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Nonlinear Control for the Longitudinal Dynamics of a Small Scale Helicopter

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Abstract— **A nonlinear control scheme is proposed for the trajectory tracking problem of a small scale helicopter's longitudinal dynamics. The control scheme is based on a control design procedure that constructs static stabilizers for nonlinear systems which are linearizable by dynamic feedback. The flatness properties of the helicopter's longitudinal dynamics are exploited to design the desired trajectory. Sufficient conditions are given to guarantee asymptotic convergence to zero of the tracking error assuming that all the helicopter's parameters are known and that all states are measured. Numerical simulations are provided to illustrate the properties of the proposed controller.**

I. INTRODUCTION

Because of the introduction of new rotor configurations for rotary wing vehicles, the helicopter with the traditional rotor configuration has been a little bit relegated as benchmark vehicle to test new nonlinear controllers. The academic market was invaded by the multi-rotor kind vehicles which avoid the mechanical complications of the swash plate. However, in terms of power consumption efficiency and reliability, the classical helicopter is even nowadays the best flying machine to perform flight missions such as surveillance, inspection, recognition and rescue. Moreover, the dynamic model's complexities represent a challenge for nonlinear control designers.

Research on nonlinear control design techniques for autonomous flight of small scale helicopters, is strongly based on the dynamic model. Frequently, this model is simplified so that it fits a certain structure which allows applying the desired nonlinear control strategy. This approach has been successfully followed for nonlinear control designs for the helicopter due to the complexity of the complete dynamic model. The principal complexity of the dynamic model of the helicopter is the coupling between the control signals of the helicopter's Cartesian dynamic and the pitch and roll rotational dynamics [18]. Hence, in [13] and [19] for instance, the Authors disregard this coupling and then design nonlinear controllers applying nonlinear control techniques such as exact and partial dynamic feedback linearization and robust backstepping. In [15], the Authors do take into account the coupling between the Cartesian dynamic and the rotational dynamics, however, the helicopter's dynamic model is simplified disregarding the lateral dynamic. Then, the Authors design a trajectory tracking controller by means of linearizing the helicopter's longitudinal model around the desired trajectory. Besides, the Liouvillian characteristics of the model are used to have the desired trajectory planning.

It turns out that the longitudinal helicopter dynamic is similar to the PVTOL lateral aircraft model. For the PVTOL the nonlinear control literature is extent. For instance, in [4] different control strategies for the PVTOL aircraft are compared. In [9], a two step control strategy is proposed to design a trajectory tracking control of the PVTOL aircraft. First, the controller is developed using the flat output [16] associated to the model and, then a desired trajectory is constructed, which is bounded by the pitch angle. In [3], a reduced PVTOL model is obtained when an input is eliminated in the translational equations so that the roll displacement and the lateral acceleration are decoupled. A dynamic extension is also made to the model so that a state feedback control can be designed for trajectory tracking; the control strategy is based on forwarding.

In the present work, a nonlinear control scheme is proposed for the trajectory tracking problem of a small scale helicopter's longitudinal dynamics including the longitudinal rotor dynamics. The control scheme is based on the strategy proposed in [14] where a static regulator is designed for nonlinear systems which are linearizable by dynamic state feedback. Besides, the flatness characteristics of the helicopter's longitudinal dynamics are used to construct the desired trajectory. Sufficient conditions are given to guarantee asymptotic convergence to zero of the tracking error assuming that all the helicopter's parameters are known and that all signals are measured. Also, some numerical simulations are given to show the performance of the controller.

The article is organized as follows. In section II, the small scale helicopter model is described. In section III, the external forces and torques applied to the small scale helicopter are explained. Also, the reduction and simplifications made on the complete model are presented; this allows to obtain the small scale helicopter's longitudinal dynamics which are used for control design purposes. The design of nonlinear control scheme is presented in section IV. Some numerical simulations are presented in section V to show the performance of the helicopter's longitudinal dynamic behavior in closed loop. Finally, some conclusions are given

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in section VI.

II. FUSELAGE DYNAMIC MODEL

Let us consider an inertial reference frame and a body fixed reference frame to specify the translational and rotational position and velocity of a small scale helicopter. The inertial reference frame, also known as earth fixed frame, $f^I = [x^I\ y^I\ z^I\ ]$, is located at a specific position over the earth, where the axis x^I is in the direction of the north, the axis y^I is in the direction of the east and z^I is in the direction to center of the earth. The body fixed reference frame, $f^b = [x^b\ y^b\ z^b\ ]$, has its origin at the helicopter center of gravity (C.G.). The direction of z^b points downwards, while the axis x^b points to longitudinal direction and y^b points to lateral direction of the vehicle. These coordinate systems are depicted in Fig. 1.

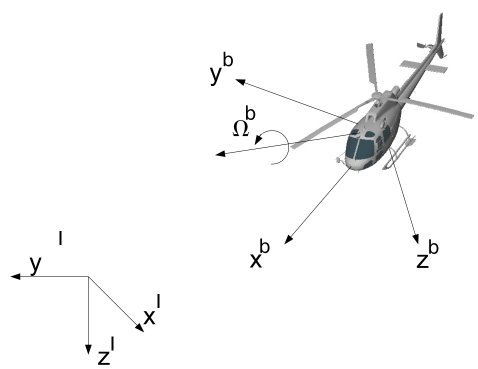

Fig. 1. Coordinate system of the helicopter.

The fuselage is modelled as a rigid body in a three-dimensional motion with respect to the body-fixed reference frame [17]. These are described by

$$m\dot{V}^b + m(\Omega^b \times V^b) \quad = f^b \tag{1}$$

$$I\dot{\Omega}^b + (\Omega^b \times I\Omega^b) \quad = \tau^b \tag{2}$$

where $V^b = [u\ v\ w]^T$ is the translational velocity, with u, v, and w being the longitudinal, lateral and vertical velocities respectively, $\Omega^b = [p\ q\ r]^T$ is the angular velocity with p, q, and r being the angular velocities around the x^b, y^b, and z^b axis respectively. The external forces, $f^b = [X\ Y\ Z]^T$, and external torques, $\tau^b = [L\ M\ N]^T$, are expressed in the body-fixed reference frame, where X, Y, and Z indicate the components of the main rotor thrust in the longitudinal, lateral and vertical direction respectively, as well as the tail rotor thrust in the lateral direction. L, M, and N are the torques applied around the x^b, y^b, and z^b axis, respectively. Finally, m and I are the mass and the inertia tensor of the small scale helicopter, respectively.

The angular velocities are related with the helicopter attitude through the Euler angles in the sequence of rotation ZYX. The Euler angles are represented as $\Theta = [\ \phi\ \ \theta\ \ \psi\ ]$, where the ϕ, θ, and ψ are the roll, pitch, and yaw of the aircraft, the angular velocities are expressed as [7].

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & -\sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}$$

In the aircraft literature, the Euler angles are commonly used to parameterize the vehicles attitude, even though they do not provide a global parameterization with respect to angular velocity neither an unique representation at certain attitudes.

A. Position and Velocities respect to the Inertial Reference Frame

The velocities in (1) are expressed with respect to a body-fixed frame and they do not indicate the position relative to the inertial reference frame. The rotation matrix, R, provides a relation between both coordinate systems, this is

$$R = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix}$$

where the notation $s\phi = \sin\phi$, $c\phi = \cos\phi$, $s\theta = \sin\theta$, $c\theta = \cos\theta$, $s\psi = \sin\psi$, and $c\psi = \cos\psi$ is used. The external forces in (1) can be represented in the inertial reference frame as

$$f^I = R^T f^b \tag{4}$$

Therefore we can express the translational position, $p^I = [x\ y\ z]$ and velocities, $v^I = [\dot{x}\ \dot{y}\ \dot{z}]$, with respect to the inertial reference frame by [13]

$$\begin{aligned} \dot{p}^I &= v^I \\ \dot{v}^I &= \frac{1}{m} f^I \end{aligned} \tag{5}$$

where x, y and z are the longitudinal, lateral and vertical displacement, respectively.

B. Main Rotor and stabilizer bar Dynamics.

The main rotor dynamics together with the stabilizer bar dynamics are represented by two differential equations of first order, which describe the rotor inclination due to the aerodynamics effects. These equations are given by [1]

$$\begin{aligned} \tau_f \dot{a} &= -a\left(1 + \frac{\mu^2}{2}\right) - \tau_f q + \frac{p}{\Omega} + A_b b + A_{lat}\delta_{lat} \\ &\quad + A_{lon}\delta_{lon}\left(1 + 3\mu^2\right) + A_{lon}k_c c\left(1 + 3\mu^2\right) + \lambda_{1c} \\ \tau_f \dot{b} &= -b\left(1 + \frac{\mu^2}{2}\right) - \tau_f p - \frac{q}{\Omega} - B_a a + B_{lon}\delta_{lon} \\ &\quad + B_{lat}\delta_{lat}\left(1 + \mu^2\right) + B_{lat}k_d d\left(1 + \mu^2\right) + \lambda_{1s} \end{aligned} \tag{6}$$

where Ω is the rotational velocity of the main rotor, a and b are the longitudinal and lateral tilt of the tip-path-plane. d and c are the longitudinal and lateral tilt of the stabilizer bar. A_{lon} and B_{lat} are constant terms while δ_{lat} and δ_{lon} are the lateral and longitudinal cyclic inputs. λ_{1c} and λ_{1s} are the longitudinal and the lateral component of the non-dimensionalized induced velocity. The term $A_b = -B_a$ is the stiffness number defined as $A_b = \frac{8}{\gamma}\left(\lambda_\beta^2 - 1\right)$ where the term λ_β is given by $\lambda_\beta = 1 + \frac{k_\beta}{I_\beta \Omega^2}$, being k_β the stiffness constant in the joint of the blade with the mast and I_β is the moment of the inertia with respect to flapping axes. μ is the advance ratio given by $\mu = \frac{u}{R_b \Omega}$. Finally, τ_f is the time constant of the main rotor tilt, given by $\tau_f = \frac{16}{\gamma \Omega}$ where γ is the Lock Number, defined as $\gamma = \frac{\rho c_b C_{l\alpha} R_b^4}{I_\beta}$. In this expression ρ is the air density, c_b is the blade chord, $C_{l\alpha}$ is the thrust coefficient, and R_b is the rotor radius.

III. External Forces and Moments

The external forces applied to the helicopter are mainly due to the thrust and reaction torque of the main and tail rotors and the gravitational force. There are other forces which are generally dismissed such as the aerodynamic forces due to the fuselage. The external forces can be expressed in the body-fixed reference as [6]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{7}$$

where X_M, Y_M, and Z_M are the components of the thrust of the main rotor, T_{MR}, while Y_T is the component of the thrust of the tail rotor in the direction of y^b axis.

The thrust of the main rotor is considered perpendicular to the rotor the tip-path-plane according to [8]. Then, the tilt of tip-path-plane can be described using the orientation of the thrust. The mathematical description of the thrust vector has the form [3]

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} -\sin a \cos b \\ \cos a \sin b \\ -\cos a \cos b \end{bmatrix} T_{MR} \approx \begin{bmatrix} -a \\ b \\ -1 \end{bmatrix} T_{MR} \tag{8}$$

The tail rotor dynamics is disregarded and it is assumed that the whole thrust of the tail rotor is vertical to its disc. The direction of the thrust of the tail rotor depends on the placement of the tail rotor in the rear of the helicopter. This is, when the helicopter is seen from above, the tail rotor can be placed on the right or left of the body helicopter. Also the thrust of the tail rotor T_T is equal to the force Y_T.

On other hand, the external torque depends on the thrust of the main and tail rotors, the stiff at the joint of the hub and the blade and the reaction torque of the main and the tail rotors. The mathematical expression for such a torque is given by [6]

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} L_M \\ M_M + M_T \\ N_M \end{bmatrix} + \begin{bmatrix} Y_M h_M + Z_M y_M + Y_T h_T \\ -X_M h_M + Z_M l_M \\ -Y_M l_M - Y_T l_T \end{bmatrix}$$

The terms $(l_M,\, y_M,\, h_M)$ and $(l_t,\, y_t,\, h_t)$ are distances from the center of gravity to the point of application of the thrust of the main and the tail rotors. The torques L_M, M_M, and N_M are given by [6]

$$\begin{aligned} L_M &= k_\beta b - Q_M \sin(a) \\ M_M &= k_\beta a + Q_M \sin(b) \\ N_M &= -Q_M \cos(a)\cos(b) \\ M_T &= -Q_T \end{aligned}$$

where $k_\beta b$ and $k_\beta a$ are the torques induced by the rotor stiffness, Q_M is the reaction torque of the main rotor and Q_T is the reaction torque due to tail rotor.

A. Reduction of the dynamic model

A dynamic model reduction is carried out in order to obtaine a model restricted to the $x^I z^I$ plane. As a first step, the angular motions corresponding to roll and yaw, the external forces in the lateral direction, the lateral traslation are assumed equal zero, this is $y = 0$ m, $v = 0$ m/s, $Y = 0$ N, $\phi = \psi = 0$ rad, $p = r = 0$ rad/s, and $L = N = 0$ N. The rotor and the stabilizer bar are made compatible with the motion in the $x^I z^I$ plane by considering $b = 0$. The reduced model expressed in inertial coordinates is described by the equations

$$\begin{aligned} m\ddot{x} &= \sin(\theta) Z_M + \cos(\theta) X_M \\ m\ddot{z} &= mg + \cos(\theta) Z_M - \sin(\theta) X_M \\ I_{yy}\ddot{\theta} &= -l_h X_M + l_M Z_M + k_\beta a - Q_T \\ \tau_f \dot{a} &= -a\left(1 + \frac{\mu^2}{2}\right) - \tau_f \dot{\theta} + A_{lon}\delta_{lon}\left(1 + 3\mu^2\right) \\ &\quad + A_{lon} k_c c (1 + 3\mu^2) + \lambda_{1c} \end{aligned} \tag{9}$$

The aim of the model reduction is to design a simple control scheme for trajectory tracking purposes. A more simplified model is obtained by considering Q_T, k_β, and l_M equal zero. Then $X_M = -aT_{MR}$ and $Z_M = -T_{MR}$ are substituted in (9), obtaining

$$\begin{aligned} m\ddot{x} &= -\sin(\theta) T_{MR} - \cos(\theta) a T_{MR} \\ m\ddot{z} &= mg - \cos(\theta) T_{MR} + \sin(\theta) a T_{MR} \\ I_{yy}\ddot{\theta} &= l_h a T_{MR} \\ \tau_f \dot{a} &= -a\left(1 + \frac{\mu^2}{2}\right) - \tau_f \dot{\theta} + A_{lon}\delta_{lon}\left(1 + 3\mu^2\right) \end{aligned} \tag{10}$$

where $[x, \dot{x}] \in \mathscr{R}^2$, $[z, \dot{z}] \in \mathscr{R}^2$ and $[\theta, \dot{\theta}] \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathscr{R}$.

Considering that the rotor dynamics response is faster than the fuselage dynamics response, the model of the rotor dynamics can be written as

$$0 = -a\left(1 + \frac{\mu^2}{2}\right) - \tau_f \dot{\theta} + A_{lon}\delta_{lon}\left(1 + 3\mu^2\right)$$

Thus, the angle of rotor inclination can be computed as

$$a = \frac{2}{2+\mu^2}\left[-\tau_f\dot{\theta} + A_{lon}(1+3\mu^2)\delta_{lon})\right] \tag{11}$$

By making the following assignment

$$\begin{aligned}\sigma_1(\mu) &= \frac{2\tau_f}{2+\mu^2}\\ \sigma_2(\mu) &= \frac{2A_{lon}(1+3\mu^2)}{2+\mu^2}\end{aligned}$$

equation (11) takes the form

$$a = -\sigma_1(\mu)\dot{\theta} + \sigma_2(\mu)\delta_{lon}$$

and the dynamics described by equation (10) becomes

$$\begin{aligned}m\ddot{x} &= -\sin(\theta)T_{MR} - \cos(\theta)\left(-\sigma_1\dot{\theta} + \sigma_2\delta_{lon}\right)T_{MR}\\ m\ddot{z} &= mg - \cos(\theta)T_{MR} + \sin(\theta)\left(-\sigma_1\dot{\theta} + \sigma_2\delta_{lon}\right)T_{MR}\\ I_{yy}\ddot{\theta} &= l_h\left(-\sigma_1\dot{\theta} + \sigma_2\delta_{lon}\right)T_{MR} \qquad (12)\end{aligned}$$

which constitutes the reduced model to be used in the design of the control scheme described in the section IV.

IV. Control Strategy

Let us consider the reduced model (12), and assume that all the parameters are known and that all the states are measured. This system is differentially flat in accordance to [9] and the flat outputs are given by

$$\begin{aligned}P_x &= x + \frac{I_{yy}}{l_h m}\sin(\theta) \qquad (13)\\ P_z &= z + \frac{I_{yy}}{l_h m}\cos(\theta)\end{aligned}$$

We now define the error variables z_1 and x_1 as functions of the flat outputs, this is z_1 and z_2 are now expressed as

$$z_1 = P_z - P_{zd}; \quad x_1 = P_x - P_{xd}$$

Thus

$$z_2 = \dot{z}_1 = \dot{P}_z - \dot{P}_{zd}; \quad x_2 = \dot{x}_1 = \dot{P}_x - \dot{P}_{xd}$$

from which it is straightforward to verify that

$$\begin{aligned}\dot{z}_2 &= g - \frac{\cos(\theta)}{m}\left[T_{MR} + \frac{I_{yy}}{l_h}\dot{\theta}^2\right] - \ddot{P}_{zd} \qquad (14)\\ \dot{x}_2 &= -\frac{\sin(\theta)}{m}\left[T_{MR} + \frac{I_{yy}}{l_h}\dot{\theta}^2\right] - \ddot{P}_{xd}\end{aligned}$$

Note that the decoupling matrix associated to system (14) is singular. Following the result in [9], it is possible to use a dynamic extension to get a full range decoupling matrix. However, a different approach based on the results found in [14] is followed in this note. For doing this, we close the loop of the z_1, z_2 dynamics by defining

$$T_{MR} = -\frac{I_{yy}}{l_h}\dot{\theta}^2 + \frac{m}{\cos(\theta)}\left[g - \ddot{P}_{zd} + \gamma_z(z_1,z_2,t)\right] \tag{15}$$

Substituting (15) into (14) gives

$$\begin{aligned}\dot{z}_1 &= z_2\\ \dot{z}_2 &= -\gamma_z(z_1,z_2,t)\\ \dot{x}_1 &= x_2\\ \dot{x}_2 &= -\tan(\theta)\left[g - \ddot{P}_{zd} + \gamma_z(z_1,z_2,t)\right]\end{aligned}$$

The next step of the control design procedure is to define the second control input. Therefore, lets us introduce the variables x_3 and x_4 defined as

$$\begin{aligned}x_3 &= \dot{x}_2 = \ddot{P}_x - \ddot{P}_{xd}\\ x_4 &= \dot{x}_3 = P_x^{(3)} - P_{xd}^{(3)}\end{aligned}$$

Then, by taking the time derivative of x_4, one gets

$$\dot{x}_4 = P_x^{(4)} - P_{xd}^{(4)} \tag{16}$$

where

$$\begin{aligned}P_x^{(4)} = &-2\left(1+\tan^2(\theta)\right)\left[-P_{zd}^{(3)} + \dot{\gamma}_z\right]\dot{\theta}\\ &-\tan(\theta)\left[-P_{zd}^{(4)} + \gamma_z^{(2)}\right]\\ &-\left(g - \ddot{P}_{zd} + \gamma_z\right)\left(1+\tan^2\theta\right)\left[\frac{l_h}{I_{yy}}(-\sigma_1\dot{\theta} + \sigma_2\delta_{lon})T_{MR}\right]\\ &-\left(g - \ddot{P}_{zd} + \gamma_z\right)\left(1+\tan^2\theta\right)(2\tan(\theta)\dot{\theta}^2) - P_{xd}^{(4)}\end{aligned}$$

Let us now, set the input δ_{lon} as

$$\delta_{lon} = \frac{1}{\sigma_2}\left[\sigma_1\dot{\theta} + \frac{I_{yy}}{l_h T_{MR}}\left(-2\tan(\theta)\dot{\theta}^2 - A\right)\right] \tag{17}$$

where A is given by

$$A = \frac{A_1 + A_2 + v_2}{\left(g - \ddot{P}_{zd} + \gamma_z\right)\left(1+\tan^2\theta\right)} \tag{18}$$

with v_2 a new input signal, and A_1, A_2 given by

$$\begin{aligned}A_1 &= 2\left(1+\tan^2\theta\right)\left(-P_{zd}^{(3)} + \dot{\gamma}_z\right)\dot{\theta}\\ A_2 &= \tan\theta\left(-P_{zd}^{(4)} + \gamma_z^{(2)}\right) + P_{xd}^{(4)}\end{aligned}$$

The resultant closed-loop system when substituting (17) into (16)

$$\begin{aligned}\dot{z}_1 &= z_2\\ \dot{z}_2 &= -\gamma_z(z_1,z_2,t)\\ \dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= v_2\end{aligned} \tag{19}$$

Then the feedback v_2 is proposed to be of the form

$$v_2 = -k_4x_4 - k_3x_3 - k_2x_2 - k_1x_1 \tag{20}$$

so that x_1, x_2, x_3 and x_4 can be made to approach zero by means of an adequate selection of parameters k_1, k_2, k_3 and k_4. It is important to notice that the term $g - \ddot{P}_{zd} + \gamma_z$ in equation (18) must never cross zero. Therefore one assumes that $\gamma_z(z_1, z_2, t)$ satisfies the condition

$$\gamma_z(z_1, z_2, t) > \ddot{P}_{zd} - g$$

Thus a bounded control based on [5] is proposed, this is

$$\gamma_z = \frac{\varepsilon_z}{2}\left[\tanh\left(\frac{2\lambda_{z1}}{\varepsilon_z} z_1\right) + \frac{1}{2}\tanh\left(\frac{4\lambda_{z2}}{\varepsilon_z} z_2\right)\right] \quad (21)$$

This can be achieve by means of a proper choice of the λ_{z1}, λ_{z2}, and ε_z constants.

The systems in a closed loop form is then written as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\gamma_z(z_1, z_2, t) \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 \end{aligned} \quad (22)$$

The following result can then be stated.

Proposition. Consider the small helicopter model (12) and assume that its parameters are known and that all the states can be measured. Consider the controller defined by (15) and (17) where the control T_{MR} is assumed to be different from zero. Then, there exists constants λ_{z_1}, λ_{z_2}, ε_z, k_1, k_2, k_3 and k_4, such that the closed loop dynamics of the tracking error are locally asymptotically stable.

Remark. It is important to notice that the model of the small helicopter's model is based on the fact that the rotor dynamics is faster than the fuselage dynamics. Also, the variables x, z and θ are indirectly controlled via the flat output.

V. Simulation results

Some simulations were carried out in order to evaluate the performance of the control strategy proposed. The values of the parameters used were m=4.9 kg, l_h=0.294 m, I_{yy}=0.027 kg-m^2 and g=9.81 m/s. The control scheme assures that the flat output converges to zero asymptotically. However, the original objective was to control the position of the helicopter in the $x^I y^I$ plane. If one proposes x_d and z_d, the flat outputs can not be explicitly defined since one has two equations and three variables. A solution to this problem can be obtained through the generation of a desired trajectory for x_d and z_d by means of Bézier polynomials while θ_d is obtained from the solution of the differential equation.

$$\ddot{\theta}_d = -\frac{m l_h}{I_{yy}}\left(\ddot{x}_d \cos\theta_d + (g - \ddot{z}_d)\sin\theta_d\right)$$

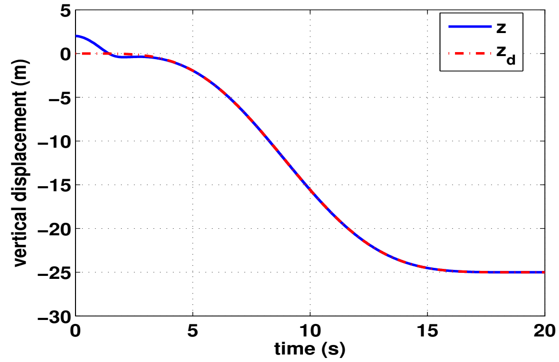

Fig. 2. Tracking of the trajectory in the axes z^I.

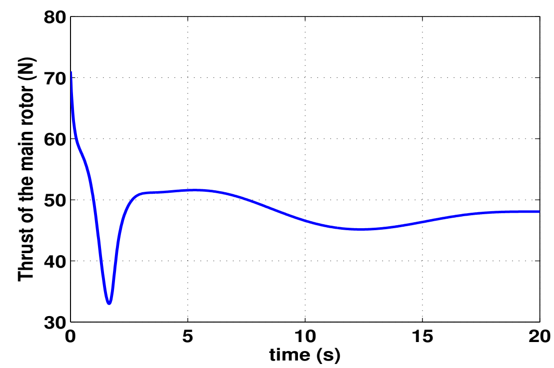

Fig. 3. Control input of T_{MR}.

This equation is obtained from the model (10). Besides, a variable change is made, such that $u_1 = T_{MR}$, $u_2 = aT_{MR}$, $x_d = x$, $z_d = z$ and $\theta_d = \theta$. The input u_1 is obtained from the first equation of (10). Therefore, $u_1 = -\frac{M}{\sin(\theta_d)}(\ddot{x}_d + \frac{1}{M}\cos(\theta_d)u_2)$. This expression is substituted in the second equation of (10) to obtain $\ddot{y}_d = g + \frac{\cos(\theta_d)}{\sin(\theta_d)}\ddot{x} + \frac{1}{M\sin(\theta_d)}u_2$. Finally, u_2 is substituted into the third equation of (10).

The gains of the feedback controller (20) were choosen as $k_1 = 16$, $k_2 = 32$, $k_3 = 24$ and $k_4 = 8$, while the gains of the bounded control (21) were chosen as $\varepsilon_z = 10$, $\lambda_{z1} = 80$ and $\lambda_{z2} = 18$.

The simulation results show how the desired trajectory is followed from the given initial condition to final condition. Fig. 2 shows the elevation of the small scale helicopter from an altitude of 2 m to -25 m. in a time of 20 seconds. The signal control is shown in Fig. 3.

The trajectory tracking with respect to the x^I axis is shown in Fig. 4, where the small scale helicopter starts with -8 m. as initial condition and reaches a position of 20 m., in a time of 20 seconds. The control input (17) is shown Fig. 5.

Fig. 6 shows the trajectory tracking of the pitch in a time of 20 s. This tracking is not perfect because θ is controlled indirectly through flat output with a -0.1 rad initial condition.

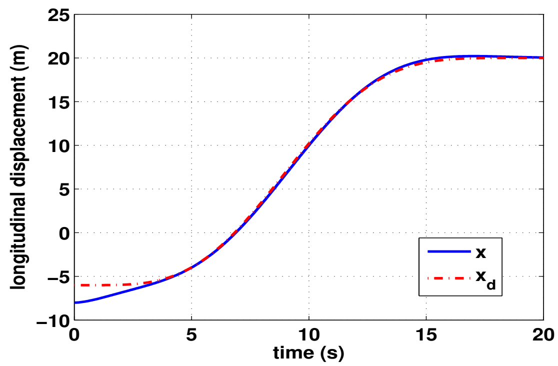

Fig. 4. Tracking of the trajectory in the axes x^I.

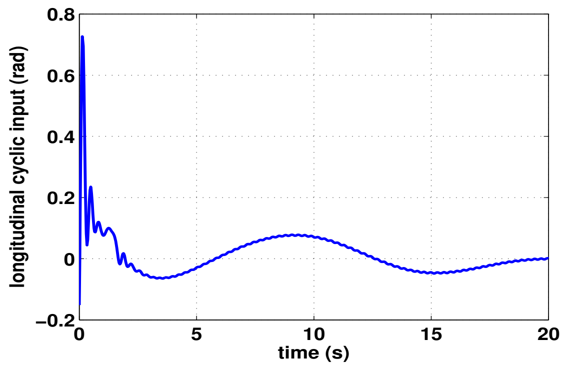

Fig. 5. Control input of δ_{lon}.

VI. CONCLUSIONS

This article presents a nonlinear control scheme for the trajectory tracking of the longitudinal dynamics in a small scale helicopter. The scheme is based on a previous results for nonlinear systems which are linearizable by dynamic feedback. The flatness characteristics of the longitudinal dynamics were also used to generate the desired trajectory. Some sufficient conditions were found to assure asymptotic tracking convergence. The simulation results obtained show a good performance of the control scheme.

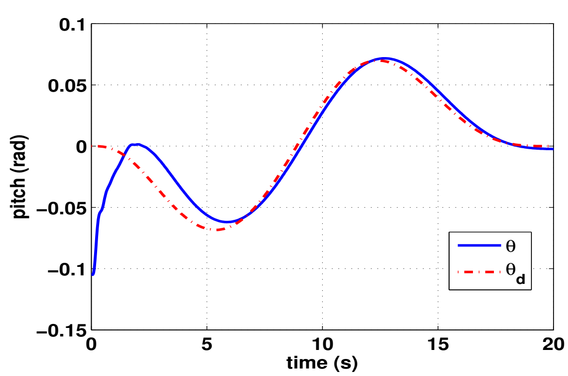

Fig. 6. Tracking of the trajectory of the pitch.

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