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Synchronization of Dynamical Networks Under Sampling

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Abstract— **Motivated by the increasing interest in networked multi-agent systems and the wide number of applications in decentralized distributed control of smart grids, we address the problem of synchronization when each node (e.g., a microgrid, a distributed generator) is modeled as a linear-time continuous system whose output measurements are sent through communication links. However, the inclusion of a communication infrastructure adds new challenges to control strategies and some problems may arise such as time delays, packet losses, sampling period, just to name a few. In this work, we consider that data is sampled with homogeneous sampling periods. Then, using the concept of average passivity, we define the conditions for synchronizability when all nodes are identical and unstable dynamics are present. Additionally, results are extended to the case of non-uniform agents, and some simulations of synchronization in smart grids are introduced.**

I. Introduction

Control of multi-agent systems has attracted great attention over the last decade. The goal is that a collection of agents with locally sensed information and limited inter-agent communication achieve a collective objective [1]. One important research area in multi-agent systems is the problem of synchronization of dynamic networks in which agents share their local information (e.g., sensed state of a distributed process or an opinion in a social network) to their neighbors, making the dynamics converge to each other over time (i.e., the state of all agents synchronizes). This kind of problems has been modeled for a wide number of applications in many research fields [2], such as synchronization of chemical reactors [3] or biochemical plants [4], and synchronization of multiple Kuramoto oscillators, [5], just to name a few.

Synchronization involves intrinsically a communication aspect between agents, which is commonly described using graph theory. However, these communications between agents cannot be considered as instantaneous or ideal in real applications, due to the fact that agents are spatially distributed and information exchange has to be done using communication networks [6]. In particular, for synchronization of dynamic networks, several works have included communication limitations. In [7], the effects of time-delays in the communication between agents are considered, and in [8], some conditions for synchronization are introduced when random packet losses are induced. On the other hand, in [9] [10] sampled-measurements have been addressed using different models. However, developments on synchronization under sampling only consider that the dynamics of each agent are stable, and an open research area lies in the analysis of synchronization under sampling with unstable dynamics. Furthermore, sampling should also be considered only in the information that each agent receives from its neighbors, i.e., each agent sends its sampled state measurement through a communication network to its neighbors, but each one has *real-time* knowledge about its own state.

A particular case of synchronization with communication constraints is present in the smart grids. Smart grids are power grids that are composed by microgrids (MGs) and a communication infrastructure that makes the grid smarter providing an insight of the states of the network [11]. Microgrids are systems that can have distributed energy resources and associated loads, and can form intentional islands in the electrical distribution systems. However, an islanded microgrid requires the ability to connect to the grid, which requires that the frequency (which depends on the power generated and consumed) of the MG has to be synchronized with the frequency of the power network [12]. The problem increases when there are several isolated MGs at time. Then, all of them have to be synchronized in order to avoid blackouts or overloads when they connect back to the network, or even when they are connected between them [13]. Moreover, if any MG presents any failure, synchronization has to be assured.

In this work, we address the problem of synchronization of MGs, and we focus on the limitations produced by sampling, where each microgrid or agent sends sampled-information to its neighbors. Each agent represents a microgrid whose behavior is modeled as a linear-time invariant (LTI) continuous system (e.g., linear models of photovoltaic generators [14] or fuel-diesel generator [15]) that may present unstable dynamics because of failures or overloads. Furthermore, we use a concept called the *average passivity* introduced by [16], in order to obtain some conditions for synchronization under sampling. The motivation for defining average passivity lies in the fact that passivity is well defined for continuous-time systems, but for the discrete-time case (when sampling is present), usual passivity is lost if the input/output link is not defined [17]. Then, dissipative and stability properties can be preserved with respect to some suitably defined sampled-data output using average passivity. We present a novel synchronization scheme under sampling, for identical and non-identical nodes, and we introduce some synchronization conditions based on average passivity. Besides, the results in this work can be extended for general synchronization problems when sampling and unstable dynamics are considered.

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The paper is organized as follows. In Section II, the problem of synchronization is formulated and some conditions that assure synchronizability are given. Synchronization under sampling for homogeneous agents, and its extension to heterogeneous nodes is presented in Section III. In Section IV an example of synchronization of distributed generators is presented, and finally some conclusions are drawn in Section V.

II. Problem Formulation

Synchronization of dynamic networks are commonly described using concepts of graph theory to model the communication between agents, and classical formulation of dynamic systems. In this work, the communication topology in smart grid distributed control problems can be represented as a graph and each microgrid possesses its own dynamics. We give here some necessary definitions adapted from [18] to follow the developments in this work.

A. Preliminaries

1) Communication Topology: Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ represents an undirected graph, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of nodes or vertices, and $\mathcal{E} = \{(i,j)|i,j \in \mathcal{V}\}$ is the set of pairs called edges. If a pair $(i,j) \in \mathcal{E}$, then i,j are said to be adjacent. The adjacency matrix $\mathcal{A}_\mathcal{G} = [a_{ij}]$ is the symmetric matrix $N \times N$, where $a_{ij} = 1$ if (i,j) are adjacent, $a_{ij} = 0$ otherwise, and $a_{ii} = 0$ for all $i \in \mathcal{V}$. For the i^{th} node, the set of neighbors is $\mathcal{N}_i = \{j|(i,j) \in \mathcal{E}\}$, and the degree of a vertex d_i is the number of neighbors that are adjacent to i, i.e., $d_i = \sum_{j=1}^{N} a_{ij}$ or $|\mathcal{N}_i|$. A sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{r-1}, i_r)$ is called a path from node i_1 to node i_r. The graph $\mathcal{G}$ is said to be connected if for any $i, j \in \mathcal{V}$ there is a path from i to j. The degree matrix is $\mathcal{D} = diag(d_1, d_2, \ldots, d_N)$, and the Laplacian of $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}_\mathcal{G}$, which has the row sum property.

2) Kronecker product: The Kronecker product , denoted by $\otimes$, is an operation of two matrices of arbitrarily size resulting in a block matrix, and it facilitates the manipulation of matrices [19]. Let consider a matrix $E \in \mathcal{R}^{n\times m}$ and $F \in \mathcal{R}^{p\times q}$. The Kronecker product is an $np \times mq$ block matrix

$$E \otimes F = \begin{bmatrix} e_{11}F & \ldots & e_{1m}F \\ \vdots & \ddots & \\ e_{n1}F & \cdots & e_{nm}F \end{bmatrix}$$

and it possesses some important properties: i) $(E \otimes F)(Q \otimes R) = (EQ \otimes FR)$, ii) $(E \otimes F)^\top = E^\top \otimes F^\top$, and iii) $I_N \otimes E = \text{diag}(E, E, \ldots, E)$, for I_N an identity matrix of size $N \times N$.

B. Synchronization of a Dynamic Network

Consider the multi-agent model for N identical agents where each one is described by the continuous-time LTI dynamics as follows

$$\begin{aligned} \dot{x}_i(t) &= Ax_i + Bu_i \\ y_i(t) &= Cx_i \end{aligned} \tag{1}$$

for $i = 1, \ldots, N$, where each $x_i \in \mathcal{R}^n, u_i \in \mathcal{R}^n$, and $y_i \in \mathcal{R}^n$ represent respectively the state, the control input, and the output of the i^{th} agent. $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times n}$, and $C \in \mathcal{R}^{n\times n}$ are the state, the input, and the output matrices. According to [20], the control protocol for synchronization is typically defined by

$$u_i = k_c \sum_{j=1}^{N} a_{ij}(y_j - y_i) \tag{2}$$

where $k_c > 0$ is a positive scalar gain and a_{ij} is an element of the adjacency matrix $\mathcal{A}_\mathcal{G}$, called the outer-coupling matrix.

The dynamics of the network are $\dot{\mathbf{x}} = (I_N \otimes A)\mathbf{x} - k_c(\mathcal{L} \otimes B)\mathbf{y}$, where $\mathbf{x} = [x_1, x_2, \ldots, x_N]^\top$, $\mathcal{L}$ is the Laplacian describing the communication topology, and I_N is the identity matrix of size $N \times N$. As $\mathbf{y} = [Cx_1, Cx_2, \ldots, Cx_N]^\top$, then

$$\dot{\mathbf{x}} = (I_N \otimes A - k_c(\mathcal{L} \otimes BC)\mathbf{x} \tag{3}$$

The dynamic network in (3) is said to achieve synchronization if $\lim_{t\to\infty} |y_i - y_j| = 0$, which is equivalent to the convergence of $\mathbf{x}$ to the output synchronization manifold $M = \{\mathbf{x}|y_1(x_1) = y_2(x_2) = \ldots = y_N(x_N).\}$.

In the next section, we introduce some conditions for synchronization of the system in Equation (3) for the special case when the individual dynamics of each agent have unstable modes and sampling is not considered.

C. Synchronization Analysis Without Sampling

Synchronization of a LTI dynamic network can be reached based on the Kalman-Yakubovich-Popov (KYP) lemma and some conditions in the Laplacian of the graph [21] [20] . However, it usually implies that the dynamics of each node have to be stable, and, if the dynamics of each agent have unstable parts, we need to establish another conditions in order to assure synchronizability.

Proposition 2.1: Let us consider the synchronization problem for identical nodes where the agent dynamics are described by Equations (1) and (2). All agents synchronize their outputs if the graph describing the communication topology is connected, and if there exists a symmetric positive definite P such that

$$A - k_c\lambda_i BB^\top P < 0, \; B^\top P = C \tag{4}$$

for $i = 1, \ldots, N$, and λ_i is the i^{th} eigenvalue of the Laplacian $\mathcal{L}$.

Proof:

Let us denote the average state of all agents by $\bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_i$, and the vector $\bar{\mathbf{x}} = \frac{1}{N}(\mathbf{1}\mathbf{1}^\top \otimes I_n)\mathbf{x} = [\bar{x}, \bar{x}, \ldots, \bar{x}]^\top$, where $\mathbf{1}\mathbf{1}^\top$ is a matrix of ones, and I_n is the identity matrix of size $n \times n$. The deviation of each agent from the average state is $\xi_i = x_i - \bar{x}$. Since B is equal for all agents, synchronization implies that $\lim_{t\to\infty} \xi_i = \frac{1}{N}\sum_{j=1}^{N} \lim_{t\to\infty} (x_i - x_j) = 0$ Now, we define $\xi = \mathbf{x} - \bar{\mathbf{x}}$, and its derivative is then $\dot{\xi} = \dot{\mathbf{x}} - \dot{\bar{\mathbf{x}}}$ where

$$\dot{\bar{\mathbf{x}}} = \frac{1}{N}(\mathbf{1}\mathbf{1}^\top \otimes A)\mathbf{x} - \frac{k_c}{N}(\mathbf{1}\mathbf{1}^\top \mathcal{L} \otimes BC)\mathbf{x}. \tag{5}$$

Subtracting (5) from (3), leads to

$$\dot{\xi} = ((I_N \otimes A) - k_c(\mathcal{L} \otimes BC))\,\xi \tag{6}$$

Now, we can select a vector $\phi_i \in \mathcal{R}^{N\times 1}$ such that $\phi_i^\top \mathcal{L} = \lambda_i \phi_i$ and it forms an unitary matrix $\Phi = [\phi_1, \phi_2, \ldots, \phi_N]$ (i.e., $\Phi^\top \Phi = I_N$). Due to the row sum property of the Laplacian, the first eigenvalue is zero, which implies that

$\phi_1 = \mathbf{1}/\sqrt{N}$ and $\Phi^\top \mathcal{L} \Phi = \text{diag}(0, \lambda_2, \ldots, \lambda_N)$. Hence, using the properties of the Kronecker product we obtain that

$$(\Phi \otimes I_n)^\top (I_N \otimes A - k_c \mathcal{L} \otimes BC)(\Phi \otimes I_n)$$
$$= \text{diag}(A, A - k_c\lambda_2 BC, \ldots, A - k_c\lambda_N BC) \quad (7)$$

Let us define $\tilde{\xi} = (\Phi \otimes I_n)^\top \xi$. Since Φ is an unitary matrix, then $\xi = (\Phi \otimes I_n)\tilde{\xi}$. Hence, we partition $\tilde{\xi}$ in two parts, such that $\tilde{\xi} = [\tilde{\xi}_1, \tilde{\xi}_2]^\top$ where $\tilde{\xi}_1$ corresponds to the first element of $\tilde{\xi}$. Since all elements of ϕ_1 are equal, then $\tilde{\xi}_1 = \frac{1}{\sqrt{N}} \sum_{i=1}^N \xi_i = 0$. In view of Equations (6) and (7), it yields that

$$\dot{\tilde{\xi}}_2 = \text{diag}(A - k_c\lambda_2 BC, \ldots, A - k_c\lambda_N BC)\tilde{\xi}_2 \quad (8)$$

As the whole matrix is composed by blocks in its diagonal, we need to assure that $A - k_c\lambda_i BC < 0$. If $C = B^\top P$, for a symmetric positive definite P, then, selecting a proper k_c we assure that the second term will compensate the unstable dynamics of A, and $\lim_{t\to\infty} \tilde{\xi}_2 = \mathbf{0}$. Besides, due to $\Phi \otimes I_N$ is nonsingular, $\tilde{\xi}_2 = \mathbf{0}$ implies that $\xi = 0$ and synchronization is achieved. ■

The synchronization condition depends only on the eigenvalues of the Laplacian and k_c. Then, increasing connectivity (or k_c) makes possible to the dynamic network to achieve synchronization. However, synchronization does not assure stability, and if the individual dynamics of any agent have unstable modes, synchronization may be achieved, but *stability is lost*. Next, we will show that including data-sampling into the network, we can obtain synchronization and stability of the dynamic network.

III. Synchronization Under Sampling

Communication between agents depends on the structure of the graph and the information that each agent has to deliver to its neighbors through a communication channel. Consequently, measurements are sampled, and different synchronization conditions need to be developed.

Now, we will analyze the synchronization problem under sampling and we will introduce some conditions in order to assure synchronization and stability based on the average passivity concept.

A. Synchronization With Data-measurements

Let us consider the synchronization problem introduced in Section II. In order to ease the following definitions, we can rewrite the multi-agent synchronization representation, such that

$$\dot{x}_i(t) = Ax_i(t) - Bk_c \sum_{j=1}^N a_{ij} C x_i(t) + Bk_c \sum_{j=1}^N a_{ij} y_j(x_j)$$

which can be reduced to

$$\dot{x}_i(t) = \hat{A} x_i(t) + B u_i^{\mathcal{N}_i} \quad (9)$$

where $\hat{A} = A - k_c d_i BC$ for $d_i = \sum_{j=1}^N a_{ij}$, and $u_i^{\mathcal{N}_i} = k_c \sum_{j=1}^N a_{ij} y_j(x_j)$ is the input that depends on the information received from the neighbors of agent i. Until now, we have not changed the synchronization problem defined in the above section. However, as the information that each agent receives from its neighbors is spatially distributed, this information has to be sent through a communication network using sampled-measurements with a sampling period δ, according to Figure 1. We assume that the sampling period is homogenous for all agents. This can be achieved using special protocols such as the *IEEE 1588* or *IEEE Precision Clock Synchronization Protocol for Networked Measurement and Control Systems*, which allows data transmission using a common sampling period [22].

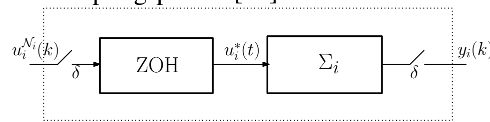

Fig. 1. Sampled data scheme for a dynamical system Σ_i, where the imput u_i^* is a piecewise control signal

The discrete-time controller $u_i^{Nj}(k)$ is calculated using discrete-time signals. Then, this controller signal is retained using a zero order holder (ZOH), such that it is constant during each sampling period (i.e., it is constant in the interval $[\delta k, \delta(k+1))$, where k are the sampling instants and the controller u_i^* is a piecewise controller).

Recalling that $\mathbf{x} = [x_1, x_2, \ldots, x_N]^\top$, the vector containing the state vectors of all N agents, $\mathcal{D} = \text{diag}(d_1, d_2, \ldots, d_N)$, where $d_i = \sum_{j=1}^N a_{ij}$, and $\mathbf{U}^* = [u_1^*, \ldots, u_N^*]$, the synchronization dynamical equation of the network for a time period $r = [k\delta, (k+1)\delta)$ can be written as

$$\dot{\mathbf{x}}(r) = ((I_N \otimes A) - k_c(\mathcal{D} \otimes BC))\,\mathbf{x}(r) + k_c\,((\mathcal{A}_\mathcal{G} \otimes B))\,\mathbf{y}^* \quad (10)$$

for $\mathbf{y}^* = [y_1^*, \ldots, y_N^*]$ being the vector of the sampled-data outputs after the ZOH.

Using the properties of the Kronecker product, and defining $\hat{\mathbf{A}} = \text{diag}(A - k_c d_1 BC, A - k_c d_2 BC, \ldots, A - k_c d_N BC)$, $\mathbf{B} = \text{diag}(B, B, \ldots, B)$, and $\mathbf{U}^* = k_c(\mathcal{A}_\mathcal{G} \otimes I_n)\mathbf{y}^*$, the continuous-time representation can be described by

$$\dot{\mathbf{x}}(r) = \hat{\mathbf{A}}\mathbf{x}(r) + \mathbf{B}\mathbf{U}^* \quad (11)$$

According to [17], the relationship between the LTI continuous sampled-data system and its discrete-time counterpart is $\hat{\mathbf{A}}^\delta = e^{\delta \hat{\mathbf{A}}}$, $\mathbf{B}^\delta = \int_\mathbf{0}^\delta \mathbf{e}^{\tau \hat{\mathbf{A}}} \mathbf{B}$, such that

$$\mathbf{x}(k+1) = \hat{\mathbf{A}}^\delta \mathbf{x}(k) + \mathbf{B}^\delta \mathbf{U}(k)$$
$$\mathbf{y}^\delta(\mathbf{x}(k)) = \mathbf{C}^\delta \mathbf{x}(k) \quad (12)$$

where $\mathbf{C}^\delta$ is described in the next section [1].

If there are not sampled-measurements, synchronization of Equation (11) depends on the conditions stated in Proposition 2.1. For the synchronization under sampling case, the analysis of stability and synchronization is given by the analysis of Equation (12), which can be done using passivity. However, as it is well known, the inclusion of sampled-measurements provokes passivity to be lost in the discrete-time representation of the system [17]. Then, a new passivity concept is introduced in such a way that passivity of sampled-data systems is assured [16].

Next, we introduce some definitions of average passivity adapted from [23] in order to obtain some conditions for synchronization and stability.

B. Average Passivity

The classical ideas of passivity for LTI continuous-time systems are well defined based on the Kalman-Yakubovich-

[1] The use of the super-index δ indicates that it is part of a sampled-data model, with a sampling period δ.

Popov (KYP) lemma, for systems of the form

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_c(t)$$
$$y_c(t) = C_c x_c(t) + D_c u_c(t) \quad (13)$$

with $A_c \in \mathcal{R}^{p\times p}$, $B_c \in \mathcal{R}^{p\times q}$, $C_c \in \mathcal{R}^{p\times q}$, and $D_c \in \mathcal{R}^{q\times q}$. The system is passive [24] with a quadratic storage function $V_c = \frac{1}{2}x_c^\top P_c x_c$, if there exists a matrix $P_c > 0$, such that

$$P_c A_c + A_c^\top P_c \leq 0,\ C_c = B_c^\top P_c,\ D_c + D_c^\top \geq 0 \quad (14)$$

However, due to the inclusion of a communication infrastructure and the use of sensors, the output of a continuous-time system is sampled with a finite sampling period $\delta > 0$, and the dynamics of the system under sampling are described by

$$x_d(k+1) = A_d x_d(k) + B_d u_d(k)$$
$$y_d(k) = C_d x_d(k) + D_d u_d(k) \quad (15)$$

where $A_d = e^{\delta A_c}$, $B_d = \int_0^\delta e^{\tau A_c} B_c d\tau$, $C_d = C_c$, and $D_d = D_c$.
The passivity condition for the system in Equation (15) is given by the KYP lemma with a storage function $V(x_d(k)) = x_d^\top(k) P_d x_d(k)$ [17], such that

$$A_d^\top P_d A_d = P_d,\ B_d^\top P_d A_d = C_d\ , D_d + D_d^\top = B^\top P_d B_d$$

However, when the system does not have a direct input/output link (i.e., $D_d = 0$), implies that $B_d = 0$ and $C_d = 0$, which means that passivity is preserved only if $y_d(k) = 0$. Besides, as the system in (15) is a representation of a continuous-time system, then $C_d = C_c$, which is neither satisfied.

The motivation of defining average passivity introduced in [16] lies in the fact that the sampled equivalent model of a passive continuous-time system does not satisfy a standard discrete-time dissipation inequality as it is shown above. Then, average passivity is based on the analysis of a LTI-continuous system under a piecewise control that depends on discrete-time dynamics, such that passivity conditions have to be satisfied over each time interval $[\delta k, \delta(k+1))$ for $k \in \mathcal{Z}_+$. Defining $x_c(\delta k)$ as an initial state and $x_c(\delta(k+1))$ as a final state that evolves under a constant input u_c^*, then the total energy stored in the system is given by the sum of the energy stored during each time interval. If $\delta \to 0$, then the total storage function is described by the common passivity definition. The next definitions adapted from [16] [23] are introduced

Definition 3.1: The sampled-data system represented by Equation (15) is average passive in the quadratic form if, for all $x_d(k)$ and for the output $y_d(x_d(k), u_d(k)) = C_d x_d(k)$, the following inequality holds

$$V(x_d(k+1)) - V(x_d(k)) \leq u_d(k)^\top y_{d_{av}}(x_d(k), u_d(k)), \quad (16)$$

where $y_{d_{av}}(x(k), u(k)) := \frac{1}{\delta u_d}\int_0^{\delta u_d} y_d(x_d(k), v)dv = C_d A_d x_d(k) + \frac{1}{2}C_d B_d u_d(k)$ Additionally, if
$V(x_d(k+1)) - V(x_d(k)) \leq u_d^\top(k) y_{d_{av}}(x_d(k), u_d(k)) - \psi(x_d(k))$ for $\psi(x(k)) > 0$, the system is *average strictly passive*, and the origin is asymptotically stable.

Definition 3.2: Let us consider the continuous-time LTI system in Equation (13), which is stable with matrix $P_d > 0$ according to (14). Then, the sampled-data equivalent is average passive if the output is given by $y_d(x_d(k)) = B_d^\top P_d x_d(k)$.
The reader is refereed to [16] [23] for more details on average passivity.
Next, average passivity is introduced for the synchronization problem in such a way synchronizability under sampling can be preserved.

C. Synchronization and Stability Under Sampling Using Average Passivity

In order to define the synchronization and stability conditions for the sampled representation in (12), common passivity conditions cannot be defined according to the following lemma.

Lemma 3.1: If the continuous-time LTI synchronization problem is passive according to Equation (4), then passivity is lost under sampling using the control protocol in (9) when the common discrete-time representation is taken into account, such that $C^\delta = C$ and $\mathbf{y}^\delta(\mathbf{x}(k)) = C\mathbf{x}(k)$

Proof

According to Section III-B, passivity is lost under sampling when the direct input/output link is not considered. If we have the continuous time synchronization model in Equation (3), and the system is subjected to a piecewise control input as given in (11), then passivity for the discrete-time representation in Equation (12) do not hold when $C = C^\delta = B^\top P$. ■

Now, in the following theorem we state some conditions to assure synchronizability of the LTI continuous-time systems with unstable dynamics under sampling recalling the average passivity properties in Section III-B.

Theorem 3.1: Let us consider the multi-agent synchronization model in Equation (11), where the dynamics of each agent are unstable. Let assume that there exists a positive definite $P = P^\top$ such that $\hat{A}^\top P + P\hat{A} < 0$, for $\hat{A} = A - k_c d_i B B^\top P$. If sampling is taken into account for some sampling period $0 < \delta < \delta^*$, agents send data to its neighbors such that $\mathbf{y}(k)^\delta = \mathbf{B}^{\delta^\top}\mathbf{P}\mathbf{x}(k)$ for $\mathbf{P} = I_N \otimes P$, and the graph is connected, then, all agents synchronize.

Proof

Let us define the matrix $\mathbf{P} = \text{diag}(P, P, \ldots, P)$. Then, for the sampled-data representation in (12), there exists a Lyapunov function $V(\mathbf{x}(k+1)) - V(\mathbf{x}(k)) = \frac{1}{2}\left(\mathbf{x}^\top(k+1)\mathbf{P}\mathbf{x}(k+1) - \mathbf{x}^\top(k)\mathbf{P}\mathbf{x}(k)\right) = \frac{1}{2}\mathbf{x}^\top(k)\left(\hat{\mathbf{A}}^{\delta^\top}\mathbf{P}\hat{\mathbf{A}}^\delta - \mathbf{P}\right)\mathbf{x}(k) + \mathbf{U}^\top(k)\mathbf{B}^{\delta^\top}\mathbf{P}\hat{\mathbf{A}}^\delta\mathbf{x}(k) + \frac{1}{2}\mathbf{U}^\top(k)\mathbf{B}^{\delta^\top}\mathbf{P}\mathbf{B}^\delta\mathbf{U}(k)$. According to definition 3.1, $\mathbf{y}_{av} = \mathbf{C}^\delta\hat{\mathbf{A}}^\delta\mathbf{x}(k) + \frac{1}{2}\mathbf{C}^\delta B^\delta\mathbf{U}(k)$. Then, if $\mathbf{C}^\delta = \mathbf{B}^{\delta^\top}\mathbf{P}$, we have that $V(\mathbf{x}(k+1)) - V(\mathbf{x}(k)) = \mathbf{U}^\top(k)\mathbf{y}_{av} + \frac{1}{2}\mathbf{x}^\top(k)\left(\hat{\mathbf{A}}^\delta\mathbf{P}\hat{\mathbf{A}}^\delta - \mathbf{P}\right)\mathbf{x}(k)$.

The condition of the continuous-time dynamics ($\hat{\mathbf{A}}^\top\mathbf{P} + \mathbf{P}\hat{\mathbf{A}} \leq 0$) for the non-sampled case ensures negativity of the term $\hat{\mathbf{A}}^{\delta^\top}\mathbf{P}\hat{\mathbf{A}}^\delta - \mathbf{P}$. Hence, according to definition 3.2, the system is average strictly passive, the origin is asymptotically stable, and all trajectories will tend to the synchronization manifold $M^\delta = \left\{\mathbf{x} | y_1^\delta = y_2^\delta = \ldots y_N^\delta = 0\right\}$. ■

Remark 3.1: **[Synchronization for heterogeneous agents:]** The aforementioned theorem establishes some conditions to assure synchronization of all agents. However, in real applications, such that the synchronization of MGs in smart grids, agents (MGs) are not homogeneous. Then, we can extend our previous result for the case when heterogeneous agents are considered, and the i^{th} agent dynamics are described by the matrices A_i, B_i, C_i with a control protocol $u_i = k_{c_i} \sum_{j=1}^{N} a_{ij}(y_j - y_i)$. Then, $\hat{A}_i = A_i - k_{c_i} d_i B_i B_i^\top P_i$, for $P_i > 0$ and $P_i^\top = P_i$ such that $\hat{A}_i^\top P_i + P_i \hat{A}_i < 0$, and $\hat{\mathbf{A}} = \text{diag}(\hat{A}_1 \ldots, \hat{A}_N)$, $\mathbf{B} = \text{diag}(B_1, B_2, \ldots, B_N)$, and $\mathbf{P} = \text{diag}(P_1, \ldots, P_N)$. According to this, if the sampled output is given by $\mathbf{y}^\delta(k) = \mathbf{B}^{\delta^\top} \mathbf{P}\mathbf{x}(k)$ the system is stable and all agents synchronize to the synchronization manifold $M^\delta = \{\mathbf{x} | y_1^\delta = y_2^\delta = \ldots y_N^\delta = 0\}$.

IV. Simulation Results

In order to illustrate the usefulness of our result, synchronization of microgrids is introduced. We consider that each microgrid is of low voltage and works in islanded mode. There are 5 MGs (nodes), each one modeled as a synchronous generator associated to a load as depicted in Figure 2a. The objective is that all MGs work to a desired frequency, controlling the power that the generator has to produce. Although there is not a physical connection between microgrids, there is a communication infrastructure associated to a communication graph, such that each MG only receives information from its neighbors (Figure 2b). We assume that the communication network works with the IEEE 1588 protocol in order to assure homogenous sampling periods.

A simplified dynamical linear model of a microgrid based on [15] is considered assuming that the frequency variations are low. Then, the dynamic model for the synchronous generator are represented by a transfer function $G_g(s) = \frac{P_g(s)}{U_g(s)} = \frac{1}{T_g s+1}$, where T_g is the time constant of the generator, P_g is the difference between the generated power and the demanded power, and U_g is the input of the generator. Besides, in low voltage MG, a relation between real power generated and frequency variations can be defined. Then, the behavior in frequency for the low voltage MG is described by $\frac{\Delta f(s)}{P_g(s)} = \frac{1}{Ms+D}$ where D is the damping constant, and M is the inertia constant. Then, the matrix representation is given

$$\begin{bmatrix} \Delta\dot{f} \\ \dot{P}_g \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{D}{M} & \frac{1}{M} \\ 0 & -\frac{1}{Tg} \end{bmatrix}}_{A} \begin{bmatrix} \Delta f \\ P_g \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{Tg} \end{bmatrix}}_{B} U$$

where U is the vector of control inputs, given by $[0, U_g]^\top$, and the sensed outputs are $[\Delta_f, Pg]$. According to [25], when there are failures in a MG, any eigenvalue of A can become positive (e.g., the damping constant becomes negative), provoking unstable behaviors in the system. Then, we assume that each MG posses its own dynamics described by the aforementioned model, and they are isolated. We also assume that failures have occurred in two of them, making them unstable. Then, synchronization protocol defined in (11) should be able to control the power generated for the heterogeneous MGs, maintaining the frequency stable.

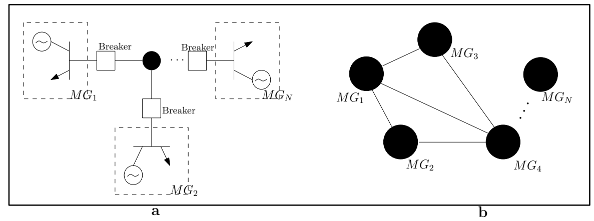

Fig. 2. a) Model of several microgrids that can work in islanded mode if the breaker is open; b) Communication topology between the microgrids.

Figure 3 illustrates the Δ_{f_i} and the P_{g_i} of each MG when sampling is not taking into account and $M = 0.2puMW - s/Hz$, $T_g = 2s$. The damping constant is such that $D/M = i/4$ for the i^{th} node. A failure has occurred in two MGs making them unstable. Since the communication between

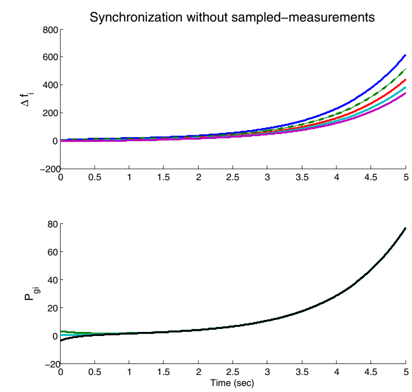

Fig. 3. Synchronization of the change in frequency and the power of all MGs, when sampled-measurements are not considered.

agents is not ideal, we need to considere a communication infrastructure, such that each agent sends sampled data to its neighbors. If we send data according to Theorem 3.1, for $k_c = 5$ we can assure that even with a failure in any node, the system is synchronized. Figure 4 shows that synchronization can be achieved when sampled-data measurements are included.

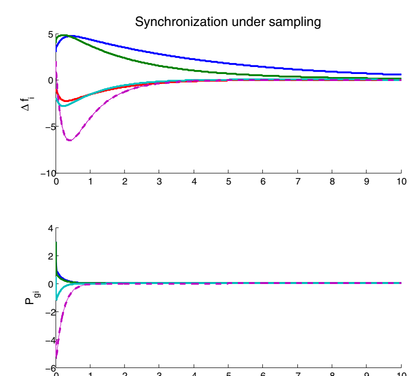

Fig. 4. Synchronization of the change in frequency and the power of all MGs, when sampled-measurements are considered.

Here, we can observe an application in smart grids, where the frequency of each MG has to be synchronized, such that the change in frequency is equal for each node. Besides, we also need that the difference between the generated power and the demanded power is zero, such that the generator only produce the necessary energy to supply the demand. However, failures in the system can provoke that the dynamics of some nodes become unstable, and, without a proper control, the frequency change of all nodes get unstable and unsynchronized as depicted in Figure 3. Usually, control of microgrids is made using PI controllers, where the dynamics of the controller, produced by the integral part, makes the dynamics of the microgrid more complex [15]. Besides, the control of a microgrid does not consider the states of the other microgrids, and synchronization is only achieved with optimal conditions (i.e., when there are not failures and all frequency differences tend to zero thanks to the PI controller). Hence, using the proposed synchronization protocol with sampling data measurements, we can assure *synchronization and stabilization* of the system even when some failures appear, using a very simple control that depends only on basic mathematical operations (e.g., sums and subtraction). The only condition that has to be satisfied is that the graph has to be connected, in such a way all agents receives information from at least one neighbor.

V. Conclusions

The synchronization of multi-agent networked system has been addressed for LTI continuous-time dynamics, where agents are distributed and share information using sampled-data measurements, with homogeneous sampling periods. The motivation is given by the smart grids that include distributed generators and a communication infrastructure, where the frequency of all isolated MGs must be synchronized in order to avoid cascade failures or blackouts. However, we considered the case when any MG presents unstable dynamics due to some internal failures. Then, we have proposed a novel synchronization scheme including sampling-measurements based on the concept average passivity for sampled-data systems for identical and non-identical nodes, and we have established the conditions to assure synchronization and stability of the dynamical network. The simulation results illustrate the case when there is not sampling but some internal failures provokes the system to get unsynchronized and unstable. However, when sampling is included, the system is stable and the output states are synchronized. These developments are not limited to synchronization of microgrids and they can be used for the synchronization of dynamical networks when the information that each node receives from its neighbors is sampled. In future work, the extension for nonlinear systems may be developed and some applications based on Kuramoto oscillators under sampling may be introduced.

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