ANTI-WINDUP STRATEGY FOR LINEAR SYSTEMS WITH AMPLITUDE AND DYNAMICS RESTRICTED ACTUATOR

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Abstract

This paper addresses the problem of the determination of regions of stability for linear systems with amplitude and successive dynamics restricted actuator through anti-windup strategies. The objective by designing anti-windup gains is to guarantee the stability of the closed-loop system and the respect of the controlled output constraints for a region of admissible initial states as large as possible. Based on the modeling of the closed-loop system resulting from the controller plus the anti-windup loop as a linear system with a dead-zone and dynamics restricted nonlinearities, constructive stability conditions are formulated by using quadratic and Lure approach associated with the Finsler's lemma. Numerical procedures are discussed.

1 Introduction

Physical, safety or technological constraints generally induce that the control actuators cannot provide unlimited amplitude signals neither unlimited speed of reaction. That means that the control systems are generally subject to amplitude and dynamics actuator saturations. The control problems of combat aircraft prototypes and launchers offer interesting examples of the difficulties due to these major constraints. Neglecting both amplitude and dynamics actuator limitations can be source of undesirable even catastrophic behaviors for the closed-loop system (as the lost of the closed-loop stability) [4]. For these reasons, the study of the control problem or analysis stability problem with respect to systems subject to both amplitude and rate actuator saturations has received the attention of many researchers in the last years (see, for example, [20], [9], [11]).

The anti-windup fits the approach consisting in taking into account the effect of saturations in a second step after a previous design performed disregarding the saturation terms. The objective then consists in introducing control modifications in order to recover, as much as possible, the performance induced by a previous design carried out on the basis of the unsaturated system. In particular, anti-windup schemes have been successfully applied in order to avoid or minimize the windup of the integral action in PID controllers, largely applied in the industry. In this case, most of the related literature focuses on the performance improvement in the sense of avoiding large and oscillatory transient responses (see, among others, [2], [1], [7]).

More recently, a special attention has been paid to the influence of the anti-windup schemes in the stability and the performances of the closed-loop system (see, for example, [3], [5], [12], [14], [15], [17], [19]). Several results on the anti-windup problem are concerned with achieving global stability properties. Since global results cannot be achieved for open-loop unstable linear systems in the presence of actuator saturation, local results have to be developed. In this context, a key issue concerns the determination of domains of stability for the closed-loop system. Most of the local results available in the anti-windup literature do not provide explicit characterization of the domain of stability. It is worth to notice that the basin of attraction is modified by the anti-windup loop. If the resulting basin of attraction is not sufficiently large, the system can present a divergent behavior depending on its initialization and the action of disturbances.

In this paper, we consider the structure of the observer-based anti-windup [2, 1]. With respect to this structure, we can cite [12] in which passivity arguments are invoked or still [18] in which a local $\mathcal{H}_\infty$ design is provided in terms of matrix inequalities. More recently, in [6], some constructive conditions are proposed both to determine suitable anti-windup gains and to quantify the closed-loop region of stability in the case of amplitude saturation actuator. Differently from the papers cited above, in this paper we focus our attention on linear systems with amplitude and successive dynamics restricted actuator and bounded controlled outputs. Our aim is the characterization of stability regions for this class of systems through anti-windup strategies. Especially, we are interested in the anti-windup gains design in order to ensure the closed-loop stability for regions of admissible initial states as large as possible. Based on the modeling of the closed-loop system resulting from the controller plus the anti-windup loop as a linear system with a dead-zone and dynamics restricted nonlinearities, constructive stability conditions are formulated by using quadratic and Lure approaches associated with the Finsler's lemma. Numerical procedures based on the solution of some iterative convex optimization problems with LMI constraints are proposed for computing the anti-windup gains that lead to the maximization of the size of the associated region of stability. At our knowledge, the current paper constitutes the first study with respect to the considered class of systems.

Notations. For any vector $x \in \Re^n$, $x \succeq 0$ means that all the components of x, denoted $x_{(i)}$, are nonnegative. For two vectors x, y of $\Re^n$, the notation $x \succeq y$ means that $x_{(i)} - y_{(i)} \geq 0, \forall i = 1,\ldots,n$. $\mathbf{1}$ and $\mathbf{0}$ denote respectively the identity matrix and the null matrix of appropriate dimensions. The elements of a matrix $A \in \Re^{m\times n}$ are denoted by $A_{(i,j)}$, $i = 1,\ldots,m$, $j = 1,\ldots,n$. $A_{(i)}$ denotes the ith row of matrix A. $|A|$ is the matrix constituted from the absolute value of each element of A. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A. $1_m \triangleq [1\ldots1]' \in \Re^m$. For any vector u of $\Re^m$ one defines each component of $sat_{u_0}(u)$ by $sat_{u_0}(u_{(i)}) = sign(u_{(i)})min(u_{0(i)}, |u_{(i)}|)$, $i = 1,\ldots,m$.

2 Problem Statement

In this paper, we consider a class of nonlinear systems which are obtained by cascading linear systems with actuator containing some nonlinearities of saturation type. The actuator under consideration is a dynamic system containing amplitude and dynamics restrictions, that is, it is described via successive time-derivatives of the input of the plant. By setting $x_a = \begin{bmatrix} u' & \dot{u}' & \ldots & u^{(q-1)\prime} \end{bmatrix}' \in \Re^{mq}$ and $y_a = sat_{u_0}(u) \in \Re^m$ where $u^{(q)}$ denotes the q-order time-derivative of u, the model of the actuator reads as follows:

$$\left\{ \begin{array}{l} \dot{x}_a(t) = A_a x_a(t) + B_{a0} sat_{u_0}(C_a x_a(t)) \\ +\sum_{j=1}^{q-1} B_{aj} sat_{u_j}(sat_{u_0}(C_a x_a(t))^{(j)}) + B_{aq} y_c(t) \\ y_a(t) = sat_{u_0}(C_a x_a(t)) \end{array} \right. \quad (1)$$

where x_a is the state of the actuator, y_a is the measured output of the actuator and $y_c \in \Re^{n_{cp}}$ is the output of the controller. Matrices A_a, B_{aj}, $j = 0,\ldots,q$, and C_a are defined by:

$$A_a = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & & & \ddots & & \vdots \\ \vdots & & \ldots & & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \ldots & & \ldots & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \ldots & & \ldots & \mathbf{0} & \mathbf{0} \end{bmatrix} \in \Re^{mq\times mq} \quad (2)$$
$$B_{aj} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \ldots & \ldots & \mathbf{0} & T_j' \end{bmatrix}' \in \Re^{mq\times m}$$
$$C_a = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \ldots & \ldots & \mathbf{0} & \mathbf{0} \end{bmatrix} \in \Re^{m\times mq}$$

Such a model is the type of actuator encountered in the control of launchers (see [16] in which $m = 1$ and $q = 2$). In (1), the positive vectors u_0 and u_j, $j = 1,\ldots,q-1$, may be viewed as bounds on the position and the successive dynamics of the actuator state. Thus, it clearly appears that one cannot have simultaneously position and dynamics saturation.

The plant is a linear continuous-time system defined as:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + By_a(t) \\ y(t) & = & Cx(t) \\ z(t) & = & C_2 x(t) \end{array} \right. \quad (3)$$

where $x \in \Re^n$, $y(t) \in \Re^p$ and $z(t) \in \Re^l$ are the state, the measured output and the controlled output vectors, respectively. Matrices A, B, C and C_2 are real constant matrices of appropriate dimensions. $y_a \in \Re^m$ is both the output of the actuator and the input of the plant.

Without saturation terms, that is with $sat_{u_0}(C_a x_a) = C_a x_a = u$ and $sat_{u_j}(sat_{u_0}(C_a x_a(t))^{(j)}) = C_a x_a^{(j)} = u^{(j)}$, $j = 1,\ldots,q-1$, system (1)-(3) is linear and reads:

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ \dot{x}_a(t) = (A_a + \begin{bmatrix} B_{a0} & \ldots & B_{aq-1} \end{bmatrix}) x_a(t) + B_{aq} y_c(t) \\ y(t) = Cx(t) \\ z(t) = C_2 x(t) \end{array} \right. \quad (4)$$

Under the (A,B)-controllability and (C,A)-observability assumptions, we assume that an n_c-order dynamic output stabilizing controller has been determined to stabilize the linear system (4) and is described as follows:

$$\left\{ \begin{array}{rcl} \dot{\eta}(t) & = & A_c \eta(t) + B_c y(t) \\ y_c(t) & = & C_c \eta(t) + D_c y(t) \end{array} \right. \quad (5)$$

where $\eta(t) \in \Re^{n_c}$ is the controller state, $u_c(t) = y(t)$ is the controller input and $y_c(t) \in \Re^{n_{cp}}$ is the controller output.

Furthermore, due to the presence of the saturation terms, in order to mitigate the undesirable effects of windup, caused by input saturation (due to y_a measurable variable of the actuator), an anti-windup term $(sat_{u_0}(C_a x_a(t)) - C_a x_a(t))$ can be added to the controller [14] through adequate gain. Thus, considering the dynamic controller and this anti-windup strategy, the closed-loop system reads:

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bsat_{u_0}(C_a x_a) \\ \dot{x}_a(t) = A_a x_a(t) + B_{a0} sat_{u_0}(C_a x_a(t)) \\ +\sum_{j=1}^{q-1} B_{aj} sat_{u_j}(sat_{u_0}(C_a x_a(t))^{(j)}) + B_{aq} y_c(t) \\ \dot{\eta}(t) = A_c \eta(t) + B_c y(t) + E_c(sat_{u_0}(C_a x_a) - C_a x_a(t)) \\ y(t) = Cx(t) \\ y_c(t) = C_c \eta(t) + D_c y(t) + F_c(sat_{u_0}(C_a x_a) - C_a x_a(t)) \\ z(t) = C_2 x(t) \end{array} \right. \quad (6)$$

where E_c and F_c are the two anti-windup gains to be determined. It is worth noticing in system (6) that if the ith component of the amplitude saturation is effective (i.e., $|C_{a(i)} x_a| > u_{0(i)}$) then the corresponding component of the successive dynamics saturation does not affect the system (i.e., $sat_{u_j}(sat_{u_0}(C_{a(i)} x_a(t))^{(j)}) = 0$, $j = 1,\ldots,q-1$).

Problem 1 *Determine anti-windup gains E_c and F_c, and a set S_0 such that:*
1. The asymptotic stability of the closed-loop system (6) is ensured for any $\begin{bmatrix} x(0)' & x_a(0)' & \eta(0)' \end{bmatrix}' \in S_0$, where S_0 is as large as possible.
2. For any $\begin{bmatrix} x(0)' & x_a(0)' & \eta(0)' \end{bmatrix}' \in S_0$ the measured output z takes values in the set $\mathcal{Z}_0$ defined by:

$$\mathcal{Z}_0 = \{z \in \Re^l; -z_0 \preceq z \preceq z_0,\ z_{0(i)} > 0, i = 1,\ldots,l\} \quad (7)$$

The implicit objective in Problem 1 is to compute E_c and F_c for enlarging the basin of attraction of the closed-loop system.

3 Preliminaries

Let us first define the q nonlinearities ϕ_0 and ϕ_j, $j=1,\dots,q-1$:

$$\phi_0(C_a x_a(t)) = y_a(t) - C_a x_a(t) = sat_{u_0}(C_a x_a(t)) - C_a x_a(t) \quad (8)$$

$$\begin{array}{rcl}\phi_j(C_a x_a(t)) & = & sat_{u_j}(y_a^{(j)}(t)) - y_a^{(j)}(t) \\ & = & sat_{u_j}(sat_{u_0}(C_a x_a(t))^{(j)}) - sat_{u_0}(C_a x_a(t))^{(j)}\end{array} \quad (9)$$

From the definition of ϕ_0, one gets

$$sat_{u_0}(C_a x_a(t))^{(j)} = \phi_0^{(j)}(C_a x_a(t)) + C_a x_a^{(j)}(t) \quad (10)$$

Therefore, the system (6) can be written in a compact form. For this, define the extended state vector

$$\xi(t) = \left[\begin{array}{ccc} x(t)' & x_a(t)' & \eta(t)' \end{array}\right]' \in \Re^{n+mq+n_c} \quad (11)$$

and the following matrices of appropriate dimensions

$$\begin{array}{l}\mathbb{A} = \left[\begin{array}{ccc} A & BC_a & \mathbf{0} \\ B_{aq}D_c C & (A_a + \left[\begin{array}{ccc} B_{a0} & \dots & B_{aq-1}\end{array}\right]) & B_{aq}C_c \\ B_c C & \mathbf{0} & A_c \end{array}\right] \\ \mathbb{B}_0 = \left[\begin{array}{c} B \\ B_{a0} \\ \mathbf{0}\end{array}\right]; \mathbb{B}_j = \left[\begin{array}{c} \mathbf{0} \\ B_{aj} \\ \mathbf{0}\end{array}\right]; \mathbb{B}_q = \left[\begin{array}{c} \mathbf{0} \\ B_{aq} \\ \mathbf{0}\end{array}\right] \\ \mathbb{R} = \left[\begin{array}{c} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1}\end{array}\right]; \mathbb{K} = \left[\begin{array}{ccc} \mathbf{0} & C_a & \mathbf{0}\end{array}\right]; \mathbb{C}_2 = \left[\begin{array}{ccc} C_2 & \mathbf{0} & \mathbf{0}\end{array}\right]\end{array} \quad (12)$$

for $j=1,\dots,q-1$. Thus, the closed-loop system reads:

$$\left\{\begin{array}{rcl}\dot{\xi}(t) & = & \mathbb{A}\xi(t) + (\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_q F_c)\phi_0(\mathbb{K}\xi(t)) \\ & & + \sum_{j=1}^{q-1}\mathbb{B}_j(\phi_j(\mathbb{K}\xi(t)) + \phi_0^{(j)}(\mathbb{K}\xi(t))) \\ z(t) & = & \mathbb{C}_2\xi(t)\end{array}\right. \quad (13)$$

In the sequel for simplicity, $\phi_0(\mathbb{K}\xi(t))$, $\phi_j(\mathbb{K}\xi(t))$ and $\phi_0^{(j)}(\mathbb{K}\xi(t))$, $j=1,\dots,q-1$ will be denoted ϕ_0, ϕ_j and $\phi_0^{(j)}$. Note that in the absence of saturation one gets $\phi_0=\mathbf{0}$, $\phi_j=\mathbf{0}$ and $\phi_0^{(j)}=\mathbf{0}$ and by hypothesis the matrix $\mathbb{A}$ is assumed to be asymptotically stable.

The nonlinearity ϕ_0 is decentralized, memoryless and satisfies the following sector condition [13] for any diagonal positive definite matrix T_0:

$$\phi_0' T_0(\phi_0 + \Lambda_0\mathbb{K}\xi) \leq 0 \quad (14)$$

provided that ξ takes values in $S(\mathbb{K}, u_0^\lambda)$

$$S(\mathbb{K}, u_0^\lambda) = \left\{\xi \in \Re^{n+mq+n_c}; |\mathbb{K}_{(i)}\xi| \leq \frac{u_{0(i)}}{1-\lambda_{0(i)}}, i=1,\dots,m\right\} \quad (15)$$

where Λ_0 is a positive diagonal matrix with $\Lambda_{0(i,i)} = \lambda_{0(i)}$.

From the definition of each component of ϕ_j, $j=1,\dots,q-1$, one verifies that for any diagonal positive definite matrix T_j, $j=1,\dots,q-1$:

$$\phi_j' T_j(\phi_j + \Lambda_j\mathbb{K}\xi^{(j)}) \leq 0 \quad (16)$$

provided that ξ takes values in $S(\mathbb{K}, u_j^\lambda)$

$$S(\mathbb{K}, u_j^\lambda) = \left\{\xi \in \Re^{n+mq+n_c}; |\mathbb{K}_{(i)}\xi^{(j)}| \leq \frac{u_{j(i)}}{1-\lambda_{j(i)}}, i=1,\dots,m\right\} \quad (17)$$

where Λ_j is a positive diagonal matrix with $\Lambda_{j(i,i)} = \lambda_{j(i)}$.

Furthermore, we can express inherent properties relating ϕ_0, ϕ_j and $\phi_0^{(j)}$.

Lemma 1 *The nonlinearities ϕ_0, ϕ_j and $\phi_0^{(j)}$ satisfy the following properties for $j=1,\dots,q-1$:*

$$(\mathbb{K}\xi^{(j)} + \phi_0^{(j)})'\phi_0^{(j)} = 0 \; ; \; (\mathbb{K}\xi^{(j)} + \phi_0^{(j)})'\phi_0 = 0 \quad (18)$$

$$\phi_j'\phi_0 = 0 \; ; \; \phi_j'\phi_0^{(j)} = 0 \; ; \; \phi_j'\phi_0^{(j+1)} = 0 \quad (19)$$

4 Stability conditions

Some conditions in terms of matrix inequalities are now presented by using both quadratic Lypunov and Lure Lyapunov functions. For this, define the vectors Φ_0^d, Φ_1 and U_1:

$$\Phi_0^d = \left[\begin{array}{c}\dot{\phi}_0 \\ \phi_0^{(2)} \\ \vdots \\ \phi_0^{(q-1)}\end{array}\right]; \Phi_1 = \left[\begin{array}{c}\phi_1 \\ \phi_2 \\ \vdots \\ \phi_{q-1}\end{array}\right]; U_1 = \left[\begin{array}{c}u_1 \\ u_2 \\ \vdots \\ u_{q-1}\end{array}\right] \quad (20)$$

and the augmented matrices

$$\mathcal{B}_1 = \left[\begin{array}{ccc}\mathbb{B}_1 & \dots & \mathbb{B}_{q-1}\end{array}\right]; \mathbb{D} = \left[\begin{array}{c}\mathbb{K}\mathbb{A} \\ \vdots \\ \mathbb{K}\mathbb{A}^{q-1}\end{array}\right] \quad (21)$$

4.1 Quadratic approach

Consider a quadratic candidate Lyapunov function $V(\xi)$:

$$V(\xi) = \xi' P\xi \text{ with } P = P' > \mathbf{0} \quad (22)$$

Proposition 1 *If there exist matrices of appropriate dimensions $P=P'>\mathbf{0}$, E_c, F_c, F, G, H, J, L, diagonal matrices $\mathbb{N}_3$, $\mathbb{N}_4$ and $\mathbb{N}_5$, diagonal positive matrices T_0, $\mathbb{T}_1$, $\mathbb{N}_2$, Λ_0, $\mathbb{L}_1$, positive scalar γ satisfying*[1]

$$\left[\begin{array}{ccc} M_1 & \star & \star \\ M_2 & M_4 & \star \\ M_3 & M_5 & M_6\end{array}\right] < \mathbf{0} \quad (23)$$

$$\left[\begin{array}{cc} P & \star \\ (1-\lambda_{0(i)})\mathbb{K}_{(i)} & \gamma u_{0(i)}^2\end{array}\right] \geq \mathbf{0}, \; i=1,\dots,m \quad (24)$$

$$\left[\begin{array}{cc} P & \star \\ (1-\mathbb{L}_{1(i)})\mathbb{D}_{(i)} & \gamma U_{1(i)}^2\end{array}\right] \geq \mathbf{0}, \; i=1,\dots,mq \quad (25)$$

[1] The symbol $\star$ stands for symmetric blocks in matrix inequalities.

$$\begin{bmatrix} P & \star \\ \mathbb{C}_{2(i)} & \gamma z_{0(i)}^2 \end{bmatrix} \geq \mathbf{0},\ i = 1,\dots,l \tag{26}$$

$$\mathbf{0} \leq \Lambda_0 < \mathbf{1}\ ;\ \mathbf{0} \leq \mathbb{L}_1 < \mathbf{1} \tag{27}$$

with $M_1 = \begin{bmatrix} \mathbb{A}'F' + F\mathbb{A} & \star \\ P - F' + G\mathbb{A} & -G - G' \end{bmatrix}$, $M_2 = \begin{bmatrix} -T_0\Lambda_0\mathbb{K} + (\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c)'F' + H\mathbb{A} + \mathbb{N}_3\mathbb{D} & -H + (\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c)'G' \\ \mathcal{B}_1'F' + J\mathbb{A} - \mathbb{T}_1\mathbb{L}_1\mathbb{D} & \mathcal{B}_1'G' - J \end{bmatrix}$, $M_3 = \begin{bmatrix} L\mathbb{A} + \mathcal{B}_1'F' + \mathbb{N}_2\mathbb{D} & -L + \mathcal{B}_1'G' \end{bmatrix}$, $M_4 = \begin{bmatrix} -2T_0 + (\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c)'H' + H(\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c) & \star \\ \mathbb{N}_4 + \mathcal{B}_1'H' + J(\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c) & -2\mathbb{T}_1 + \mathcal{B}_1'J' + J\mathcal{B}_1 \end{bmatrix}$, $M_5 = \begin{bmatrix} \mathcal{B}_1'H' + L(\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c) + \mathbb{N}_3 & \mathcal{B}_1'J' + L\mathcal{B}_1 + \mathbb{N}_5 \end{bmatrix}$, $M_6 = -2\mathbb{N}_2 + \mathcal{B}_1'L' + L\mathcal{B}_1$, *then the gains* E_c, F_c *and the set* $S_0 = \{\xi \in \mathfrak{R}^{n+mq+n_c}; \xi'P\xi \leq \gamma^{-1}\}$ *solve Problem 1.*

Proof. Consider the quadratic function $V(\xi) = \xi'P\xi$ with $P = P' > \mathbf{0}$ (defined in (22)). Throughout the proof, the dependence of t of the vectors considered is omitted for ease of notation. Hence, we want to satisfy simultaneously

$$\begin{array}{l} \mathcal{L}_0 = \dot{V}(\xi) = \dot{\xi}'P\xi + \xi'P\dot{\xi} < 0 \\ \mathcal{L}_1 = -2\phi_0'T_0(\phi_0 + \Lambda_0\mathbb{K}\xi) \geq 0 \\ \mathcal{L}_{1j} = -2\phi_j'T_j(\phi_j + \Lambda_j\mathbb{K}\xi^{(j)}) \geq 0,\ j = 1,\dots,q-1 \\ \mathcal{L}_{2j} = 2(\mathbb{K}\xi^{(j)} + \phi_0^{(j)})'N_{2j}\phi_0^{(j)} = 0,\ j = 1,\dots,q-1 \\ \mathcal{L}_{3j} = 2(\mathbb{K}\xi^{(j)} + \phi_0^{(j)})'N_{3j}\phi_0 = 0,\ j = 1,\dots,q-1 \\ \mathcal{L}_{4j} = 2\phi_j'N_{4j}\phi_0 = 0,\ j = 1,\dots,q-1 \\ \mathcal{L}_{5j} = 2\phi_j'N_{5j}\phi_0^{(j)} = 0,\ j = 1,\dots,q-1 \end{array} \tag{28}$$

where T_0, T_j and N_{2j} are diagonal positive matrices. N_{3j}, N_{4j} and N_{5j} are diagonal matrices.

Consider now from (20) and (21) the following definitions:

$$\zeta = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \\ \phi_0 \\ \Phi_1 \\ \Phi_0^d \end{bmatrix};\ \mathcal{Q} = \begin{bmatrix} \mathbf{0} & P & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ P & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix};\ \mathcal{Y} = \begin{bmatrix} F \\ G \\ H \\ J \\ L \end{bmatrix}$$
$$\mathcal{X} = \begin{bmatrix} \mathbb{A} & -\mathbf{1} & (\mathbb{B}_0 + \mathbb{R}E_c + \mathbb{B}_qF_c) & \mathcal{B}_1 & \mathcal{B}_1 \end{bmatrix}$$

The satisfaction of $\dot{V}(\xi) < 0$ along the trajectories of the closed-loop system (13) is equivalent to satisfy:

$$\zeta'\mathcal{Q}\zeta < 0,\ \forall\zeta \text{ such that } \mathcal{X}\zeta = 0,\ \zeta \neq 0$$

Before using the Finsler's lemma [8], note some useful properties. In particular from the definitions of matrices given in (2), (12) and (21), one can verify that $\mathbb{K}\xi^{(j)} = \mathbb{K}\mathbb{A}^j\xi$. From this, we can re-write in a more compact form the expressions of $\mathcal{L}_{1j}$, $\mathcal{L}_{2j}$, $\mathcal{L}_{3j}$, $\mathcal{L}_{4j}$ and $\mathcal{L}_{5j}$ by defining $\mathbb{T}_1 = diag(\begin{bmatrix} T_1 & \dots & T_{q-1} \end{bmatrix})$, $\mathbb{L}_1 = diag(\begin{bmatrix} \Lambda_1 & \dots & \Lambda_{q-1} \end{bmatrix})$, $\mathbb{N}_k = diag(\begin{bmatrix} N_{k1} & \dots & T_{kq-1} \end{bmatrix})$, $k = 2,3,4,5$.

By applying the Finsler's lemma [8], it follows that if there exists a matrix $\mathcal{Y}$ such that

$$\zeta'(\mathcal{Q} + \mathcal{Y}\mathcal{X} + \mathcal{X}'\mathcal{Y}' + \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\mathbb{K}'\Lambda_0T_0 + \mathbb{D}'\mathbb{N}_3 & -\mathbb{D}'\mathbb{L}_1\mathbb{T}_1 & \mathbb{D}'\mathbb{N}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -T_0\Lambda_0\mathbb{K} + \mathbb{N}_3\mathbb{D} & \mathbf{0} & -2T_0 & \mathbb{N}_4 & \mathbb{N}_3 \\ -\mathbb{T}_1\mathbb{L}_1\mathbb{D} & \mathbf{0} & \mathbb{N}_4 & -2\mathbb{T}_1 & \mathbb{N}_5 \\ \mathbb{N}_2\mathbb{D} & \mathbf{0} & \mathbb{N}_3 & \mathbb{N}_5 & -2\mathbb{N}_2 \end{bmatrix})\zeta < 0 \tag{29}$$

then $\forall\xi \in S(\mathbb{K}, u_0^\lambda) \cap S(\mathbb{K}, u_j^\lambda)$, $j = 1,\dots,q-1$, it follows that $\dot{V}(\xi) < 0$. Hence, provided that relation (23) is verified and $\xi \in S(\mathbb{K}, u_0^\lambda) \cap S(\mathbb{K}, u_j^\lambda)$, $j = 1,\dots,q-1$, it follows that (29) is verified and therefore $V(\xi)$ is a stricly decreasing Lyapunov function for the closed-loop system. □

In Proposition 1, there appear some nonlinearities in particular due to the product between the multipliers (F, G, H, J and L) and the gains of the anti-windup E_c and F_c. Moreover, the satisfaction of relation (23) means that $G' + G > \mathbf{0}$ and therefore matrix G must be nonsingular. From this fact and a suitable choice of multipliers with an adequate change of variables simplify a major part of the inequalities of Proposition 1.

Corollary 1 *If there exist matrices of appropriate dimensions* $W = W' > \mathbf{0}$, S, Z_1, Z_2, V_1, V_2, *diagonal positive matrices* Λ_0, $\mathbb{L}_1$, R_0, R_1, R_2, *positive scalar* γ *satisfying*

$$\begin{bmatrix} S\mathbb{A}' + \mathbb{A}S' & \star & \star & \star & \star \\ W - S + \mathbb{A}S' & -S - S' & \star & \star & \star \\ -\Lambda_0\mathbb{K}S' + R_0\mathbb{B}_0' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}_q' & R_0\mathbb{B}_0' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}_q' & -2R_0 & \star & \star \\ R_1\mathcal{B}_1' - \mathbb{L}_1\mathbb{D}S' & R_1\mathcal{B}_1' & V_1 & -2R_1 & \star \\ R_2\mathcal{B}_1' + \mathbb{D}S' & R_2\mathcal{B}_1' & \mathbf{0} & V_2 & -2R_2 \end{bmatrix} < \mathbf{0} \tag{30}$$

$$\begin{bmatrix} W & \star \\ (1 - \lambda_{0(i)})\mathbb{K}_{(i)}S' & \gamma u_{0(i)}^2 \end{bmatrix} \geq \mathbf{0},\ i = 1,\dots,m \tag{31}$$

$$\begin{bmatrix} W & \star \\ (1 - \mathbb{L}_{1(i)})\mathbb{D}_{(i)}S' & \gamma U_{1(i)}^2 \end{bmatrix} \geq \mathbf{0},\ i = 1,\dots,m(q-1) \tag{32}$$

$$\begin{bmatrix} W & \star \\ \mathbb{C}_{2(i)}S' & \gamma z_{0(i)}^2 \end{bmatrix} \geq \mathbf{0},\ i = 1,\dots,l, \tag{33}$$

$$\mathbf{0} \leq \Lambda_0 < \mathbf{1}\ ;\ \mathbf{0} \leq \mathbb{L}_1 < \mathbf{1} \tag{34}$$

then the gains $E_c = Z_1R_0^{-1}$, $F_c = Z_2R_0^{-1}$ *and the* $S_0 = \{\xi \in \mathfrak{R}^{n+mq+n_c}; \xi'S^{-1}W(S')^{-1}\xi \leq \gamma^{-1}\}$ *solve Problem 1.*

4.2 Lure approach

Let us now consider a Lure candidate Lyapunov function $V(\xi)$:

$$\begin{array}{l} V(\xi) = \xi'P\xi - 2\sum_{i=1}^m \int_0^{\mathbb{K}_{(i)}\xi} \phi_{0(i)}(\sigma)N_{0(i,i)}d\sigma \\ -2\sum_{j=1}^{q-1}\sum_{i=1}^m \int_0^{\phi_{0(i)}^{(j)} + \mathbb{K}_{(i)}\xi^{(j)}} \phi_{j(i)}(\sigma)N_{j(i,i)}d\sigma \end{array} \tag{35}$$

with $P = P' > \mathbf{0}$, N_0 and N_j diagonal positive matrices. One gets $V(\xi) > 0$ for all $\xi \in S(\mathbb{K}, u_0^\lambda) \bigcap_{j=1}^{q-1} S(\mathbb{K}, u_j^\lambda)$, $\xi \neq 0$.

Proposition 2 *If there exist matrices of appropriate dimensions* $P = P' > \mathbf{0}$, E_c, F_c, F, G, H, J, L, *diagonal matrices* $\mathbb{N}_3$, $\mathbb{N}_4$ *and* $\mathbb{N}_5$, *diagonal positive matrices* T_0, $\mathbb{T}_1$, $\mathbb{N}_0$, $\mathbb{N}_1$, $\mathbb{N}_2$, Λ_0, $\mathbb{L}_1$, *positive scalar* γ *satisfying relations (24), (25), (26), (27) and*

$$\begin{bmatrix} M_1 & \star & \star \\ \tilde{M}_2 & M_4 & \star \\ M_3 & M_5 & M_6 \end{bmatrix} < \mathbf{0} \tag{36}$$

with M_1, M_3, M_4, M_5, M_6 defined in Proposition 1 and $\tilde{M}_2 = \begin{bmatrix} -T_0\Lambda_0\mathbb{K} + (\mathbb{B}_0+\mathbb{R}E_c+\mathbb{B}_qF_c)'F' + H\mathbb{A} + \mathbb{N}_3\mathbb{D} & -H + (\mathbb{B}_0+\mathbb{R}E_c+\mathbb{B}_qF_c)'G' - \mathbb{N}_0\mathbb{K} \\ \mathcal{B}_1'F' + J\mathbb{A} - \mathbb{T}_1\mathbb{L}_1\mathbb{D} & \mathcal{B}_1'G' - J - \mathbb{N}_1\mathbb{D} \end{bmatrix}$ *then the gains E_c, F_c and the set $S_0 = \{\xi \in \Re^{n+mq+n_c}; V(\xi) \leq \gamma^{-1}\}$ solve Problem 1.*

Proof. The proof follows the same lines as that one of Proposition 1 by considering the Lure function defined in (35). Hence, note that the time-derivative of this function $V(\xi)$ reads:

$$\begin{array}{rcl} \dot{V}(\xi) & = & \dot{\xi}'P\xi + \xi'P\dot{\xi} - 2\phi_0'N_0\mathbb{K}\dot{\xi} - 2\Phi_1'\mathbb{N}_1\mathbb{D}\dot{\xi} - 2\Phi_1'\mathbb{N}_1\dot{\Phi}_0^d \\ & = & \dot{\xi}'P\xi + \xi'P\dot{\xi} - 2\phi_0'N_0\mathbb{K}\dot{\xi} - 2\Phi_1'\mathbb{N}_1\mathbb{D}\dot{\xi} \end{array}$$

where $\mathbb{N}_1 = diag([N_1, \ldots, N_{q-1}])$. Indeed, by definition of different nonlinearities one can prove that $\Phi_1'\mathbb{N}_1\dot{\Phi}_0^d = \mathbf{0}$. Therefore, one can remark that the new matrix Q is

$$Q = \begin{bmatrix} \mathbf{0} & P & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ P & \mathbf{0} & -\mathbb{K}'N_0 & -\mathbb{D}'\mathbb{N}_1 & \mathbf{0} \\ \mathbf{0} & -N_0\mathbb{K} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbb{N}_1\mathbb{D} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

From the use of this matrix the proof is similar to that one of Proposition 1. Moreover, from (35) one has $\xi'P\xi \leq V(\xi)$ and the following inclusion $\{\xi \in \Re^{n+m(q-1)+N_c}; V(\xi) \leq \gamma^{-1}\} \subset \{\xi \in \Re^{n+m(q-1)+N_c}; \xi'P\xi \leq \gamma^{-1}\}$ holds. Hence, conditions (24), (25), (26) allow to verify that $\xi \in S(\mathbb{K}, u_0^\lambda) \cap S(\mathbb{K}, u_j^\lambda)$, $j = 1, \ldots, q-1$. □

As in Corollary 1, an adequate change of variables and of multipliers exhibits the two gains E_c and F_c.

Corollary 2 *If there exist matrices of appropriate dimensions $W = W' > \mathbf{0}$, S, Z_1, Z_2, V_1, V_2, diagonal positive matrices Λ_0, $\mathbb{L}_1$, R_0, R_1, R_2, positive scalar γ satisfying relations (31), (32), (33), (34) and*

$$\begin{bmatrix} S\mathbb{A}' + \mathbb{A}S' & \star & \star & \star & \star \\ W - S + \mathbb{A}S' & -S - S' & \star & \star & \star \\ -\Lambda_0\mathbb{K}S' + R_0\mathbb{B}_0' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}_q' & R_0\mathbb{B}_0' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}_q' - \mathbb{K}S' & -2R_0 & \star & \star \\ R_1\mathcal{B}_1' - \mathbb{L}_1\mathbb{D}S' & R_1\mathcal{B}_1' - \mathbb{D}S' & V_1 & -2R_1 & \star \\ R_2\mathcal{B}_1' + \mathbb{D}S' & R_2\mathcal{B}_1' & \mathbf{0} & V_2 & -2R_2 \end{bmatrix} < \mathbf{0} \quad (37)$$

then the gains $E_c = Z_1R_0^{-1}$, $F_c = Z_2R_0^{-1}$ and the $S_0 = \{\xi \in \Re^{n+mq+n_c}; V(\xi) \leq \gamma^{-1}\}$, with $P = S^{-1}W(S')^{-1}$, $N_0 = R_0^{-1}$ and $\mathbb{N}_1 = R_1^{-1}$, solve Problem 1.

Note that if the satisfaction of relation (37) implies that the Lure function $V(\xi)$ defined in (35) verifies $\dot{V}(\xi) < 0$ along the trajectories of the closed-loop system (13), the inclusion $\{\xi \in \Re^{n+mq+n_c}; V(\xi) \leq \gamma^{-1}\} \subset \{\xi \in \Re^{n+mq+n_c}; \xi'P\xi \leq \gamma^{-1}\}$ does not imply that the quadratic function $\xi'P\xi$ is a decreasing function along the trajectories of the closed-loop system (13). Moreover, by setting $N_0 = \mathbf{0}$ and $\mathbb{N}_1 = \mathbf{0}$, condition (36) (resp. (37)) is equivalent to (23) (resp. (30)). Moreover, an interesting fact appearing in relation (37) with respect to the equivalent one obtained via classical quadratic approach (see [10]) is that there is no nonlinearities between matrices $N_0 = \mathbf{0}$, $\mathbb{N}_1 = \mathbf{0}$ of the Lure function and the anti-windup gains.

5 Numerical procedure

Some relations of Corollaries 1 or 2 are bilinear in decision variables Λ_0 and S, and $\mathbb{L}_1$ and S. A way to overcome the computational difficulty of directly solving BMI conditions consists in using relaxation schemes, that is to fix one of the variables and seek for the other ones. In this case, the relations become linear. Moreover, the implicit objective is to maximize the region of stability of the closed-loop system over the choice of the anti-windup gains.

5.1 Quadratic approach

From Proposition 1 and Corollary 1, the region of stability associated to the closed-loop system (13) is the ellipsoid $S_0 = \{\xi \in \Re^{n+mq+n_c}; \xi'P\xi \leq \gamma^{-1}\}$. By noting that the volume of S_0 is proportional to $\sqrt{det(\frac{P^{-1}}{\gamma})}$, it is then possible to maximize its size by minimizing the function $log(det(\gamma P))$. By definition of P it follows: $det(\gamma P) = det(\gamma S^{-1}W(S')^{-1}) = \gamma^{n+m(q-1)+n_c} det(S^{-1})det(W)det((S')^{-1}) = \gamma^{n+mq+n_c}\frac{det(W)}{det(S)^2}$ or $log(det(\gamma P)) = (n+mq+n_c)log(\gamma) + log(det(W)) - 2log(det(S))$. Hence, an algorithm based on some relaxation schemes can be considered. Let us underline that the maximization of $trace(\Lambda_0)$ and $trace(\mathbb{L}_1)$ implies to maximize the set $S(\mathbb{K}, u_0^\lambda) \cap S(\mathbb{K}, u_j^\lambda)$, $j = 1, \ldots, q-1$, which contains the set of interest S_0. Therefore that would add some degrees of freedom to maximize the size of S_0. Nevertheless, it is important to note that, in general, the better solution is not obtained for $\Lambda_0 \to \mathbf{1}$ and $\mathbb{L}_1 \to \mathbf{1}$ excepted in the case where matrix A is not strictly unstable. Moreover, since matrix $\mathbb{A}$ is supposed to be asymptotically stable, there always exist a solution in the case $\Lambda_0 = \mathbf{0}$ and $\mathbb{L}_1 = \mathbf{0}$. Due to the form of the actuator (1), remind us that one cannot have simultanesously $\phi_{0(i)} \neq 0$ and $\phi_{j(i)} \neq 0$. Indeed, when $\phi_{0(i)} \neq 0$ one gets $\phi_{j(i)} = 0$, and therefore one gets $\lambda_{0(i)} \neq 0$ and $\mathbb{L}_{1(i)} = \lambda_{j(i)} = 0$. This fact means that the numerical tests of relations (30), (31) and (32) will be done by removing some lines and columns in matrix inequality (30).

5.2 Lure approach

With respect to the Lure function defined in (35), one gets:

$$V(\xi) \leq \xi'(P + \mathbb{K}'N_0\mathbb{K})\xi + (\Phi_0^d + \mathbb{D}\xi)'\mathbb{N}_1(\Phi_0^d + \mathbb{D}\xi) \quad (38)$$

Recall that by definition of $\Phi_0^d + \mathbb{D}_{(i)}\xi$, one gets, $\forall i = 1, \ldots, m$:

$$\phi_{0(i)}^{(j)} + \mathbb{K}_{(i)}\xi^{(j)} = \begin{cases} 0 & \text{if} \quad |\mathbb{K}_{(i)}\xi| > u_{0(i)} \\ \mathbb{K}_{(i)}\xi^{(j)} & \text{if} \quad |\mathbb{K}_{(i)}\xi| \leq u_{0(i)} \end{cases}$$

Thus in a certain way, one can consider that the right part of the inequality (38) evolves as $\xi'(P + \mathbb{K}'N_0\mathbb{K} + \mathbb{D}'\mathbb{N}_1\mathbb{D})\xi$. From this, in order to maximize the size of the set $S_0 = \{\xi \in \Re^{n+mq+n_c}; V(\xi) \leq \gamma^{-1}\}$, we use the same concept as in the previous subsection. Hence, note that $det(\gamma(P + \mathbb{K}'N_0\mathbb{K} + \mathbb{D}'\mathbb{N}_1\mathbb{D})) = det(\gamma S^{-1}(W + S\mathbb{K}'N_0\mathbb{K}S' + S\mathbb{D}'\mathbb{N}_1\mathbb{D})(S')^{-1}) = \gamma^{n+m(q-1)+n_c}det(S^{-1})det(W +$

$S\mathbb{K}' N_0 \mathbb{K} S' + S\mathbb{D}' \mathbb{N}_1 \mathbb{D} S') det((S')^{-1}) = \gamma^{n+m(q-1)+n_c} \frac{det(W+S\mathbb{K}'N_0\mathbb{K}S'+S\mathbb{D}'\mathbb{N}_1\mathbb{D}S')}{det(S)^2}$ which gives still $log(det(\gamma(P + \mathbb{K}' N_0 \mathbb{K} + \mathbb{D}' \mathbb{N}_1 \mathbb{D}))) = (n + m(q-1) + n_c) log(\gamma) + log(det(W + S\mathbb{K}' N_0 \mathbb{K} S' + S\mathbb{D}' \mathbb{N}_1 \mathbb{D} S')) - 2log(det(S))$. The main difficulty when minimizing the above expression resides in the part $log(det(W + S\mathbb{K}' N_0 \mathbb{K} S' + S\mathbb{D}' \mathbb{N}_1 \mathbb{D} S'))$. We can first remark that the minimization of this term is very close to the minimization of $trace(W + S\mathbb{K}' N_0 \mathbb{K} S' + S\mathbb{D}' \mathbb{N}_1 \mathbb{D} S')$. Hence, by recalling that from Corollary 2 $N_0 = R_0^{-1}$ and $\mathbb{N}_1 = R_1^{-1}$, we can consider the following optimization problem $\min\{(n+mq+n_c)log(\gamma) + log(det(W)) - 2log(det(S)) + trace(Y)\}$ subject to relations (37), (31), (32), (33), (34) and $\begin{bmatrix} Y - W & \star & \star \\ \mathbb{K}S' & R_0 & \star \\ \mathbb{D}S' & 0 & R_1 \end{bmatrix} \geq \mathbf{0}$.

From the additional constraints involving Y it appears that $Y \geq W + S\mathbb{K}' N_0 \mathbb{K} S' + S\mathbb{D}' \mathbb{N}_1 \mathbb{D} S'$ and therefore the minimization of $trace(Y)$ corresponds to the minimization of $trace(W + S\mathbb{K}' N_0 \mathbb{K} S' + S\mathbb{D}' \mathbb{N}_1 \mathbb{D} S')$. From the above optimization problem, the same type of algorithm than in the quadratic case can be considered. Another interesting idea can be to consider a solution obtained from the quadratic case to initialize an algorithm based on Proposition 2: see [10].

6 Concluding remarks

In this paper, we have addressed the problem of designing anti-windup gains in order to obtain a region of stability, as large as possible, for linear systems with amplitude and dynamics restricted actuator. The anti-windup strategy developed consisted in adding the part due to the saturation of the output actuator (measured part) both in the state evolution and in the output of the controller. Theoretical constructive conditions have been provided in order to associate the synthesized anti-windup gain to a region of stability, while controlled output constraints were satisfied. From these conditions, algorithms based on the solution of LMI-based problems have been proposed in order to optimize the size of the region of stability over the choice of the anti-windup gains E_c and F_c. Since the solution $E_c = F_c = \mathbf{0}$ is always admissible, we can conclude that anti-windup gains can always be used in order to obtain larger regions of stability.

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