

# Model reference based robust tuning of five-parameter 2DoF PID controllers for first-order plus dead-time models

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**Abstract**—The aim of this paper is to present a robust tuning method for two-degree-of-freedom proportional integral derivative controllers for first-order plus dead-time controlled processes. A two-degree-of-freedom Ideal PID with filter control algorithm is selected and its complete set of parameters (included the filter time constant) is obtained. This is based on the use of a model reference optimization procedure with servo and regulatory control closed-loop target transfer functions. The designer is allowed to select a maximum sensitivity  $M_S$  in the range from 2.0 to 1.4.

## I. INTRODUCTION

As has been widely reported [1], the proportional integral derivative (PID) still is the control algorithm most extensively used at processes industry.

The design procedure for a closed-loop PID control system is usually based on low-order linear models identified at its normal operating point. The most used model for self-regulated processes is of first-order plus dead-time (FOPDT). Due to the nonlinear characteristics found in most industrial processes, it is necessary to anticipate the changes in the process characteristics when the operating point changes, assuming certain robustness requirements for the control system.

Starting with Ziegler and Nichols [2] a great number of other tuning procedures have been developed for the PID controller and its variations, as revealed in O'Dwyer's handbook [3]. Initially, only the control system performance was taken into account in the controller design as in the classical tuning rules in [4]–[6], among others.

Later, the control system robustness to the changes in the controlled process characteristics was introduced into the controller design considering the control-loop gain and phase margins [7] or by mean of the maximum of the magnitude of the sensitivity function ( $M_S$ ) [8].

For overdamped controlled processes there are methods such as kappa-tau [9] and AMIGO [1] that provide tuning rules by which to design two-degree-of-freedom (2DoF) PID closed-loop control systems with a specific robustness, but only for one or two maximum sensitivity values. Besides, there is no guarantee that the selected design robustness level will be obtained with either of these two tuning methods [10].

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An alternative tuning method for 2DoF proportional integral derivative controllers is presented in this communication. The proposed approach explicitly considers the trade-off between the performance and robustness of a control system. The distinctive feature of the resultant tuning procedure is the incorporation of the desired robustness level as measured with the maximum sensitivity,  $M_S$ , which is the explicit and only design parameter. The designer may select the desired robustness  $M_S$  level for the control system in the range from 2.0 to 1.4.

## II. PROBLEM FORMULATION

Consider the closed-loop control system in Fig. 1 where  $P(s)$  and  $C(s)$  are the controlled process model and the controller transfer functions, respectively. In this system,  $r(s)$  is the set-point,  $u(s)$  the controller output signal,  $d(s)$  the load-disturbance,  $y(s)$  the process controlled variable,  $n(s)$  the measurement-noise, and  $y'(s)$  the noise contaminated feedback signal. It is assumed that the disturbance enter at process input (load-disturbance).

Problem faced is obtaining controller  $C(s)$  parameters  $\theta_c$  that best match target closed-loop responses to  $r(s)$  and  $y(s)$  step changes with a specified robustness level for first-order plus dead-time models  $P(s)$  with parameters  $\theta_p$ .

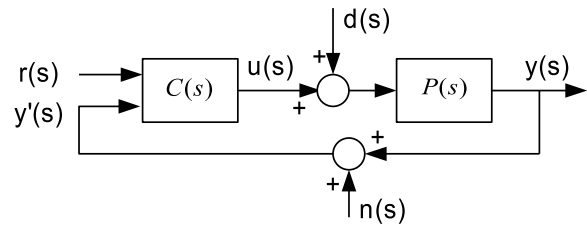


Fig. 1. Closed-loop control system

### A. 2DoF Proportional Integral Derivative Controller

As a first controller option consider the standard two-degree-of-freedom (2DoF) proportional integral derivative (PID) control algorithm whose output is given by the following [9]:

$$u(s) = K_p \left\{ e_p(s) + \frac{1}{T_i s} e_i(s) + \frac{T_d s}{\alpha T_d s + 1} e_d(s) \right\}, \quad (1)$$

with

$$e_p(s) = \beta r(s) - y'(s), \quad (2)$$

$$e_i(s) = r(s) - y'(s), \quad (3)$$

$$e_d(s) = \gamma r(s) - y'(s), \quad (4)$$

where  $K_p$  is the controller gain,  $T_i$  the integral time constant,  $T_d$  the derivative time constant,  $\beta$  and  $\gamma$  the set-point weights, and  $\alpha$  the derivative filter constant.

To avoid an extreme instantaneous change in the controller output signal when a set-point step change occurs normally  $\gamma$  is set to zero [11]. In this case (1) reduces to

$$u(s) = K_p \left( \beta r(s) - y'(s) + \frac{1}{T_i s} [r(s) - y'(s)] \right) - K_p \left( \frac{T_d s}{\alpha T_d s + 1} \right) y'(s), \quad (5)$$

that is denoted as  $PID_2$ .

In general the controller output (5) is given by the following:

$$u(s) = C_r(s)r(s) - C_y(s)y(s). \quad (6)$$

The closed-loop control system output to a change in its inputs is given by

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \quad (7)$$

where

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \quad (8)$$

is the transfer function from set-point to controlled process variable, the *servo control* closed-loop transfer function, and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \quad (9)$$

is the one from load-disturbance to controlled process variable, the *regulatory control* closed-loop transfer function which are related by

$$M_{yr}(s) = C_r(s)M_{yd}(s). \quad (10)$$

### B. Overdamped Controlled Process Models

The overdamped controlled processes are represented by a first-order plus dead-time (FOPDT) linear model given by the transfer function

$$P(s) = \frac{Ke^{-Ls}}{Ts + 1}, \quad \tau_o = \frac{L}{T}, \quad (11)$$

where  $K$  is the model gain,  $T$  the main time constant,  $L$  the dead-time, and  $\tau_o$  the model *normalized dead-time* ( $0.1 \leq \tau_o \leq 2.0$ ). The parameters of the controlled process model (11) are  $\theta_p = \{K, T, L, \tau_o\}$ .

### III. CLOSED-LOOP TRANSFER FUNCTIONS TARGETS

For the proposed model reference robust tuning (MoReRT) procedure the desired control system responses to a step change in its inputs (set-point and load-disturbance) need to be specified.

### A. Overdamped Closed-loop Response Targets

In the 2DoF proportional integral controllers ( $PI_2$ ) tuning presented in [12] the regulatory control closed-loop transfer function target is selected nonoscillatory; for a smooth response; and with no steady-state error, given by

$$M_{yd}^t(s) = \frac{(T_i/K_p)se^{-Ls}}{(\tau_c T s + 1)^2}, \quad (12)$$

where  $\tau_c$  is the dimensionless design parameter.

From (10) the servo control closed-loop transfer function is

$$M_{yr}(s) = \frac{(\beta T_i s + 1)e^{-Ls}}{(\tau_c T s + 1)^2}. \quad (13)$$

Then, to have a response to a set-point step change without oscillation and overshoot, and with no steady-state error, the servo control closed-loop transfer function target is selected as

$$M_{yr}^t(s) = \frac{e^{-Ls}}{\tau_c T s + 1}. \quad (14)$$

Equation (14) implied that  $\beta \rightarrow \tau_c T / T_i$ .

The selection of the closed-loop transfer functions targets (12) and (14) seeks also to obtain, as a side effect, a smooth controller output.

### B. Underdamped Closed-loop Response Targets

In addition to the above it is shown in [13] that it is possible to modify the control system performance to a load-disturbance and set-point step changes without affecting its robustness selecting the regulatory control closed-loop transfer function target with two underdamped dominant poles given by

$$M_{yd}^t(s) = \frac{(T_i/K_p)se^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1}. \quad (15)$$

From (10) the new servo-control closed-loop transfer function is

$$M_{yr}(s) = \frac{(\beta T_i s + 1)e^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1}. \quad (16)$$

Selecting  $\beta = \tau_c T / T_i$  (16) is

$$M_{yr}^t(s) = \frac{(\tau_c T s + 1)e^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1}. \quad (17)$$

The control system have now two design parameters, the *closed-loop relative speed*  $\tau_c$  and the *damping ratio*  $\zeta$ .

The global control system output target is

$$y^t(s) = \frac{(\tau_c T s + 1)e^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1} r(s) + \frac{(T_i/K_p)se^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1} d(s). \quad (18)$$

### IV. $PID_2$ PERFORMANCE ANALYSIS

First consider the  $PID_2$  (5) with five adjustable parameters,  $\theta_c = \{K_p, T_i, T_d, \alpha, \beta\}$ , and a noise-free control system ( $y'(s) = y(s)$ ).

## A. Derivative Filter Constant

1. Considering  $\alpha$  as a tunable parameter following cases were analyzed:

FOPDT process models with  $0.1 \leq \tau_o \leq 2.0$  and robustness levels  $M_S^t \in \{2.0, 1.6, 1.4\}$ .

In all these cases  $\alpha \rightarrow 0$ . This is, the Standard PID (with derivative filter) tends to be an Ideal PID (with a nonproper transfer function).

The derivative mode gain at high frequencies is  $1/\alpha$  then, if  $\alpha$  is very low the feedback signal noise, if any, will be amplified.

2. Effect of  $\alpha$  Selected in the Range from 0.05 to 0.20

In commercial controllers  $\alpha$  normally is fixed, or adjustable, in the range from 0.05 to 0.20 [1].

Now for the same controlled process and robustness level as  $\alpha$  decrease the control system performance measured with the integrated absolute error increase but the control effort total variation increase. In all cases the performance is not very sensitive to the changes in the derivative filter constant.

Considering the above and the possible noise amplification the default value  $\alpha = 0.1$  [11] is recommended, for a maximum derivative mode high frequency noise amplification limit of 10 (20 db).

Then, the  $PID_2$  adjustable parameters reduce to four,  $\theta_c = \{K_p, T_i, T_d, \beta\}$ .

## B. Closed-loop Transfer Functions Targets Damping Ratio

In [13] was shown than in the proportional integral ( $PI_2$ ) controller case using an underdamped closed-loop output with damping ratios  $\zeta$  in the range from 0.7 to 0.8 it is possible to improve the control system performance, specially the servo-control response, but adversely affecting the control effort characteristics.

It is also shown that the underdamped responses impose a restriction to the obtainable robustness, specially if a low damping ratio target is specified.

The tests with the  $PID_2$  controllers with  $\alpha = 0.1$  show the same behavior and that a  $\zeta \approx 0.7$  would provide a good balance for the performance (integrated absolute error) control effort total variation trade-off.

## V. 2DOF IDEAL PID CONTROLLER WITH FILTER

An alternative PID algorithm implementation is the Ideal PID with filter given by the output equation [9]:

$$u(s) = K_p^* \left( \beta^* + \frac{1}{T_i^* s} \right) r(s) - K_p^* \left( 1 + \frac{1}{T_i^* s} + T_d^* s \right) \left( \frac{1}{T_f s + 1} \right) y'(s). \quad (19)$$

In this case the filter is applied only to the feedback signal (just as a "measurement noise filter").

In the following we will use the 2DoF Ideal PID with filter given by (19) denoted by  $PID_{2F}$ . Its parameters are  $\theta_c^* = \{K_p^*, T_i^*, T_d^*, T_f, \beta^*, \gamma^* = 0\}$ . Then, the  $PID_{2F}$  controller have five adjustable parameters.

It is possible to obtain a Standard 2DoF PID controller ( $PID_2$ ) (5) equivalent to the Ideal PID with filter ( $PID_{2F}$ ) (19) using the following relations from [14]:

$$K_p = F_c K_p^*, \quad (20)$$

$$T_i = F_c T_i^*, \quad (21)$$

$$T_d = \frac{T_d^*}{F_c} - T_f, \quad T_d^* > F_c T_f, \quad (22)$$

$$\alpha = \frac{F_c T_f}{T_d^* - F_c T_f}, \quad (23)$$

$$\beta = \frac{\beta^*}{F_c}, \quad (24)$$

$$\gamma = 0, \quad (25)$$

$$F_c = 1 - \frac{T_f}{T_i^*}, \quad T_i^* > T_f. \quad (26)$$

The  $PID_{2F}$  is a more general control algorithm and not always an equivalent  $PID_2$  may be obtained as shown in (22) and (26).

## A. Closed-loop Transfer Functions Targets

To take into account the filter of the  $PID_{2F}$  controller the global control system output target  $y^t(s)$  (18) is modified as follow:

$$y^t(s) = \frac{(\beta^* T_i^* s + 1)(T_f s + 1)e^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1} r(s) + \frac{(T_i^*/K_p^*)s(T_f s + 1)e^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta \tau_c T s + 1} d(s). \quad (27)$$

## VI. CONTROLLER DESIGN

In what follows the complete set of  $PID_{2F}$  controller parameters  $\theta_c^* = \{K_p^*, T_i^*, T_d^*, T_f, \beta^*\}$  are obtained considering, at the same time, the regulatory control and the servo-control performance, to obtain a controller with a targeted *servo/regulatory performance combination* that will also produce a closed-loop control system with a specific robustness level.

The closed-loop control system response target (27) can be rewritten in the time domain as follows:

$$y^t(t) = y_r^t(t) + y_d^t(t), \quad (28)$$

where  $y_r^t(t)$  is the servo-control step response target and  $y_d^t(t)$ , the regulatory control step response target.

## A. Controller Optimization

For the controller design, the following overall cost functional is optimized:

$$J_T(\tau_c, \zeta, \theta_c^*, \theta_p) \doteq \int_0^\infty [y_r^t(\tau_c, \zeta, \theta_p, t) - y_r(\theta_c^*, \theta_p, t)]^2 dt + \int_0^\infty [y_d^t(\tau_c, \zeta, \theta_c^*, \theta_p, t) - y_d(\theta_c^*, \theta_p, t)]^2 dt, \quad (29)$$

where  $y_r^t$  and  $y_d^t$  are the servo control target response and the regulatory control target response, respectively, and  $y_r$

and  $y_d$  the corresponding control system responses with the controller with variable parameters.

The optimum controller parameters  $\theta_c^o$  are such that

$$J_T^o \doteq J_T(\tau_c, \zeta, \theta_c^o, \theta_p) = \min_{\theta_c^*} J_T(\tau_c, \zeta, \theta_c^*, \theta_p). \quad (30)$$

Note that  $\theta_c^o = \theta_c^o(\theta_p, \tau_c, \zeta)$ .

Moreover, for each  $\theta_c^o$  set obtained, the closed-loop control system robustness is measured using the maximum sensitivity  $M_S$ , which is defined as follows:

$$M_S \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C_y(j\omega)P(j\omega)|}. \quad (31)$$

Standard  $M_S$  values range from 2.0 (minimum robustness) to 1.4 (high robustness).

In the proposed one-step optimization of (29) the control system target response is given by (27) where its design parameters,  $\tau_c$  and  $\zeta$ , are restricted by the minimum target robustness,  $M_S^t$ . The optimization problem stated is a *model match problem* instead of a performance optimization problem as in the traditional two-step  $PI_2/PID_2$  controller optimization procedure. Matching the selected  $y_{yd}^t$  depends on  $(K_p^*, T_i^*, T_d^*, T_f)$  but matching the selected  $y_{yr}^t$  depends on all the controller parameters  $(K_p^*, T_i^*, T_d^*, T_f, \beta^*)$ . So, the one-step approach will provide a better overall optimization.

Using the controlled process model gain,  $K$ , and time constant,  $T$ , as well as the transformation  $\hat{s} = Ts$ , the controlled process (11) and the  $PID_{2F}$  controller (19) transfer functions can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{e^{-\tau_o \hat{s}}}{\hat{s} + 1}, \quad (32)$$

$$\hat{C}_r(\hat{s}) = \kappa_p^* \left( \beta^* + \frac{1}{\tau_i^* \hat{s}} \right), \quad (33)$$

$$\hat{C}_y(\hat{s}) = \kappa_p^* \left( 1 + \frac{1}{\tau_i^* \hat{s}} + \tau_d^* \hat{s} \right) \left( \frac{1}{\tau_f \hat{s} + 1} \right), \quad (34)$$

where

$$\kappa_p^* = K_p^* K, \quad \tau_i^* = \frac{T_i^*}{T}, \quad \tau_d^* = \frac{T_d^*}{T}, \quad \tau_f = \frac{T_f}{T}, \quad (35)$$

are the normalized gain, integrating time, derivative time and filter time constant of the controller, respectively.

The normalized dead-time of the model  $\tau_o$  is selected in the range from 0.1 to 2.0 to consider both lag dominant and dead-time dominant controlled processes.

To select the target closed-loop damping ratio  $\zeta$  a performance (integrated absolute error) to control effort total variation use trade-off analysis is made. As with the  $PID_2$  case  $\zeta = 0.70$  provides a good balance of these two response characteristics.

Starting with a low closed-loop relative speed  $\tau_c$  value, that produces a control system with fast responses but with low robustness (high  $M_S$  value), during the optimization process the design parameter ( $\tau_c$ ) is iteratively increased in such a way that the robustness level of the resulting closed-loop system is raised to a specific target  $M_S^t$  value selected in the range from 2.0 to 1.4.

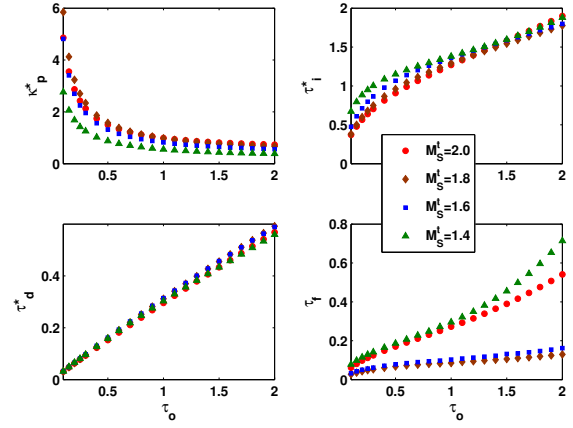


Fig. 2.  $PID_{2F}$  Controller Parameters for FOPDT models

For optimization of (29) a Nelder-Mead simplex-based algorithm [15] is used.

From the optimization results all controllers set-point weights  $\beta = 0$ .

Figure 2 shows the optimum normalized controller parameters ( $\kappa_p^*$ ,  $\tau_i^*$ ,  $\tau_d^*$ ,  $\tau_f$ ) obtained for FOPDT models with normalized dead-time  $\tau_o$  in the range from 0.1 to 2.0, and robustness targets levels  $M_S^t \in \{2.0, 1.8, 1.6, 1.4\}$ .

This shows the influences of the controlled process model normalized dead-time ( $\tau_o$ ) and the desired robustness ( $M_S^t$ ) over the controller parameters required to meet the target step responses.

### B. Tuning Equations

The controller parameters obtained from the optimization procedure are used to fit the controller parameter equations for the four robustness levels considered.

The optimum normalized controller parameters (35) can be obtained with the following equations:

$$\kappa_p^* = a_0 + a_1 \tau_o^{a_2}, \quad (36)$$

$$\tau_i^* = b_0 + b_1 \tau_o^{b_2}, \quad (37)$$

$$\tau_d^* = c_1 \tau_o^{c_2}, \quad (38)$$

$$\tau_f = d_0 + d_1 \tau_o^{d_2}, \quad (39)$$

$$\beta^* = 0. \quad (40)$$

Constants  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$  in (36) to (39) are shown in Table I.

### C. MoReRT Control System Robustness

The robustness obtained with (36) to (39) for each normalized dead-time in the range analyzed are shown in Fig. 3. As can be seen proposed tuning equations provide a control system with the selected target robustness, except for a very low normalized dead-time ( $\tau_o = 0.1$ ).

TABLE I  
CONSTANTS FOR TUNING RELATIONS (36) TO (39)

$M_S^t$	2.0	1.8	1.6	1.4
$a_0$	0.3745	0.3067	0.2458	0.1047
$a_1$	0.6237	0.6719	0.5753	0.4553
$a_2$	-0.8575	-0.9161	-0.8997	-0.7672
$b_0$	0.0825	-0.336	-0.4981	0.4383
$b_1$	1.199	1.639	1.874	0.9669
$b_2$	0.5727	0.3507	0.2744	0.5062
$c_1$	0.2948	0.3091	0.3114	0.2997
$c_2$	0.9495	0.9471	0.9327	0.9075
$d_0$	0.06395	0	0.01938	0.1074
$d_1$	0.2104	0.08934	0.0864	0.1868
$d_2$	1.137	0.4708	0.6525	1.641

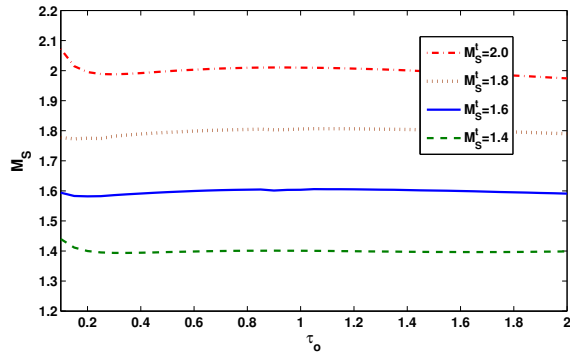


Fig. 3. Robustness of MoReRT Controllers

#### D. Controller High Frequency Gain

In industrial applications generally PI control has been preferred rather than PID argued that the derivative mode amplifies the high frequency measurement noise. The noise amplification depends on the controller normalized high frequency gain that are given by

$$\kappa_{PI}^\infty = \kappa_p = K_p K, \quad (41)$$

for a PI controller, by

$$\kappa_{PID_2}^\infty = \kappa_p \left(1 + \frac{1}{\alpha}\right) = 11K_p K, \quad (42)$$

for a 2DoF Standard PID (with  $\alpha = 0.1$ ), and by

$$\kappa_{PID_{2F}}^\infty = \frac{\kappa_p^* T_d^*}{\tau_f} = \frac{K_p K T_d^*}{T_f}, \quad (43)$$

for and Ideal PID with filter ( $PID_{2F}$ ).

The normalized high frequency gain of the  $PID_{2F}$  controller tuned with the proposed method is shown in Fig. 4.

### VII. EVALUATION OF THE MORERT TUNING

Before compare the proposed tuning with other robust tuning rules we analyze the robustness obtained with following methods: SSC [16], an analytically deducted (servo control) method for 1DoF Ideal PID controllers; TGF [17], based

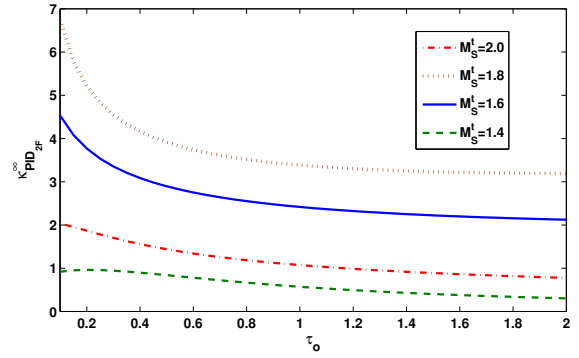


Fig. 4.  $PID_{2F}$  Controller Normalized High Frequency Gain

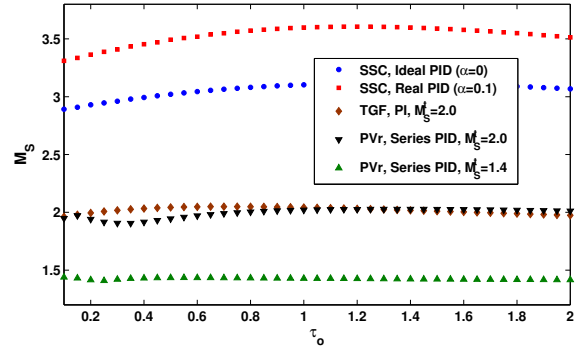


Fig. 5. Robustness accomplishment

on a performance and robustness optimization for 2DoF PI controllers; and PVr [18], based on a regulatory control performance (integrated absolute error) and robustness optimization for 1DoF Series PID controllers. The robustness of these methods is shown in Fig. 5.

As can be seen the SSC method is not robust, the TGF and the PVr methods produce system with robustness near to the target values.

Consider now the fourth-order controlled process proposed as benchmark in [19] and given by the transfer function:

$$P_e(s) = \frac{1}{(s+1)(0.5s+1)(0.25s+1)(0.125s+1)}. \quad (44)$$

Using a two-point identification procedure the parameters of a FOPDT model approximation of (44) are:  $K = 1$ ,  $T = 1.247$ , and  $L = 0.691$  ( $\tau_o = 0.554$ ).

Controllers parameters, robustness ( $M_S^m$ , with the model), performance (measured with the integrated absolute error,  $J_{ed}$  and  $J_{er}$ ) and control effort total variation ( $TV_{ud}$  and  $TV_{ur}$ ) to unitary step changes, of control systems obtained with proposed tuning and PVr method are listed in Table II. For PVr tuning a Series PID controller with the derivative mode applied to the feedback signal only is used (an ‘‘Industrial’’ PID).

Control system outputs to a 20% step set-point change followed by a 10% step load-disturbance change are shown in Fig. 6.

TABLE II  
EXAMPLE- CONTROLLER PARAMETERS AND PERFORMANCE

$M_S^t$	MoReRT			PVr†	
	2.0	1.6	1.4	2.0	1.4
$K_p$	1.409	1.225	0.821	0.922	0.581
$T_i$	1.169	1.366	1.440	0.542	0.609
$T_d$	0.210	0.224	0.219	0.443	0.405
$T_f$	0.214	0.098	0.222	—	—
$\beta$	0	0	0	1	1
$K^\infty$	1.383	2.813	0.8075	9.22	5.81
$M_S^m$	2.001	1.599	1.397	1.958	1.434
$J_{ed}$	1.113	1.116	1.754	0.821	1.251
$TV_{ud}$	1.742	1.173	1.021	1.714	1.290
$J_{er}$	2.171	2.383	2.972	1.655	1.893
$TV_{ur}$	2.117	1.209	1.008	3.011	1.684

† Series PID

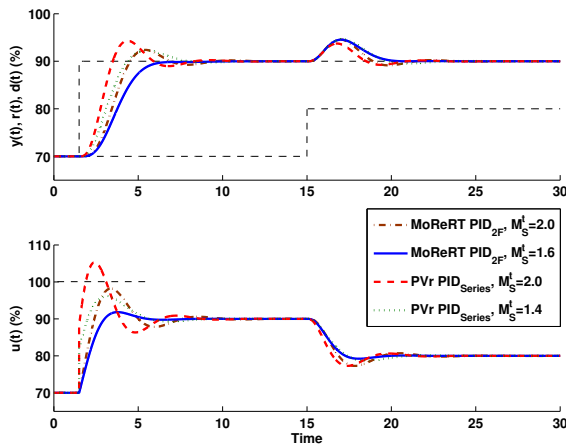


Fig. 6. Robustness accomplishment

From the table data it can be seen that target robustness are better accomplished by the MoReRT method. The PVr tuning provides more performance (it is based on a regulatory control performance optimization with robustness restriction procedure) but with higher servo control controller output changes. If a set-point weight or filter is added to the PVr controller (not provided by the method) its servo control performance will be affected.

It is important to note that the PVr Series PID controllers high frequency gains are about seven times the corresponding to the  $PID_{2F}$  MoReRT controllers. The proposed controllers provide a better measurement noise filtering capability.

### VIII. CONCLUSIONS

The proposed MoReRT tuning method for 2DoF proportional integral derivative controller with filter ( $PID_{2F}$ ) guarantees the design robustness level for first-order plus dead-time models with  $0.1 \leq \tau_o \leq 2.0$  using only one design parameter, which is the required closed-loop control system robustness as measured with the maximum sensitivity  $M_S$ . Tuning equations were obtained for  $M_S \in$

$\{2.0, 1.8, 1.6, 1.4\}$ , allowing the designer to select the required robustness.

The low high frequency gain of the resulting  $PID_{2F}$  controllers allows to reduce the unwanted effects of the measurement noise.

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### REFERENCES

- [1] K. Åström and T. Hägglund, *Advanced PID Control*. Research Triangle Park, NC 27709, USA: ISA - The Instrumentation, Systems, and Automation Society, 2006.
- [2] J. G. Ziegler and N. B. Nichols, "Optimum settings for Automatic Controllers," *ASME Transactions*, vol. 64, pp. 759–768, 1942.
- [3] A. O'Dwyer, *Handbook of PI and PID Controller Tuning Rules*, 2nd ed. Imperial College Press, London, UK, 2006.
- [4] G. H. Cohen and G. A. Coon, "Theoretical considerations of retarded control," *ASME Transactions*, vol. 75, pp. 827–834, 1953.
- [5] A. M. López, J. A. Miller, C. L. Smith, and P. W. Murrill, "Tuning controllers with error-integral criteria," *Instrumentation Technology*, vol. 14, pp. 57–62, 1967.
- [6] A. Rovira, P. W. Murrill, and C. L. Smith, "Tuning controllers for setpoint changes," *Instrumentation & Control Systems*, vol. 42, pp. 67–69, 1969.
- [7] C.-H. Lee, "A Survey of PID Controller Design Based on Gain and Phase Margins," *Int. J. of Computational Cognition*, vol. 2(3), pp. 63–100, 2004.
- [8] V. M. Alfaro, R. Vilanova, and O. Arrieta, "Maximum Sensitivity Based Robust Tuning for Two-Degree-of-Freedom Proportional-Integral Controllers," *Ind. Eng. Chem. Res.*, vol. 49, pp. 5415–5423, 2010.
- [9] K. J. Åström and T. Hägglund, *PID Controllers: Theory, Design and Tuning*. Instrument Society of America, Research Triangle Park, NC, USA, 1995.
- [10] R. Vilanova, V. M. Alfaro, O. Arrieta, and C. Pedret, "Analysis of the claimed robustness for PI/PID robust tuning rules," in *18th IEEE Mediterranean Conference on Control and Automation (MED2010)*, June 23-25, Marrakech, Morocco, 2010.
- [11] A. Visioli, *Practical PID Control*. Springer Verlag London Limited, 2006.
- [12] V. M. Alfaro and R. Vilanova, "Model-reference robust tuning of 2DoF PI controllers for first- and second-order plus dead-time processes," *Journal of Process Control*, vol. 22, pp. 359–374, 2012.
- [13] —, "Performance Analysis of Model-Reference Robust Tuned 2DoF PID Controllers for Over Damped Processes," in *IEEE 20th Mediterranean Conference on Control and Automation (MED 2012)*, 2012, July 3-6, Barcelona, Spain.
- [14] —, "Conversion Formulae and Performance Capabilities of Two-Degree-of-Freedom PID Control Algorithms," in *17th IEEE International Conference on Emerging Technologies & Factory Automation (ETFA 2012)*, 2012, september 17-21, Kraków, Poland.
- [15] The Mathworks, Inc., *Optimization Toolbox User's Guide (R2011a)*. Natick, MA, USA.: The MathWorks, Inc., 2011.
- [16] R. p. Sree, M. N. Srinivas, and M. Chidambaram, "A simple method of tuning PID controllers for stable and unstable FOPDT systems," *Computers & Chemical Engineering*, vol. 28, pp. 2201–2218, 2004.
- [17] S. Tavakoli, I. Griffin, and P. J. Fleming, "Robust PI controller for load disturbance rejection and setpoint regulation," in *IEEE Conference on Control Applications*, 2005, pp. 1015–1020, IEEE Conference on Control Applications, August 28-31, Toronto, Canada.
- [18] F. Padula and A. Visioli, "Tuning rules for optimal PID and fractional-order PID controllers," *Journal of Process Control*, vol. 21, pp. 69–81, 2011.
- [19] K. J. Åström and T. Hägglund, "Benchmark Systems for PID Control," in *IFAC Digital Control: Past, Present and Future of PID Control (PID'00)*, 2000, 5-7 April., Terrasa, Spain.