this space; in this study we use the center-average defuzzifier mapping [15].

The output of a MIMO-FLS with center-average defuzzifier, product inference, and singleton fuzzifier are of the following form:

$$y_j(x)=\frac{\sum_{p=1}^{M}\bar{y}_j{}^p\left(\prod_{i=1}^{n}\mu_{I_i^p}(x_i)\right)}{\sum_{p=1}^{M}\left(\prod_{i=1}^{n}\mu_{I_i^p}(x_i)\right)}. \qquad j=1,2,\ldots m \tag{4}$$

Where, μ_I is the membership function of the input, $\bar{y}_j^p$ is the point where the memberships function of the output $y=(y_1,\ y_2,\ldots,y_m)$ achieves its maximum value, which is assumed to be 1, M is the number of fuzzy rules, and n and m are the dimensions of the input $x=(x_1,\ x_2,\ldots,x_n)$ and the output vectors, respectively. This FLS can be viewed as a kind of neural network [7] with only three layers as shown in Figure 1.

If the $\mu_I(x_i)$ terms are fixed and the $\bar{y}^p$ terms are viewed as adjustable parameters, thus, a MIMO-FLS can be rewritten as:

$$y = \xi(x)^T \Theta \tag{5}$$

where, Θ is a parameter vector, and $\xi(x)$ the regressive matrix $\xi(x)=(\xi_1(x),\ldots,\xi_M(x))$, which the pth component of the ith regressive vector is given by:

$$\xi_p(x) = \frac{\left(\prod_{i=1}^{n} \mu_{ip}(x_i)\right)}{\sum_{p=1}^{M}\left(\prod_{i=1}^{n} \mu_{ip}(x_i)\right)} \tag{6}$$

This relation provides a justification for applying the FLS to almost any non-linear modeling problem. It also provides an explanation for the practical success of the FLS in engineering applications.

Based on the universal approximator theorem, this adaptive FLS can approximate any continuous function over a compact set to an arbitrary accuracy as demonstrated in [21]. Although $y(x)$ is a non-linear function of x, it is linear in its parameter Θ**.** Therefore, the adaptive fuzzy logic system is relatively easier to construct and to analyze.

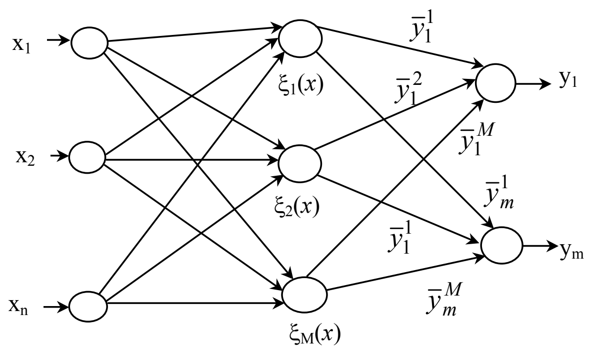

Figure 1: The structure of a MIMO-FLS.

3.2 Controller synthesis

Now, the system functions (F and G) are unknown. Hence, in order to maintain the same control law as presented in equation (2), the unknown functions should be substituted by their fuzzy approximations $\hat{F}(x/\Theta_f)$ and $\hat{G}(x/\Theta_g)$ respectively. Where Θ_f, and Θ_g are the parameter's vector and matrix respectively of the fuzzy logic systems approximating F and G respectively. Thus, the control can be given by:

$$u = \hat{G}^{-1}(x/\Theta_g)[-\hat{F}(x/\Theta_f) + y_r^{(n)} - Ps - \Omega - Ksign(s)] \tag{7}$$

It is known that this control law satisfies the attractivity condition $s^T\dot{s} \le -\eta|s| < 0$ and yields a desirable s dynamics in the presence of uncertainties and perturbations if the following form is used:

$$\dot{s} = -Ps - \Omega - Ksgn(s) + (F-\hat{F}) + (G-\hat{G})u + d \tag{8}$$

since $\hat{F}(x/\Theta_f) = \Theta_f^T\xi(x)$ and $\hat{G}(x/\Theta_g) = \Theta_g^T\xi(x)$, and after some straightforward manipulations we obtain the dynamic error equation in the matrix form:

$$\dot{s} = -Ps - \Omega - Ksgn(s) + \xi^T(x)\tilde{\Theta}_f + \xi^T(x)\tilde{\Theta}_g u + W \tag{9}$$

where, $\tilde{\Theta} = \Theta\text{-}\Theta^*$, and Θ^* is the optimal parameter [21], and,

$$W = (F^*(x/\Theta_f^*) - F(x)) + (G^*(x/\Theta_g^*) - G(x))u + d \tag{10}$$

is the sum of the external disturbances and the error due to the fuzzy approximation of $F(x)$ and $G(x)$. We note that the optimal values are only used for the mathematical tools, and are not needed for control implementation.

The adaptive FLS depicted in Figure 2 can approximate these functions as closely as possible.

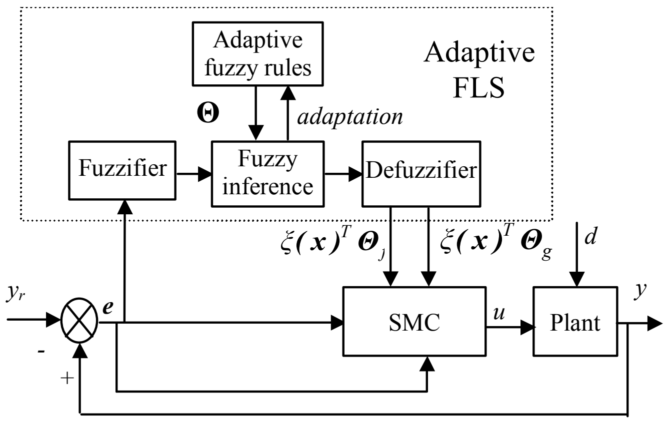

Figure 2: Adaptive fuzzy controller achieving SMC tracking.

3.3 Adaptation law and stability analysis

The control law generated must insure not only the convergence toward zero of the tracking error, but also the boundedness of all the involved variables of the closed-loop system. Let's choose the following Lyapunov function:

$$V = \frac{1}{2}s^Ts + \frac{1}{2\gamma_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2\gamma_2}tr(\tilde{\theta}_g^T\tilde{\theta}_g)$$

where γ_1 and γ_2 are the learning parameters, and tr(.) is the trace of the function (.).

The derivative of the Lyapunov equation yields to:

$$\dot{V} = s^T\dot{s} + \frac{1}{2\gamma_1}\left[\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \tilde{\theta}_f^T\dot{\tilde{\theta}}_f\right] + \frac{1}{2\gamma_2}tr\left[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g + \tilde{\theta}_g^T\dot{\tilde{\theta}}_g\right]$$

Then, substituting equation (9) and since θ^* is constant, one can write $\dot{\tilde{\theta}} = \dot{\theta}$, leading to:

$$\dot{V} = s^T\left[-Ps - \Omega - K\,sgn(s) + B[(F-\hat{F}) + (G-\hat{G})u + d]\right]$$
$$+ \frac{1}{2\gamma_1}\left[\dot{\theta}_f^T\tilde{\theta}_f + \tilde{\theta}_f^T\dot{\theta}_f\right] + \frac{1}{2\gamma_2}tr\left[\dot{\theta}_g^T\tilde{\theta}_g + \tilde{\theta}_g^T\dot{\theta}_g\right]$$

which can be written as,

$$\dot{V} = -s^T Ps - s^T \Omega - s^T K\, sgn(s) + s^T Bw +$$
$$s^T (\xi^T \tilde{\theta}_f + \xi^T \tilde{\theta}_g u) + \frac{1}{\gamma_1} \tilde{\theta}_f^T \dot{\theta}_f^T + \frac{1}{\gamma_2} tr(\tilde{\theta}_g^T \dot{\theta}_g)$$

In order to ensure the system stability, the following adaptation laws are considered:

$$\dot{\theta}_f = -\gamma_1 s \xi(x)$$
$$\dot{\theta}_{g_{ij}} = -\gamma_2 s_i^T \xi_{gi}^T(x) u_j \qquad i=1,\dots,n;\ j=1,\dots,m.$$

So, since, the function F and G, and the external disturbances are assumed to have well-known bounds, and in order to satisfy the sliding condition, one can choose $min(k_i) \geq \|w\| + \eta$, which implies that $\dot{V} \leq 0$ [7]. Hence, we can deduce that the system remains stable and that the tracking error converges to zero in a finite time.

4 Results

The proposed method is applied for the position tracking control of a two-link robot manipulator driven by DC motors. We will show that the proposed control law can enhance the performance of tracking.
The system parameters are as follows: link masses $m_1 = 5$ *kg*, $m_2 = 2.5$ *kg*, lengths $l_1 = 0.5$ *m*, $l_2 = 0.5$ *m*. The dynamic model is described by the following equation:

$$M'(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Gamma(q) + d' \qquad (11)$$

where, $q, \dot{q}, \ddot{q}$ are the angular position, velocity, and acceleration vectors respectively.
$M'(q)$ is the inertia matrix, which is symmetric positive definite and its inverse exists. The matrix $\dot{M}'(q) - 2C'(q)$ is skew, and the dynamic structure is linear in terms of suitable selected set of robot and load parameters.

$$M'(q) = \begin{bmatrix} (m_1 + m_2) l_1^2 & m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) & m_2 l_2^2 \end{bmatrix} \qquad (12)$$

where: $s_1 = sin(q_1)$; $s_2 = sin(q_2)$; $c_1 = cos(q_1)$; $c_2 = cos(q_2)$.
$C(q,\dot{q})$ is the vector of the centripetal and Coriolis forces:

$$C(q,\dot{q}) = m_2 l_1 l_2 (c_1 s_2 - s_1 c_2) \begin{bmatrix} 0 & -\dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix} \qquad (13)$$

$G_r(q)$ is the vector of gravitational force:

$$G_r(q) = \begin{bmatrix} -(m_1 + m_2) l_1 g\, s_1 \\ -m_2 l_2 g\, s_2 \end{bmatrix} \qquad (14)$$

$d' \in R^2$ represents the structural and unstructural disturbances, and $\Gamma(q)$ is the vector of torque developed at the joint side of the gearbox.
The relation between the joint position q and the motor-shaft position q_m is given by $q_m = N q$, where N is the 2×2 diagonal positive matrix of the gear ratios for the two joints.
The electrical equation of a DC motor can be written as:

$$RI + L\frac{dI}{dt} + K_b \frac{dq_m}{dt} = U \qquad (15)$$

Furthermore,

$$\Gamma(q) = NK_t I \qquad (16)$$

Substituting (16) into (15) we obtain:

$$R_n \Gamma + L_n \dot{\Gamma} + K_{bn} \frac{dq}{dt} = U \qquad (17)$$

where $K_{bn} = K_b N$, $R_n = R/NK_t$, and $L_n = L/NK_t$.
Substituting (11) into (17) after some rearrangements we obtain:

$$M(q) q^{(3)} + D(q,\dot{q},\ddot{q}) = U + R_n d' + L_n \dot{d}' \qquad (18)$$

where, $M(q) = L_n M'(q)$ and

$$D(q,\dot{q},\ddot{q}) = \left[R_n M'(q) + L_n \left(\dot{M}'(q) + C(q,\dot{q})\right)\right]\ddot{q} + R_n G_r(q) + L_n \dot{G}_r(\dot{q}) + \left[R_n C(q,\dot{q}) + L_n \dot{C}(q,\dot{q}) + K_{bn}\right]\dot{q} \qquad (19)$$

By introducing $x^T = [x_1 \ \ x_2 \ \ x_3]^T = [q \ \ \dot{q} \ \ \ddot{q}]^T$ as the state vector, equation (18) can be written as:

$$q^{(3)} = -M^{-1}(x) D(x) + M^{-1}(x) U + d \qquad (20)$$

where: $d = M^{-1}\left(R_n d' + L_n \dot{d}'\right)$

This equation (20), which represents typical dynamics of a non-linear system, is similar to (1).
Our objective now is to determine a control U to obtain a tracking error $e = q - q_d$ as small as possible, where q_d is the desired trajectory to be followed by the robot manipulator. Next, The designed procedure is applied to this plant.
In order to approximate the system, the inputs of the correspondent fuzzy system are first defined and then, their fuzzy sets. Seven gaussian membership functions are defined as follows:
$\mu_1(x_1) = 1/(1+exp(15\,(x_1+1.6)))$; $\mu_2(x_1) = exp(-2.5(x_1+1.3)^2)$
$\mu_3(x_1) = exp(-2.5(x_1+0.65)^2)$; $\mu_4(x_1) = exp(-2.5\,(x_1)^2)$;
$\mu_5(x_1) = exp(-2.5(x_1-.65)^2)$; $\mu_6(x_1) = exp(-2.5\,(x_1-1.3)^2)$;
$\mu_7(x_1) = 1/(1+exp(-15(x_1-1.6)))$.
To adjust the parameters, we choose: $\gamma_1 = 10$ and $\gamma_2 = 0.01$. The external disturbances are chosen in the following form: $d(t) = 0.1sin(2t)$. Initially the robot is supposed in rest, at the two link positions, $q_1 = \pi/4$ and $q_2 = \pi/4$. The initial positions are chosen so far to prove the efficiency of the approach.
The results of simulation of the two links robot using the control law (7) will be presented to illustrate the performances of the combination of the SMC and the FLS used as approximator for a MIMO uncertain system. Results using the *sign* function in the control law are presented in figures 3 and 4. Figure 3 (a) and (b) illustrate the simulated and the desired trajectory for the two links and their tracking errors (c) and (d) respectively. The obtained steady state error is more less than with the one given by the control law (2); this is foreseeable since the FLS acts mainly on the structural uncertainties. Despite external disturbances, the system remains stable and robust with a good time response and a weak-tracking error. Nevertheless, the chattering phenomenon persists (figure 4), it results in low control accuracy and high heat loss in electrical power circuits is inevitable. It may also excite unmodelled high frequency dynamics, which degrades the performances of the system and may even lead to instability. The phase plane depicted in figure 4 ((a) and (b)) gives the convergence of the system but unfortunately shows also the induced chattering. In order

to avoid this undesirable phenomenon, the function *sat* is used instead of the *sign* function and the control law becomes

$$u = \hat{G}^{-1}(x/\Theta_g)\left[-\hat{F}(x/\Theta_f) + y_r^{(n)} - Ps - \Omega - Ksat(s/\phi)\right].$$

Figure 5 and 6 depict the system's states behavior during the tracking and their correspondent errors. In this case, the velocities and accelerations signals are smooth and their errors converge towards zero. These results reveal the robustness of the implemented controllers and the efficiency of the combination of SMC and FLS. Figure 7 (a) and (b) show the acceleration errors, their great values at the beginning are due to the initial chosen positions. Figure 7 (c) depicts the desired and the obtained phase plane of all the states (position, velocity and acceleration), we see that the proposed control scheme results in satisfactory tracking performances. Figure 7 (d) illustrates the quadratic integral error, it constant value during the steady state implies that all the variables involved are bounded. The chattering has been greatly attenuated as shown in figure 8 (a) and (b), and the applied voltage depicted in figure 8 (c) and (d) shows a smooth behavior for the signal during the steady state however in the transient time the solicitations are more important.

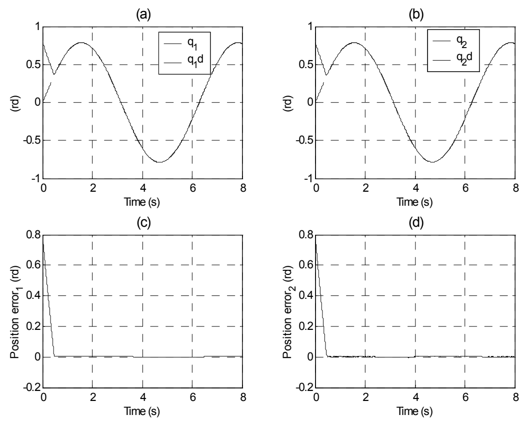

Figure 3: Simulated and desired angular position for the first link (a) the second link (b) and their respective tracking errors (c) and (d) when using *sign* function.

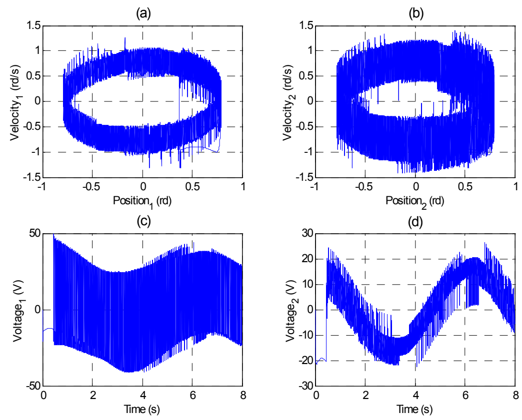

Figure 4: The phase plan (a), (b) and the applied voltage (c), (d) .

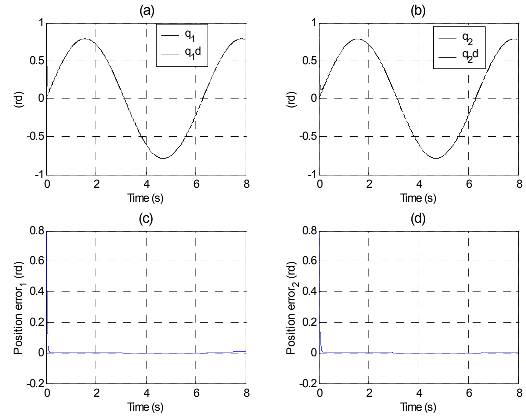

Figure 5: Simulated and desired angular position for the first link (a) the second link (b) and their respective tracking errors (c) and (d) when using *sat* function.

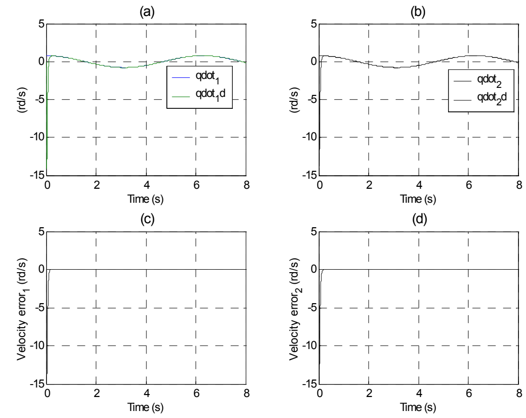

Figure 6: Simulated and desired angular velocities (a), (b) and their tracking errors (c) and (d) when using *sat* function

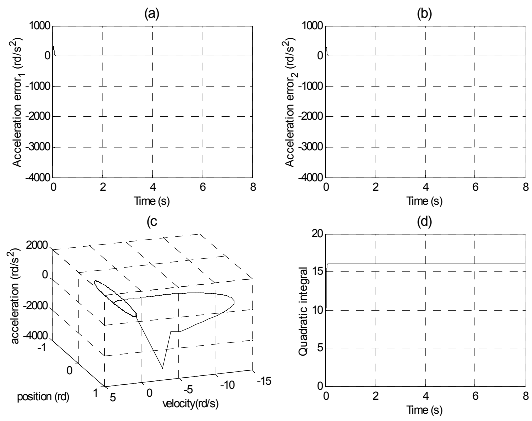

Figure 7: Accelerations errors for each robot axe (a) and (b), the simulated and desired phase plan of all the state vector (c) and quadratic integral error (d)

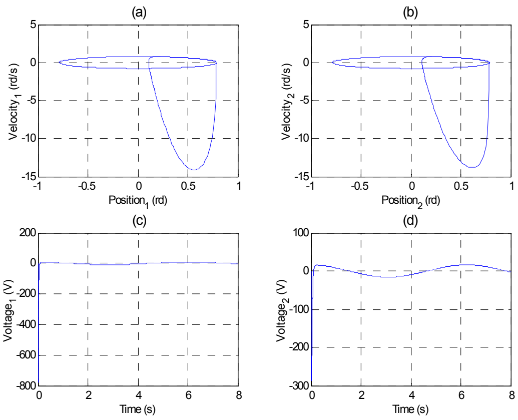

Figure 8: The phase planes (a), (b) and the applied voltage using *sat* function(c), (d).

5 Conclusion

In this paper, a combination of sliding mode control and fuzzy logic systems for the tracking trajectory problem is proposed. This approach is applied for a class of non-linear MIMO systems involving plant uncertainties and external disturbances. The tracking performances are greatly improved by the use of both the SMC and FLS. Indeed, the FLS approximates as closely as possible the model plant and the SMC attenuates the effect due to both the approximation error and the external disturbances. In this paper replacing the *sign* function by a *sat* one has reduced the chattering phenomenon. However, in this case, the boundary layer width (Φ) is chosen with a trade off between robustness and high control signal variation. To improve this choice, the authors intend, in their future work to use a fuzzy adaptation law for the chosen sliding surface.

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