

# Pricing long-term permits and scheduling of electric vehicle charging in parking lots with shared resources

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**Abstract**—With the interest from car owners in going green being at an all-time high, electric vehicles (EVs) are flooding the automobile market. One of the primary concerns in owning an EV is the availability of charging infrastructure while away from home. There has been a renewed interest in managing and pricing the usage of shared commercial EV chargers, while maximizing the operator's profits. Towards this end, we propose a combined pricing-scheduling quadratic integer programming (QIP) model that iteratively prices and schedules EV charging. A pricing module is used to accept/reject charging requests and control the right number and types (arrival-departure times, charge demand etc.) of EVs to charge. The scheduling module ensures that the demand can be met subject to price-demand sensitivity and other scheduling constraints. Once the EVs to be accepted have been finalized and their permit prices determined, the scheduling module can be run every night once the day-to-day arrival and departure times of each EV is revealed to the operator.

## I. INTRODUCTION

Electric vehicles (EV) are gaining in popularity (eg: Chevrolet Volt, Nissan Leaf, Tesla etc). According to Pike Research report, there will be more than 200,000 EVs sold in the US by 2015 [1]. A classic approach to charging these vehicles is overnight charging at homes. This has a few drawbacks – (1) large simultaneous loads on the nation's electricity grid, (2) charger installation costs, (3) insufficient charging to cover an entire day's trip. To address concerns such as range anxiety and enable large-scale adoption of EVs, EV charging stations are being located in business districts. There are three types of EV charging stations that differ in the charging rate and installation costs. The charging cost to a driver at these stations depends on the installation price of the charger. One of the primary challenges in managing chargers installed in commercial locations is demand forecasting, pricing and management. To do so in real-time poses computational and implementation challenges.

Today's EVs have two constraints: (1) A fully charged EV runs for about 50 miles on an average. Hence, there is cause for anxiety whether a trip can be completed with the remaining charge, (2) Depending on the type of the charging station, it could take from 2 to 8 hours to completely charge an EV. Besides EVs, Plug-in Hybrid Electric Vehicles (PHEVs) also have batteries and may use electric mode frequently. Although PHEV drivers may not face the range anxiety issue, some driving habits (especially in city traffic) may lead to more use of electric mode for environmental concerns and cost. The range associated with the electric

mode in PHEV's is about 10 miles. Some drivers may prefer to have enough charge available in the battery to ensure use of electric mode in their commute. With above scenarios, we believe that EV and PHEV drivers will frequently charge their vehicles in the commercial or corporate parking lots at work (or other locations). This presents a significant challenge for a parking lot operator to manage the charging process of these vehicles because: 1) commuters will have a varying arrival-departure schedule and demand, 2) utilities often offer time-varying time of day pricing schemes for electricity and 3) the charging infrastructure is an expensive and limited resource and need to be used carefully.

A manager not only needs to determine a charging schedule for each vehicle in the preferred parking location which meets the demand and deadline, but also needs to determine the fair price of charging offered to the consumers. For service organizations, this presents an opportunity to develop a novel service to manage the EV charging process under time-varying price of electricity levied by the utilities. Towards this end, we propose a combined pricing and scheduling model that simultaneously prices long-term charging permits for EVs while accounting for scheduling constraints arising from their arrival and departure times, processing times and demand for charge. More specifically, our method helps a manager (which could represent a parking lot operator, a city/county/state government, a charging servicer, a corporate/campus parking lot manager, etc.) to accept/reject charging requests, assign EVs to appropriate chargers, price long-term charging permits and schedule the charging of accepted EVs. Given a price-demand function for EV owners, a manager could use the permit price as a lever to adjust the right number of EVs that could be served given the constraints of available resources. Once the number and characteristics of EVs to be charged is known, an optimal schedule that maximizes the percentage of service level agreement met could be designed. The proposed methodology addresses the problem of managing the charging process by determining the price of long-term charging permit for EVs and charging schedule that maximizes the profit for the parking lot owner. The combined pricing-scheduling formulation is modeled as a quadratic integer programming (QIP). We consider linear price-demand curves of the form  $D = a - b * p$ . We follow two intuitive guidelines while choosing these parameters. First, the number of requests for the same charging demand level are higher for the longer duration stays in the parking lot. Second, for the same duration stay in the parking lot, the higher the demand, the commuters are willing to pay more.

**Organization:** The paper is organized as follows. In the

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next section, we briefly discuss the related work. In Section III we present our problem formulation and propose our heuristic in Section IV. In Section V we present our results on a small and large problem instances. We conclude in Section VI and discuss future work.

## II. RELATED WORK

There have been a reasonable number of papers published in the area of scheduling and/or pricing electric vehicles, although a majority of these papers consider problems applicable to home/apartment charging and the associated management of electricity grid related charging infrastructure. Sundstrom and Binding [2] study the planning of the EVs done under voltage and power constraints, while avoiding distribution grid congestion and satisfying the requirements of individual vehicle owners. They use a trip forecasting procedure to compute demand. Sundstrom et al. [3] propose alternative methods for a charging service provider to predict EV trip lengths and demand. They propose controlled charging mechanisms to better integrate the charging load with the traditional load in the electricity grid and to minimize the cost of charging behavior. A semi-Markov model is used for trip prediction and energy consumption of a single vehicle over a time period. Stein et al. [4] propose a model-based online mechanism with pre-commitment for EV charging. They assume that EV arrive dynamically and require a certain amount of charge before departure time. An online mechanism is used to schedule charging of these EVs using an optimization algorithm, when agents may misreport their preferences.

The scheduling of EVs poses an interesting challenge in estimating the desired amount of charge for each EV in advance. There has been work done in developing offline and online algorithms for scheduling EV charging. Chen and Tong [5] propose online algorithms for real-time charging of EVs in commercial spaces and pricing a single usage. The scheduling of large scale charging is formulated as a deadline scheduling problem. The proposed customer utility function is a function of arrival, departure and amount of charge. The proposed algorithm achieves the highest competitive ratio for the linear utility function. The pricing scheme maximizes the number of customers who are willing to accept a certain price. A customer is accepted if none of the prior customer requests will become unsatisfied. Chen et al. [6] study deadline scheduling for large scale charging of electric vehicles with renewable energy. The problem of scheduling for large scale charging of EV with renewable sources is considered. A new online charging algorithm is proposed by formulating the charging problem as one of deadline scheduling with admission control and variable charging capacities. It has a reserve dispatch algorithm to compensate the intermittency of renewable sources. Chen and Tong [7] propose a multiprocessor deadline scheduling for online scheduling of EV charging. This formulation assumes arbitrary arrivals, charging requests and service deadlines. An online algorithm based on threshold admission and greedy scheduling is proposed. The energy management

system considered is assumed to have access to renewable sources such as solar powers, and can supplement the renewable with purchased electricity from the grid. Gerding et al. [8] propose a novel online auction protocol that could be used by vehicle owners to bid for power and also state time windows to bid for power and also state time windows when vehicle is available for charging.

For offline scheduling Gan et al. [9] offer optimal decentralized protocol to manage the challenges faced by the grid in the integration of EV. This protocol is used for negotiating day-ahead charging schedules for EV. The goal is to shift the EV load to fill the drop in the overnight electricity demand through next day price profile broadcast and update by the utility. This requires no coordination among the EV, hence requiring low communication and computation capability. Sundstrom and Binding [10] present a service framework applicable to centralized and decentralized EV charging control. The charging service elements (user defined charging levels, prediction based charging levels, quick charge requirements and green energy requirements) can be mapped to an optimization problem which computes charging schedules. Sundstrom and Binding [11] describe an approach to optimize EV battery charging behavior with the goal of minimizing charging costs, achieving satisfactory state of energy levels and optimal power balancing. Linear and quadratic approximations of the EV batteries to plan the charging have been utilized. Caramanis and Foster [12] discuss management of EV charging to mitigate renewable generation intermittency and distribution network congestion. The authors propose a decision support methodology for the EV load aggregator to manage EV charging while engaging in energy and reserve capacity transactions in the wholesale power market.

The main contribution of our work is in the long-term pricing of charging permits in commercial charging lots. Aside from Chen and Tong [6], there has been just one other paper that discusses charging in commercial spaces. Kantarci and Mouftah [13] propose a prediction-based charging scheme which receives dynamic pricing information, predicts the market prices during the charging period and determines an appropriate time of charging with low cost. All consumers are assumed to use smart outlets that can communicate with the grid and learn the dynamic price of electricity. To the best of our knowledge, there is no work done in developing pricing schemes for long-term charging permits for commercially located charging spaces, while addressing scheduling constraints. As explained above, although there have been papers that address certain features of our work, none of them address the issue of using pricing of long-term permits to accept or reject requests. In this paper, we propose a combined pricing and scheduling model where the pricing model is used to have just the right number of EVs to charge to maximize profit for the operator. A scheduling model is utilized to ensure that the demand can be met.

### III. PROBLEM FORMULATION

First we describe our assumptions and notation. Then we propose the formulation for the combined pricing-scheduling problem.

#### A. Notation and assumptions

We are primarily interested in finding prices of long-term charging permits for arrival-departure schedules and demands of commuters. Hence, we assume that arrivals, departures and demands are appropriately discretized. This allows us to group charging requests into *bins* each with unique arrival-departure-demand tuple. Let  $K$  denote the set of such bins. Let  $T$  denote the set of time slots during a day. Each bin  $k \in K$  is represented by a tuple  $(r_k, d_k, D_k)$ , where  $r_k$ ,  $d_k$  and  $D_k$  respectively denote the arrival slot, departure slot, and demand of each vehicle in the bin in kWh. Arrival occurs at the beginning of a slot and departure happens as the end of a slot.  $p_t$  indicates the price of electricity in \$/kWh/slot for every  $t \in T$ . Under time-of-use pricing schemes,  $p_t$  during peak time slots will be higher. From the parking manager's viewpoint every request in each bin is the same. Hence, we make an important assumption that every charging request in a bin is offered the same price  $p_k$  in \$/kWh. We further assume that each bin  $k$  has a linear price-demand curve represented by  $N_k = [a_k - b_k p_k]$ , where  $N_k$  denotes the number of cars/commuters willing to pay the price  $p_k$  for charging,  $a_k$  and  $b_k$  are constant parameters and  $[\cdot]$  indicates the rounding operator. The parking manager uses the price as a lever to control the number of cars in each bin. Note that we are using the deterministic setting, *i.e.*, there is no uncertainty in the price of electricity and the parameters of the price-demand curves. Let  $M$  denote the set of chargers, with  $R_m$  indicating capacity of each charger in kWh/slot. We introduce a binary decision variable  $X_{j,k,m,t}$  associated with scheduling where:

$$X_{j,k,m,t} = \begin{cases} 1, & \text{if demand request } j \text{ in bin } k \in K \text{ is} \\ & \text{assigned to charger } m \in M \text{ in slot } t \in T \\ 0, & \text{otherwise.} \end{cases}$$

Our goal is to find the optimal  $p_k$  for each bin and  $X_{j,k,m,t}$  for each charge request in each bin.  $p_k$  is tied to the number of charge requests in each bin making the number of decision variables themselves variable. To fix this issue, we assume an upper limit on the number of requests in each bin  $k$ . Let  $J_k$  denote the limit. Then we introduce another binary decision variable  $Y_{j,k}$  where:

$$Y_{j,k} = \begin{cases} 1, & \text{if demand request } j \in J_k \text{ in bin } k \in K \\ & \text{is satisfied} \\ 0, & \text{otherwise.} \end{cases}$$

A loose choice for  $J_k$  is  $a_k$ , but in the actual implementation this can be improved a lot.  $Y_{j,k}$  variable plays the role of tying pricing and scheduling constraints together as we discuss in the next subsection. The total number of unknown

$X_{j,k,m,t}$ 's are  $K \times M \times T \times \sum_{k=1}^K J_k$  and the total number of unknown  $Y_{j,k}$ 's are  $K \times \sum_{k=1}^K J_k$ .

#### B. Pricing-scheduling QIP formulation

Below we present the quadratic integer programming (QIP) formulation of the combined pricing-scheduling problem. The objective of the formulation is to find the optimal pricing and scheduling variables  $p_k$ ,  $X_{j,k,m,t}$  and  $Y_{j,k}$  so as to maximize the profit of the parking lot manager.

$$\begin{aligned} \max \quad & \sum_k p_k D_k N_k - \sum_{j,k,m,t} p_t R_m X_{j,k,m,t} \\ & - \sum_{j,k,m,t} c |X_{j,k,m,t} - X_{j,k,m,t-1}| \quad (1) \end{aligned}$$

s.t.

$$N_k = [a_k - b_k p_k] \quad \forall k \in K \quad (2)$$

$$\sum_j Y_{j,k} = N_k \quad \forall k \in K \quad (3)$$

$$\sum_{t < r_k, m} X_{j,k,m,t} = 0 \quad \forall j \in J_k, k \in K \quad (4)$$

$$\sum_{t > d_k, m} X_{j,k,m,t} = 0 \quad \forall j \in J_k, k \in K \quad (5)$$

$$\sum_{j,k} X_{j,k,m,t} \leq 1 \quad \forall m \in M, t \in T \quad (6)$$

$$\sum_m X_{j,k,m,t} \leq Y_{j,k} \quad \forall j \in J_k, k \in K, t \in T \quad (7)$$

$$\sum_{m,t} R_m X_{j,k,m,t} \geq D_k Y_{j,k} \quad \forall j \in J_k, k \in K \quad (8)$$

$$X_{j,k,m,t} \in \{0, 1\} \quad \forall j \in J_k, k \in K, m \in M, t \in \mathcal{T} \quad (9)$$

$$Y_{j,k} \in \{0, 1\} \quad \forall j \in J_k, k \in K \quad (10)$$

$$X_{j,k,m,0} = 0 \quad \forall j \in J_k, k \in K, m \in M \quad (11)$$

$$X_{j,k,m,T+1} = 0 \quad \forall j \in J_k, k \in K, m \in M. \quad (12)$$

The objective function in (1) has three components. The first component is the revenue earned by the parking lot owner for charging the vehicles. Note that the revenue is a quadratic function of  $N_k$ . The second component is the cost of electricity for charging the vehicles. The third component is the cost associated with connecting a car to a charger or disconnecting it from a charger. We think of this as a set-up cost and could be associated with the resources for managing the process. It is also intended to minimize the number of frequent changeovers. The constraint in (2) indicates the number of vehicles in each bin  $k$  conforms with its price-demand curve. The constraint in (3) ensures that the total number of charging requests accepted for each bin  $k$  is equal to the number of cars conforming to the price-demand curve. The remaining constraints in the formulation are associated with the scheduling problem. (4) ensures that a vehicle does not get charged before its arrival. (5) ensures that a vehicle does not get charged after its departure. The constraint in (6) ensures that for a given charger and a given time slot at most one car can get charged. (7) ensures that if a vehicle request for charging is accepted, then during a given time

slot, the vehicle can get charged on at most one charger. This constraint also ensures that if the vehicle request is not accepted, all the corresponding  $X_{j,k,m,t}$  are set to 0. (8) ensures that during its stay in the parking lot a vehicle's demand is satisfied if its request is accepted.

#### IV. OUR HEURISTIC

The QIP formulation in the previous section is hard to solve. Even for a very small instance, we observed that it takes a long time for CPLEX to solve the QIP. In this section, we propose a heuristic to obtain a solution with improved runtime. We build our heuristic based on two critical observations. First, if we consider the relaxed version of the QIP, it is a convex programming problem and can be solved quickly to obtain a globally optimal solution. In the relaxed QIP, decision variables  $X_{j,k,m,t}$ 's and  $Y_{j,k}$ 's take any value in  $[0, 1]$  and we relax constraint (2) to  $N_k = a_k - b_k p_k$ . We denote this formulation as  $\text{QIP}_{\text{relax}}(\mathbf{P}, \mathbf{X}, \mathbf{Y})$  where  $\mathbf{P}$ ,  $\mathbf{X}$  and  $\mathbf{Y}$  respectively denote the set of  $p_k$ 's,  $X_{j,k,m,t}$ 's and  $Y_{j,k}$ 's. Second, if  $p_k$ 's are known, then the number of cars whose request is satisfied are known and the QIP reduces to the scheduling IP which can be solved by the CPLEX relatively quickly. In this sense,  $p_k$ 's are the complicating variables. Below we present the formulation of the scheduling IP and then present our heuristic.

##### A. Scheduling formulation

When the price of charging for each bin  $p_k$  is fixed, the number of accepted requests  $N_k$ 's are fixed. Let  $L_k$  denote the set of accepted requests for each bin  $k$  for the known price  $p_k$ . The QIP is reduced to the scheduling IP. Below we present the formulation of the scheduling IP.

$$\begin{aligned} \min \quad & \sum_{j,k,m,t} p_t R_m X_{j,k,m,t} \\ & + \sum_{j,k,m,t} c |X_{j,k,m,t} - X_{j,k,m,t-1}| \end{aligned} \quad (13)$$

s.t.

$$\sum_{t < r_k, m} X_{j,k,m,t} = 0 \quad \forall j \in L_k, k \in K \quad (14)$$

$$\sum_{t > d_k, m} X_{j,k,m,t} = 0 \quad \forall j \in L_k, k \in K \quad (15)$$

$$\sum_{j,k} X_{j,k,m,t} \leq 1 \quad \forall m \in M, t \in T \quad (16)$$

$$\sum_m X_{j,k,m,t} \leq 1 \quad \forall j \in L_k, k \in K, t \in T \quad (17)$$

$$\sum_{m,t} R_m X_{j,k,m,t} \geq D_k \quad \forall j \in L_k, k \in K \quad (18)$$

$$X_{j,k,m,t} \in \{0, 1\} \quad \forall j \in L_k, k \in K, m \in M, t \in \mathcal{T} \quad (19)$$

$$X_{j,k,m,0} = 0 \quad \forall j \in L_k, k \in K, m \in M \quad (20)$$

$$X_{j,k,m,T+1} = 0 \quad \forall j \in L_k, k \in K, m \in M. \quad (21)$$

Once the price of charging is fixed for each bin, the objective of the scheduling IP is to find an optimal charging schedule that minimizes the cost of operations for the parking

lot owner given the cost of electricity. The objective function in (13) has two components. The first component is the cost of electricity for charging vehicles. The second component is the set-up cost as discussed in the previous section. Constraints (14) and (15) ensure that a vehicle does not get charged before its arrival and after its departure. (16) ensures that for a given charger and a given time slot at most one car can get charged. (17) ensures that if a vehicle request for charging is accepted, then during a given time slot, that vehicle can get charged on at most one charger. Constraint (18) ensures that during its stay in the parking lot a vehicle's demand is satisfied.

We note that the scheduling formulation above is generic in a sense that it does not require any assumptions about typical demand or typical arrival-departure schedules. In daily operations, once the charging permit prices are fixed, the above formulation can be used to manage the scheduling process.

##### B. Our algorithm

We denote the scheduling IP formulation by  $\text{IP}_{\text{schedule}}(\mathbf{X})$ . Our algorithm follows an iterative procedure and can be intuitively explained as follows. We first begin with the relaxed QIP and obtain the corresponding optimal prices. We then solve the scheduling IP for the corresponding known demand. If it gives a feasible solution, then we are done. If the scheduling problem is infeasible, we are operating near the charging capacity. Hence, we add pricing cuts which ensure that the prices in each bin are above the current optimal by a small amount  $\epsilon$ . We obtain the new QIP with these pricing cuts and repeat the above procedure till we obtain a feasible solution to the scheduling IP. Algorithm 1 is formally presented below.

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##### Algorithm 1 Our Heuristic

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- 1:  $\text{NewQIP}_{\text{relax}}(\mathbf{P}, \mathbf{X}, \mathbf{Y}) \leftarrow \text{QIP}_{\text{relax}}(\mathbf{P}, \mathbf{X}, \mathbf{Y})$
  - 2: Solve  $\text{NewQIP}_{\text{relax}}(\mathbf{P}, \mathbf{X}, \mathbf{Y})$  to find  $\mathbf{P}^*$ .
  - 3: Find rounded demand for  $\mathbf{P}^*$  as  $N_k^* = \lceil a_k - b_k p_k^* \rceil$ .
  - 4: Solve  $\text{IP}_{\text{schedule}}(\mathbf{X})$  for rounded demands  $N_k^*$ 's.
  - 5: **if**  $\text{IP}_{\text{schedule}}(\mathbf{X})$  is feasible and yields solution  $\mathbf{X}^*$  **then**
  - 6: Find new prices for  $N_k^*$  as  $p_k' = \frac{a_k - N_k^*}{b_k}$ .
  - 7: **else**
  - 8: Add price related cuts as:  $\text{NewQIP}_{\text{relax}}(\mathbf{P}, \mathbf{X}, \mathbf{Y}) = \text{NewQIP}_{\text{relax}}(\mathbf{P}, \mathbf{X}, \mathbf{Y}) \cup \{p_k \geq p_k^* + \min(\epsilon, p_k^* - \frac{a_k}{b_k}) \forall k \in K\}$ . Go to Step 2.
  - 9: **end if**
  - 10: Return  $p_k'$  as the final prices and schedule  $\mathbf{X}^*$ .
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The term  $p_k^* - \frac{a_k}{b_k}$  in the min operator in Step 8 in Algorithm 1 ensures that the demand does not become negative. We do note that there is a gap between the output of our algorithm and the globally optimal solution. However, obtaining bounds on our algorithm and improving it is part of ongoing work. In the following section, we show the performance of our algorithm on a small-scale problem. From a scalability perspective, finding a feasible solution

to  $IP_{\text{schedule}}(\mathbf{X})$  is challenging for large-scale scheduling problems. We are currently working on using cutting plane techniques to lower the run time and improve the quality of the feasible solution found.

## V. RESULTS

We use the YALMIP toolbox [14] in the MATLAB environment along with the CPLEX solver to implement our heuristic. While implementing the heuristic, we notice that the majority of time is spent in generating the constraints. In our heuristic we solve the relaxed QIP as well as the scheduling IP multiple times. Instead of generating the constraints from scratch every time, to speed up the running time we store copies of the overlapping constraint objects and avoid the regenerating process. Below we present our results for two instances, small and large. In both cases, we use realistic values of demands and charging infrastructure capacities. In both instances below, we assume that there are 8 time slots and the price of electricity peaks to \$0.30/kWh from slots 3 to 6 and is \$0.15/kWh in the remaining slots. There are 4 demand levels at 4 kWh, 6 kWh, 8 kWh and 10 kWh. We consider Level 1 and Level 2 chargers with capacities 2 kWh and 10 kWh respectively. In each instance, we had to set up  $a_k$ 's and  $b_k$ 's of multiple price-demand curves. We follow two guidelines while setting up these parameters. First, the number of requests for the same charging demand level are higher for the longer duration stays in the parking lot. Second, for the same duration stay in the parking lot, the higher the demand, the commuters are willing to pay more.

### A. Example 1 - small instance

Figure 1 shows the experimental set up for a small instance. We consider 5 EV charger, 3 Level 1 and 2 Level 2. We consider 3 arrival-departure tuples, (1,8), (1,4) and (5,8). In total there are 12 arrival-departure-demand bins. Table I and Table II indicate the price of charging in \$/kWh and the number of requests accepted for each bin as obtained by our heuristic. We observe that the algorithm runs through two iterations as the demand exceeds the capacity in the first iteration and the scheduling IP is infeasible. The price of each bin is increased according to our heuristic. The total demand satisfied is close to the net capacity of the chargers. We observe that the final number of requests are 0 in most bins except in the bottom right corner of Table II. The corresponding prices are also high compared to the rest.

$D_k$	(1,8)	(1,4)	(5,8)
4	0.167	0.250	0.250
6	0.200	0.333	0.333
8	0.250	0.425	0.425
10	0.333	0.650	0.650

TABLE I

PRICE OF CHARGING IN \$/KWH FOR EACH BIN FOR SMALL INSTANCE

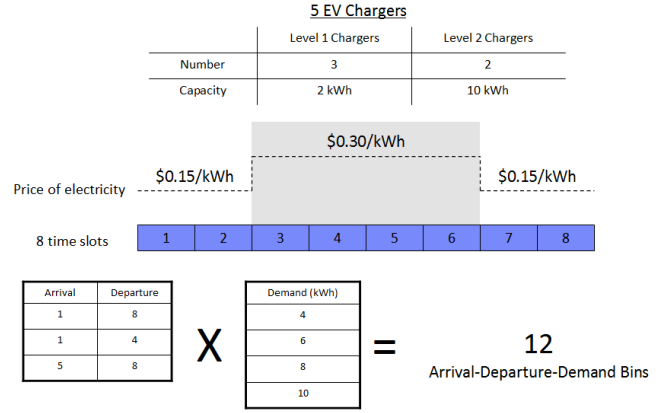


Fig. 1. Set up for small instance

$D_k$	(1,8)	(1,4)	(5,8)
4	0	0	0
6	0	0	0
8	0	3	7
10	0	3	7

TABLE II

THE NUMBER OF ACCEPTED REQUESTS IN EACH BIN FOR SMALL INSTANCE

### B. Example 2 - large instance

Figure 2 shows the experimental set up for a small instance. We consider 50 EV charger, 30 Level 1 and 20 Level 2. We consider 4 arrival-departure tuples, (1,8), (1,4), (3,6) and (5,8). In total there are 16 arrival-departure-demand bins. Table III and Table IV indicate the price of charging in \$/kWh and the number of requests accepted for each bin as obtained by our heuristic. We observe that the algorithm runs through one iteration as the demand is well below the capacity in the first iteration and the scheduling IP is feasible. We observe the following trends in the tables:

- For the same charge demand level, more flexibility, *i.e.*, longer duration stay at the parking lot corresponds to lower charging prices. This can be seen by comparing prices of (1,8) tuple with the rest.
- For the same arrival-departure, more charge demand generally means higher price of charging. There is only one exception to this in the table for (1,8) for demand level 10 kWh. This can be explained by the fact that Level 2 charger has capacity 10 kWh and can complete the task of charging demand 10 kWh in one slot, whereas for 8 kWh requires either entire one slot on Level 2 charger or multiple split slots on Level 1 charger. In that sense, 10 kWh demand provide more flexibility in this context.
- For same duration stays at the parking lot, higher average price of electricity means higher price of charging. This can be seen by comparing prices of (1,4), (3,6) and (5,8).
- Profit maximizing strategy might lead to demand and

prices that are well below the scheduling capacity. Hence, algorithm converges in 1 iteration. In future, we plan to consider examples where we maximize demand.

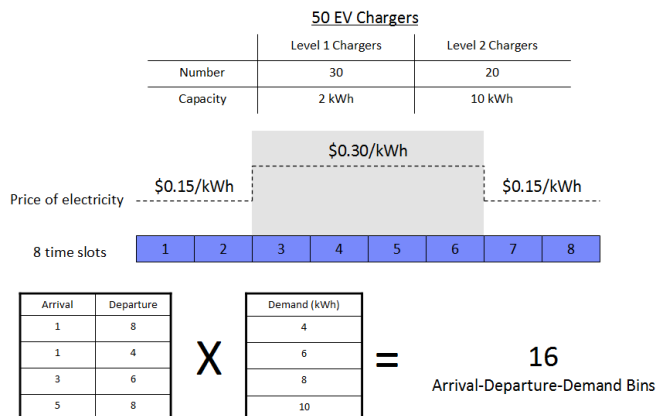


Fig. 2. Set up for large instance

$D_k$	(1,8)	(1,4)	(3,6)	(5,8)
4	0.255	0.279	0.333	0.279
6	0.276	0.333	0.419	0.333
8	0.287	0.394	0.501	0.394
10	0.278	0.424	0.583	0.424

TABLE III

PRICE OF CHARGING IN \$/kWh FOR EACH BIN FOR LARGE INSTANCE

$D_k$	(1,8)	(1,4)	(3,6)	(5,8)
4	13	13	5	13
6	15	15	10	15
8	13	14	11	14
10	8	10	8	10

TABLE IV

THE NUMBER OF ACCEPTED REQUESTS IN EACH BIN FOR LARGE INSTANCE

## VI. CONCLUSION AND FUTURE WORK

In this paper, we propose a combined pricing-scheduling QIP model that iteratively prices and schedules EVs. A pricing module is used to accept/reject charging requests, suggest alternative times and control the right number and types (arrival-departure times, charge demand etc.) of EVs to charge. The scheduling module ensures that the demand can be met subject to price-demand sensitivity and other scheduling constraints. Once the EVs to be accepted have been finalized and their permit prices determined, the scheduling module would need to be run every night once the day-to-day arrival and departure times of each EV is revealed to the operator. Since the QIP formulation is hard to solve, we propose a heuristic to obtain a solution with improved runtime. We build our heuristic based on two critical observations. First, if we consider the relaxed version

of the MIQP, it is a convex programming problem and can be solved quickly to obtain a globally optimal solution. Second, if price of charging permit ( $p_k$ 's) are known, then the number of cars whose request is satisfied are known and the QIP reduces to the scheduling IP which can be solved by the CPLEX relatively quickly. We show the performance of our heuristic on two example problems.

In future work, we plan to address management under uncertain demands and uncertain electricity prices, while minimizing grid losses and impact on the grid. Most EV's and chargers are capable of recording charging data. We plan to use real-world data to study different aspects.

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