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Integrating the RTO in the MPC: an adaptive gradient-based approach

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***Abstract*— Model Predictive Control (MPC) is the most used advanced control technique in process industries, since it ensures stability, constraints satisfaction and convergence to the setpoint. The optimal setpoint is calculated by the Real Time Optimizer (RTO), minimizing the economic objective taking into account the operational limits of the plant. Since RTO employs complex stationary nonlinear models to perform the optimization and a larger sampling time than the controller, the economic setpoints calculated by the RTO may be inconsistent for the MPC layer and the economic performance of the overall controller may be worse than expected. The aim of this work is to propose an MPC controller that explicitly integrates the RTO into the MPC control layer. The proposed strategy is based on the MPC for tracking; the optimization problem to be solved only requires one evaluation of the gradient of the economic cost function at each sampling time. Based on this gradient, a second order approximation of the economic function is obtained and used in the MPC optimization problem resulting in a convex optimization problem. Recursive feasibility and convergence to the optimal equilibrium point is ensured.**

I. Introduction

In the process industries, the control task is usually performed by means of a hierarchical control structure [1]: at the top, an economic scheduler and planner determines the whole plant production (level, quality, etc.). In the next layer, a Real Time Optimizer (RTO), computes the stationary targets minimizing an economic criterion and according to the information that it receives from the scheduler. Then, the targets computed by the RTO are sent to the MPC control level which calculates the control actions necessary to drive the plant to the targets.

The communication between the RTO and the MPC layers may be inconsistent. This is mainly due to the fact that the RTO is usually based on a complex nonlinear stationary model of the plant, while the MPC take into account a simplified dynamic model. Moreover, the RTO sampling time is larger than that of the MPC, since it optimizes only at a stationary point. As a consequence, problems that go from unreachability of the target to poor economic performance are not unusual. A proper strategy to unify these (probably competing) objectives is, hence, highly desirable.

In [2], [3] the authors propose an MPC for tracking, characterized by a modified cost function that includes an additional term, the so-called offset cost function, which minimizes the distance from an artificial steady state to the desired target, in such a way that the recursive feasibility, the convergence and the local optimality of the control strategy can be assured. Similar strategies, which includes slack variables and are formulated for input increment models, are also presented in [4] and [5].

All these approaches assume an implicit separation between transient and stationary objectives, by including additional terms to the traditional MPC cost. Since this additional terms can assume different forms (under some mild assumptions), they open the door to the idea of including stationary economic objectives in the own MPC problem. In such a way, a one-layer RTO-MPC could be proposed [6]. The drawback of this strategy is that, the high nonlinearity of the economic RTO cost function, turns the one-layer MPC cost also nonlinear and difficult to be solved. In [7] the authors present the formulation and industrial application of a combined RTO/MPC controller applied to a fluidized-bed catalytic cracker, FCC, in which the RTO economic cost function is part of the MPC cost function. In [8], the gradient of the economic objective function is included in the controller cost function, in order to obtain a computationally low-cost strategy. An enhanced formulation of this approach is presented in [9], where a suboptimal MPC strategy is presented, which ensures recursive feasibility and convergence to the (economically) optimal target, with a reduced computational cost.

In this work, the one-layer MPC strategy is improved, by adding a second order approximation of the RTO cost function to the MPC cost. In this way the optimization problem is turned into a convex problem than only requires

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one evaluation of the gradient of the economic cost function per sampling time. Recursive feasibility and convergence to the (economically) optimal steady state are ensured.

II. Problem Statement

In the control structure aimed to optimize the economic cost of the operation of the plant, the Real Time Optimizer is the responsible for calculate the optimal equilibrium point where the plant should operate. This optimal equilibrium is obtained from the solution of the following optimization problem

$$\begin{aligned}\min_{x_s,u_s} \quad & \Phi(x_s,u_s,p)\\ s.t. \quad & F(x_s,u_s)=0\\ & H(x_s,u_s)\leq 0\end{aligned}$$

where x_s and u_s denote a steady state and input of the plant, $\Phi(x_s,u_s;p)$ is the cost function of the process that depends of a set of parameters p such as unitary prices, costs, etc.; $F(x_s,u_s)=0$ is the stationary model of the plant (updated with the reconciliated data and estimated parameters) and the inequalities $H(x_s,u_s)\leq 0$ define the operational constraints of the plant.

The optimal operation point is characterized by a set of controlled variables y_s denoted as targets. Assuming that this set of controlled variables univocally defines an equilibrium point of the plant, there exists functions such that every solution of $F(x_s,u_s)=0$ is such that $x_s=g_x(y_s)$ and $u_s=g_u(y_s)$. Then substituting these ones in the RTO optimization problem, this can be rewritten as

$$\begin{aligned}\min_{y_s} \quad & f_{eco}(y_s,p)\\ s.t. \quad & h_q(y_s)\leq 0, \quad q\in\mathbb{I}_{1:n_h}\end{aligned}$$

where f_{eco} and h_q are strictly related to $\Phi(x_s,u_s,p)$, $F(x_s,u_s)$ and $H(x_s,u_s)$ but represented as functions of y_s. The optimal target that defines the economically optimal operation point is denoted as y_t. The feasible set of this optimization problem is denoted as $\mathcal{Y}_t$.

The following conditions are assumed:

Assumption 1: The functions f_{eco} and h_q are assumed to be convex and the optimal solution of the RTO optimization problem is unique. Besides $\forall y_s\in\mathcal{Y}_t$ there exists a $\mathcal{K}_\infty$ function α such that

$$f_{eco}(y_s,p)-f_{eco}(y_t,p)\geq\alpha(|y_s-y_t|)$$

□

Furthermore, the economic cost function must satisfy the following assumption.

Assumption 2: The gradient of $f_{eco}(y,p)$ and the gradient of $h_q(y)$ are Lipschitz continuous in $\mathcal{Y}_t$, that is,

$$\|\nabla_y f_{eco}(y_1;p)-\nabla_y f_{eco}(y_2;p)\|\leq\rho_f|y_1-y_2|$$

and

$$\|\nabla_y h_q(y_1)-\nabla_y h_q(y_2)\|\leq\pi_q|y_1-y_2|, \quad q\in\mathbb{I}_{1:n_h}$$

□

The role of the predictive controller is to regulate the plant to the optimal operation point by steering the controlled variables to the optimal target y_t provided by the RTO. The MPC is based on a prediction model that it is given by a linear time-invariant discrete time model

$$x^+ = Ax+Bu \tag{1}$$
$$y = Cx+Du \tag{2}$$

where $x\in\mathrm{I\!R}^n$ is the system state, $u\in\mathrm{I\!R}^m$ is the current control vector, x^+ is the successor state and $y\in\mathrm{I\!R}^p$ is the set of controlled variables of the plant. This prediction model fulfills the following hypothesis.

Assumption 3: The pair (A,B) is controllable and the state is measured at each sampling time.

The linear prediction model describes the transient of the system, while the nonlinear nature of the plant is considered in the RTO by means of the stationary model of the plant $F(x,u)=0$.

The solution of this system for a given sequence of control inputs $\mathbf{u}=\{u(0),\cdots,u(j-1)\}$ and an initial state x is denoted as $x(j)=\phi(j;x,\mathbf{u})$, where $x=\phi(0;x,\mathbf{u})$. The state of the system and the control input applied at sampling time k are denoted as $x(k)$ and $u(k)$ respectively.

The system is subject to constraints on state and input:

$$(x(k),u(k))\in Z \tag{3}$$

for all $k\geq 0$, where $X\subset\mathbb{R}^n$ and $U\subset\mathbb{R}^m$.

Assumption 4: The set Z is convex, closed and contains the origin in its interior.

We define the set of admissible equilibrium points of the prediction model as

$$\begin{aligned}\mathcal{Z}_s &= \{(x,u)\in\gamma Z \mid x=Ax+Bu\}\\ \mathcal{X}_s &= proj_x(\mathcal{Z}_s)\end{aligned}$$

where γ is a constant contained in $(0,1)$, but arbitrarily close to 1. This is added to avoid those equilibrium points where the constraints are active.

III. The two layer MPC structure

In the hierarchical optimal controller, there exists a separation of objectives, models and time-scales between the different layers. While the RTO optimizes the operation of the plant at medium time-scales, the advanced control scheme deals with the tracking and disturbance rejection problem at a faster time-scale [1]. The main disadvantages of the RTO are that the control structure exhibits a slow reaction to process variations, for instance, disturbances, due to the infrequent solution of the RTO and the existing mismatches between the model used in the RTO and the dynamic model used by the advanced controller. The model mismatch may render the economic target calculated by the RTO inconsistent with the dynamic model or the constraints used in the advanced control [10].

In order to enhance the economic performance, some methods tending to reduce the gap between the predictive controller and RTO have been proposed. One of the solutions widely used is the addition of the steady state target optimization (SSTO) with the MPC [11], [12]. In this case, the advanced control is split into two layers: in the upper level, denoted as steady state target optimizer (SSTO), the setpoint of the predictive control (x_s^*, u_s^*) is calculated by solving a mathematical programming problem as follows

$$\begin{aligned}(x_s^*, u_s^*) = \arg\min_{x_s,u_s} \quad & \ell_{eco}(y_s - y_t)\\ s.t. \quad & x_s = Ax_s + Bu_s + \hat{d}\\ & y_s = Cx_s + Du_s\\ & (x_s, u_s) \in Z\end{aligned}$$

where $\ell_{eco}(y_s - y_t)$ is a local approximation of the profit function f_{eco}, typically a linear or a quadratic function. The optimal setpoint is then calculated taking into account information from the RTO and using as plant model the prediction model of the MPC, leading to a reduction of the inconsistencies [1]. Notice that the setpoints are updated with the same time scale than the MPC and can take into account the estimated mismatches between the linear model and the plant $\hat{d}$.

In the lower level, the predictive controller is designed to regulate the plant to the desired setpoints. This is derived from the solution of the following optimization problem [13]:

$$\begin{aligned}\min_{\mathbf{u}} \quad & \sum_{j=0}^{N-1} \ell(x(j) - x_s^*, u(j) - u_s^*)\\ s.t. \quad & x(0) = x,\\ & x(j+1) = Ax(j) + Bu(j) + \hat{d}, \quad j \in \mathbb{I}_{0:N-1}\\ & (x(j), u(j)) \in Z, \quad \in \mathbb{I}_{0:N-1}\\ & x(N) = x_s^*\end{aligned}$$

where $\ell(\cdot,\cdot)$ is a positive definite function that measures the tracking error with the setpoint (x_s^*, u_s^*) and x is the state of the plant. The control law is derived by means of the receding horizon technique: $\kappa_N(x) = u^0(0;x)$ and this ensures asymptotic stability of the closed-loop system [14].

This two layer implementation has demonstrated to be a practically successful solution to the optimal operation of the plant. However it has been demonstrated that the economic performance achieved could be enhanced. On the other hand, throughout the operation of the plant, the economic target calculated by RTO may experience frequent changes derived, for instance, from changes in the profit function due to variations of the parameters p of economic criteria. This change of the economic target may lead to a loss of the feasibility of the predictive controller.

In the following section, a possible solution to these problems is presented. This controller integrates the RTO and MPC in a single layer, leading to an enhancement of the performance, and ensures stability and feasibility for any variation of the parameters p.

IV. The one layer economic MPC strategy

In this section, a controller that unifies the dynamic and economic control objectives is presented. This controller is formulated following [2], [3], but considering the offset cost function as the economic objective. The controller cost function is hence given by:

$$\begin{aligned}V_N(x, \hat{d}, p; \mathbf{u}) \quad = \quad & \sum_{j=0}^{N-1} \ell(x(j) - x(N\text{–}1), u(j) - u(N\text{–}1))\\ & + f_{eco}(y(N\text{–}1), p)\end{aligned}$$

where $\ell(z, v)$ is a positive definite function of the form $\ell(z, v) = \|z\|_Q^2 + \|v\|_R^2$ (for appropriate matrices Q and R), and $f_{eco}(y, p)$ is the cost function of the RTO.

Notice that the first term of the cost can be considered as a transitory term if the pair $(x(N\text{–}1), u(N\text{–}1))$ is forced - by means of an additional constraint in the corresponding optimization problem - to be a non-fixed (admissible) equilibrium point. Furthermore, the second term can be considered as a stationary term, in the sense that it only tries to move the equilibrium point to which the transitory term steers the system (characterized in this case by $y(N\text{–}1)$), to a point that minimizes the stationary economic objective.
For any current state x, the optimization problem $P_N(x, \hat{d}, p)$ to be solved is given by:

$$\min_{\mathbf{u}} V_N(x, \hat{d}, p; \mathbf{u}) \tag{4a}$$
$$s.t.\ x_0 = x, \tag{4b}$$
$$x(j+1) = Ax(j) + Bu(j) + \hat{d}, \quad j \in \mathbb{I}_{0:N-1} \tag{4c}$$
$$y(j) = Cx(j) + Du(j) \tag{4d}$$
$$(x(j), u(j)) \in Z, \qquad j \in \mathbb{I}_{0:N-1} \tag{4e}$$
$$x(N) = x(N-1) \tag{4f}$$
$$h_q(y(N-1)) \le 0, \quad q \in \mathbb{I}_{0:n_h} \tag{4g}$$

In this optimization problem, x, p and the estimated disturbance $\hat{d}$ are the parameters, while the input sequence $\mathbf{u} = \{u(0), \cdots, u(N-1)\}$ is the optimization variable.
As it was said, the pair $(x(N-1), u(N-1))$ defines a strictly admissible equilibrium point, such that $x(N) = x(N-1)$ is in $\mathcal{X}_s$.
The control law, following the receding horizon policy, is given by $\kappa_N(x, p) = u^0(0; x)$, where $u^0(0; x)$ is the first element of the solution sequence $\mathbf{u}^0(x)$.

Remark 1: The domain of attraction of the proposed control strategy is given by the states that can be admissibly steered in $N-1$ steps to the equilibrium set $\mathcal{X}_s$. This set is the $(N-1)$-step stabilizable set from X to $\mathcal{X}_s$.
Assuming that the estimated disturbance $\hat{d}$ is constant, if f_{eco} and $h_q(y)$ are convex functions then this controller ensures that the trajectory of the controlled systems satisfy the constraints and converges to the optimal target y_t [3]. Furthermore, if the parameters of the cost function p varies, feasibility and asymptotic stability to the (new) optimal target is preserved. If $\hat{d}$ is time-varying, its evolution must be slow enough to avoid a possible feasibility loss.

The main drawback of the aforementioned strategy, however, is neither the recursive feasibility, nor the convergence to the economic optimum, but the computational burden associated to optimization problem. In fact, the solution of the optimization problem $P_N(x,\hat{d},p)$ may require a large number of evaluations of the economic cost functions per sampling time and this may be computationally prohibitive, and so, the on-line implementation of this one-layer strategy may become impractical. To overcome this problem, instead of directly solving the complex one-layer problem, a (local) approximated problem, which is an upper bound of the original one, is proposed in this work. By means of an appropriate algorithm to update this local approximation, convergence to the target can be assured.

Remark 2: A different way to approach this problem is considering the so-called economic MPC formulation [15], in which $f_{eco}(x,u,p)$ is directly taken as stage cost of the MPC controller. This solution is mainly proposed to improve the economic optimality of the closed loop during the transient. This implies the solution of a more complex optimization problem. Besides, the stability and recursive feasibility may be lost in case of changes in the economic cost function.

V. Gradient-based strategy for the one-layer MPC

Consider that a suitable target z is chosen to calculate the approximation of $f_{eco}(y,p)$. This target must be a feasible solution of the RTO, that is, $z \in \mathcal{Y}_t$.

It can be proved that the following lemma holds

Lemma 1: Consider that f_{eco} and the functions $h_q(y)$ satisfy assumption 2 and let $z \in \mathcal{Y}_t$, then

$$\begin{aligned} f_{eco}(y,p) &\le f_{eco}(z,p) + \nabla_y f_{eco}(z,p)^T(y-z) \\ &\quad + \frac{\rho_f}{2}\|y-z\|^2 \end{aligned}$$

and

$$\begin{aligned} h_q(y) &\le h_q(z) + \nabla_y h_q(z)^T(y-z) \\ &\quad + \frac{\pi_q}{2}\|y-z\|^2, \quad q \in \mathbb{I}_{1:n_h} \end{aligned}$$

for all $y \in \mathcal{Y}_t$.

Define the approximated cost function for the proposed MPC

$$\begin{aligned} V_N^a(x,p,z;\mathbf{u}) &= \sum_{j=0}^{N-1} \ell(x(j)-x(N{-}1), u(j)-u(N{-}1)) \\ &\quad + f_{eco}(z,p) \\ &\quad + \nabla_y f_{eco}(z,p)^T(y(N-1)-z) \\ &\quad + \frac{\rho_f}{2}\|y(N-1)-z\|^2 \end{aligned}$$

Notice that this cost function is a quadratic cost function and it is an upper bound of the original cost function, that is

$$V_N(x,p,z;\mathbf{u}) \le V_N^a(x,p,z;\mathbf{u})$$

On the other hand, the convex set defined as

$$\begin{aligned} \mathcal{Y}_t^a(z) = \{y : \quad & h_j(z) + \nabla_y h_j(z)^T(y-z) \\ & + \frac{\pi_j}{2}\|y-z\|^2 \le 0,\ j \in \mathbb{I}_{1:n_h}\} \end{aligned}$$

is such that $\mathcal{Y}_t^a(z) \subseteq \mathcal{Y}_t$ for all $z \in \mathcal{Y}_t$. Notice that for all $z \in \mathcal{Y}_t$, the set $\mathcal{Y}_t^a(z)$ is non-empty since $z \in \mathcal{Y}_t^a(z)$.

Then, the approximated MPC optimization problem $P_N^a(x,\hat{d},p,z)$ is derived from replacing the original constraints by their approximated counterparts, yielding

$$\min_{\mathbf{u}} V_N^a(x,p,z;\mathbf{u}) \tag{5a}$$
$$s.t.\ x_0 = x, \tag{5b}$$
$$x(j+1) = Ax(j) + Bu(j) + \hat{d}, \quad j \in \mathbb{I}_{0:N-1} \tag{5c}$$
$$y(j) = Cx(j) + Du(j) \tag{5d}$$
$$(x(j),u(j)) \in Z, \qquad j \in \mathbb{I}_{0:N-1} \tag{5e}$$
$$x(N) = x(N-1) \tag{5f}$$
$$y(N-1) \in \mathcal{Y}_t^a(z) \tag{5g}$$

The predictive control law derived from this optimization problem is denoted as $\kappa_N^a(x,\hat{d},p,z)$. This optimization problem is a convex problem (particularly a QCQP problem) that can be efficiently solved using, for instance, interior point methods [16]. Moreover, if the set $\mathcal{Y}_t$ is a polyhedron, then the resulting optimization problem is a QP.

It is important to remark that every feasible solution of the approximated solution $P_N^a(x,\hat{d},p,z)$ is also a feasible solution of the exact optimization problem $P_N(x,\hat{d},p,z)$. Besides, the optimal solution is such that

$$V_N{}^o(x,\hat{d},p,z) \le V_N^a{}^o(x,\hat{d},p,z)$$

and then the approximated solution minimizes an upper bound of the exact cost function. The approximation error depends on the selection of the approximation point z. Then the selection of this point plays an important role in the economic optimality of the proposed controller.

In this paper we propose a control algorithm inspired by the Newton's optimization method: the approximation point $z(k)$ is taken as the optimal terminal output of the last optimization problem. Then, the control algorithm is as follows:

Algorithm 1 Newton-type Control Algorithm

Require: The estimated disturbance $\hat{d}$, the set of parameters p and an initial approximation point $z(0)$

1: Read $x(k)$
2: Solve $P_N^a(x(k),\hat{d},p,z(k))$
3: Apply $u(k) \leftarrow \kappa_N^a(x(k),\hat{d},p,z(k))$
4: Update $z(k+1) \leftarrow y^*(N-1|k)$
5: Wait for the next sampling interval

VI. Stability and convergence analysis

In this section it will be proved that the proposed controller ensures recursive feasibility for all initial state and for any value of the parameters p. Besides it will be proved that the closed-loop system converges to a stable equilibrium point and this is the one that minimizes the economic cost function, that is, $y(k)$ converges to y_t.

Now these statements are proved:

Recursive feasibility:

Assume that at $x(k)$ and $z(k)$, the optimal solution is $\mathbf{u}^o(k)$. The successor state is $x(k+1)$. Let define the sequence $\tilde{\mathbf{u}}(k+1) = \{u(1|k), \cdots, u(N-1|k), u(N-1|k)\}$. Assuming that the estimated disturbance $\hat{d}$ is constant, the sequence of predicted states for $\tilde{\mathbf{u}}(k+1)$ is $\tilde{\mathbf{x}}(k+1) = \{x(1|k), \cdots, x(N-1|k), x(N-1|k)\}$. Since $\mathbf{u}^o(k)$ is feasible, then $(\tilde{x}(j|k+1), \tilde{u}(j|k+1)) \in Z$ and $\tilde{x}(N|k+1) = \tilde{x}(N-1|k+1)$.
Besides, from the feasibility of the solution at k we have that

$$y(N-1|k) \in \mathcal{Y}_t^a(z(k)) \subseteq \mathcal{Y}_t$$

Then, since $z(k+1) = y(N-1|k) \in \mathcal{Y}_t$,

$$\tilde{y}(N-1|k+1) = y(N-1|k) \in \mathcal{Y}_t^a(z(k+1))$$

Notice that this property holds for any value of p, since the set of constraints does not depend on p.

Stability and convergence:

First it is proved that $V_N^{a\,o}(x(k), z(k), p)$ is a decreasing function. Denote

$$\begin{aligned} \Delta V_N^a(k) &= V_N^a(x(k+1), \tilde{\mathbf{u}}(k+1), z(k+1), p) \\ &\quad - V_N^{a\,o}(x(k), z(k), p) \end{aligned}$$

Then we have that

$$\begin{aligned} \Delta V_N^a(k) &= \Big(V_N^a(x(k+1), \tilde{\mathbf{u}}(k+1), z(k+1), p) \\ &\quad - V_N^a(x(k+1), \tilde{\mathbf{u}}(k+1), z(k), p)\Big) \\ &\quad + \Big[V_N^a(x(k+1), \tilde{\mathbf{u}}(k+1), z(k), p) \\ &\quad - V_N^{a\,o}(x(k), z(k), p)\Big] \\ &= \Delta V_1 + \Delta V_2 \end{aligned}$$

Taking into account that $z(k+1) = y(N-1|k)$, the first term of the rhs of this equation is equal to

$$\begin{aligned} \Delta V_1 &= f_{eco}(y(N-1|k)) - f_{eco}(z(k), p) \\ &\quad + \nabla_y f_{eco}(z(k), p)^T (y(N-1|k) - z(k)) \\ &\quad + \frac{\rho_f}{2} \|y(N-1|k) - z(k)\|^2 \end{aligned}$$

From lemma 1 we derive that $\Delta V_1 \leq 0$.

From [3], the second term of the equation is such that

$$\Delta V_2 \leq -\ell(x(k) - x(N-1|k), u(k) - u(N-1|k))$$

Therefore we have that

$$\Delta V_N^a(k) \leq -\ell(x(k) - x(N-1|k), u(k) - u(N-1|k))$$

Since $\Delta V_N^a(k)$ is an upper bound of the decrement of the optimal cost function, it is inferred that the system converges to an admissible equilibrium point $(x_\infty, u_\infty, y_\infty)$. At x_∞, the optimal solution is such that $u^o(j) = u_\infty$ and $x^o(j) = x_\infty$ and then $z(k)$ converge to y_∞, and then $(x_\infty, u_\infty) \in Z$ such that $h(y_\infty) \leq 0$.

The optimal cost function at x_∞ is $V_N^a(x_\infty, y_\infty, p) = f_{eco}(y_\infty, p)$, then taking into account that the set $\mathcal{Y}_t$ is convex and the function f_{eco} is convex, from [3] it can be proved that y_∞ is such that

$$y_\infty = \arg\min_{y \in \mathcal{Y}_t} f_{eco}(y, p)$$

that is, the optimal solution of the RTO, $y_\infty = y_t$.

VII. Example

In order to illustrate the properties of the controller, this is applied to a simple academic example. We consider a linear system given by the matrices

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This system is subject to the constraints $\|x(k)\|_\infty \leq 5$ and $\|u(k)\|_\infty \leq 0.3$. The considered economic function has been $f_{eco}(y, p) = \|y - p\|_2^3$ where p is a time-varying parameter. The proposed controller has been designed for $Q = C^T C$ and $R = 1$ and the prediction horizon is $N = 3$.

In the test the parameter p changes from $p_1 = (-10, -10)$ to $p_2 = (0, 10)$ and then back to p_1. The resulting trajectory of the system can be seen in the figure 1. In this figure, the trajectory of the states are depicted in solid line, while in dashed line the terminal state $x(N-1)$ is shown. Figure 2 shows the evolution of the input while Figure 2 shows the evolution of the economic cost function $f_{eco}(y, p)$ evaluated at the current output.

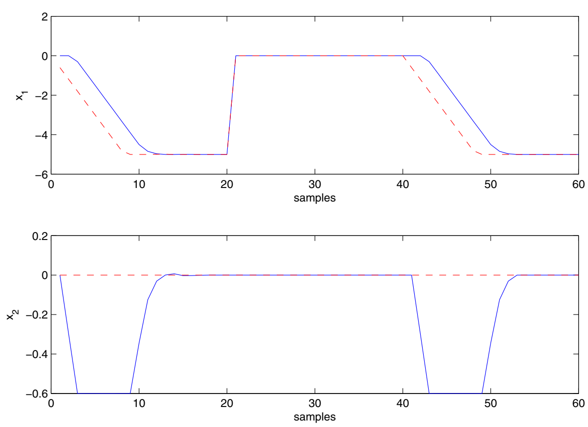

Fig. 1. Evolution of the states of the system under changes in the parameter

It has been demonstrated that the system maintains recursive feasibility under large changes in the parameter p. Furthermore, for each constant value of p, the controller steers the system to the point where the economic function is minimized and this property has been achieved by solving one single QP at each sampling time and calculating the gradient of f_{eco} once per sample.

VIII. Conclusions

In this paper a one-layer RTO/MPC formulation has been presented, in such a way that the controller cost function includes a second order approximation of the original nonlinear RTO cost function. The resulting cost is an upper bound

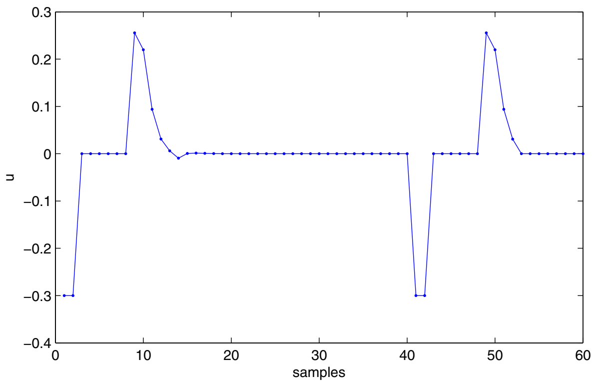

Fig. 2. Evolution of the input of the system under changes in the parameter

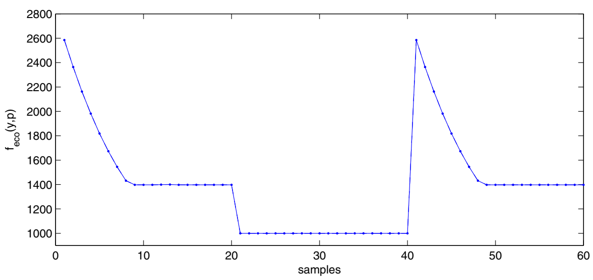

Fig. 3. Evolution of the economic function $f_{eco}(y, p)$ under changes in the parameter

of the one-layer MPC cost obtained by directly adding the RTO nonlinear economic function to the MPC cost function. Moreover, the use of EngellJPC2007EngellJPC2007this second order approximation turns the optimization control problem into a convex problem. An iterative algorithm has been presented, that ensures recursive feasibility and convergence to the economic optimum, resorting to classic ideas of the gradient-based optimization algorithms.
The controller has been designed following the MPC for tracking formulation. This fact makes the controller guarantees recursive feasibility under any change of the economic function. Furthermore, if the system is initially in an admissible steady state, then stability can be ensured even for a small prediction horizon.

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