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Constrained decomposition based control approach for linear parameter varying systems

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***Abstract*— Considering a constrained discrete-time linear time-varying system, this paper proposes a novel approach which aims at achieving high performance and enlarging the domain of attraction with respect to any particular linear controller. The main idea of the paper is to use a linear decomposition principle together with a parameter dependent Lyapunov function. At each time instant a quadratic programming problem is solved on-line. Proofs of recursive feasibility and asymptotic stability are given.**

I. Introduction

Consider the problem of regulating the following discrete-time linear time-varying (LPV) system

$$x(k+1) = A(\alpha(k))x(k) + Bu(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ are respectively, the state and control variables. The matrix $A(\alpha(k)) \in \mathbb{R}^{n\times n}$ satisfies

$$\begin{cases} A(\alpha(k)) = \sum\limits_{i=1}^{s} \alpha_i(k)A_i, \\ \sum\limits_{i=1}^{s} \alpha_i(k) = 1,\ \alpha_i(k) \geq 0 \end{cases} \tag{2}$$

where the matrices A_i are given. The case when the matrix B is also time-varying is omitted here for brevity but is addressed in a journal version of the paper.

The state vector, the control input are subject to the following polytopic constraints

$$\begin{cases} x(k) \in X,\ X = \{x \in \mathbb{R}^n : F_x x \leq g_x\}, \\ u(k) \in U,\ U = \{u \in \mathbb{R}^m : F_u u \leq g_u\} \end{cases} \tag{3}$$

where the matrices F_x, F_u and the vectors g_x, g_u are assumed to be constant with $g_x > 0$ and $g_u > 0$, such that the origin is contained in the interior of X and U. The inequalities here are component-wise. In this paper, it is assumed hereafter

- **LPV hypothesis:** $\alpha(k)$ is measurable at each time instant.

Several approaches have been reported for the constrained control of LPV system of type (1) in the model predictive control (MPC) literature. One approach [1], [2], [3], [4] is based on formulating the MPC problem in the min-max optimization framework. At each time instant, a semi-definite problem using linear matrix inequalities (LMI) constraints is solved on-line by minimizing an upper bound on the infinite horizon objective function. In [5], another approach is used, based on the explicit MPC together with Polya's relaxation theorem.

In [6], [7], [8] an interpolation among predefined linear feedback laws is proposed. The feasible region is the convex hull of the constituent terminal invariant sets, leading to a relatively large domain of attraction.

Recently, another interpolation approach was proposed in [9], [10], [11] between a global robust vertex controller and a local unconstrained robust optimal control law. By minimizing an auxiliary objective function, recursive feasibility and a robustly asymptotically stable closed-loop are guaranteed.

Note that for all the control strategies in [6], [7], [9], [10], [11], the parameters $\alpha(k)$ are not available in real-time. Therefore, the controller can be classified as robust but not scheduled, in the sense that the control action is not a function of the current vector of parameters.

In this paper, a novel linear decomposition based control method is proposed for the state feedback stabilization problem. The main idea is to use a linear decomposition between several gain-scheduled controllers. For reducing conservativeness, a parameter dependent Lyapunov function is used. The main contribution of the paper is the extension of the results in [7], [12] for constrained linear time-varying systems. Using the same set of predefined local controllers, this paper has three main special features with respect to [12].

First, it will be shown that the proposed method in this paper contains as a subset all solutions available to the method in [12].

Second, the proposed method requires fewer decision variables for the optimization problem than the method in [12].

Third, the feasible regions of the proposed method in this paper are larger than the feasible regions of the method in [12].

From the computational complexity point of view, at each time instant, the proposed scheme requires a solution of a quadratic programming problem of dimension $(r-1)n$, where n is the dimension of the state and r is the number of pre-defined controllers among which gain-scheduling is performed. In term of performance and the domain of attraction, as practice usually shows, it is enough with $r = 2$ or $r = 3$. We show that this extremely simple optimization problem is computationally comparable with a one- or two-step ahead MPC.

The paper is organized as follows. Section 2 is dedicated to some notations and basic definitions from set invariance

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theory. Section 3 presents a linear decomposition based control technique. The simulation results are evaluated in Section 4 before drawing the conclusions.

II. Notation and basic definitions

A. Notation

Throughout the paper, $\mathcal{A}^T$ denotes the transpose of matrix $\mathcal{A}$. A positive definite (negative definite) square matrix $\mathcal{A}$ is denoted by $\mathcal{A} \succ 0$ ($\mathcal{A} \prec 0$). $\mathcal{A}^{\frac{1}{2}}$ denotes its Cholesky factor, which satisfies $(\mathcal{A}^{\frac{1}{2}})^T \mathcal{A}^{\frac{1}{2}} = \mathcal{A}$. $\mathcal{I}$ and $\mathbf{0}$ denote the identity and zero matrices, respectively, of appropriate dimensions. Whenever time k is unspecified, a variable x stands for $x(k)$ for some time $k \in \mathbb{N}$.

B. Set invariance

The relationship between the constraints (3) and the dynamics (1) leads to the introduction of invariance/viability concepts. Suppose that the following gain scheduled controller

$$u(k) = K(\alpha(k))x(k)$$

exists for system (1) such that the resulting closed loop system

$$x(k+1) = (A(\alpha(k)) + BK(\alpha(k)))x(k) \tag{4}$$

is asymptotically stable.

Definition 1: (Positively invariant constraint-admissible set) [13] The set $\Omega \subseteq X$ satisfying $K(\alpha(k))\Omega \subseteq U$ is positively invariant constraint-admissible with respect to the LPV system (4) and the constraints (3) if any only if for all $x(k) \in \Omega$, it follows that $x(k+1) \in \Omega$.

The largest positively invariant set is generally called the maximal admissible set [14].

Dentition 2: (Controlled invariant set) [13] The set $\Pi \subseteq X$ is controlled invariant with respect to the LPV system (1) and constraints (3) if and only if for all $x(k) \in \Pi$, there exists an admissible control input $u(k) \in U$ such that $x(k+1) \in \Pi$.

III. Linear decomposition based control

A. The principle

Following Rossiter and Ding [7], [8], any state $x(k)$ is decomposed as

$$x(k) = x_1(k) + x_2(k) + \ldots + x_r(k) \tag{5}$$

where $x_j \in \mathbb{R}^n$ for $j = 1, 2, \ldots, r$ are slack variables. The slack variables are not fixed, but will be treated as decision variables. Extending [7], [8], we propose the control to be

$$u(k) = \sum_{j=1}^{r} K_j(\alpha(k))x_j(k) \tag{6}$$

where $K_j(\alpha(k)) = \sum\limits_{i=1}^{s} \alpha_i(k)K_{ji}$ are given such that the closed loop systems

$$x(k+1) = \Phi_j(\alpha(k))x(k)$$

are asymptotically stable, with

$$\Phi_j(\alpha(k)) = A(\alpha(k)) + BK_j(\alpha(k))$$

From equation (2), it follows that $\Phi_j(\alpha(k))$ can be expressed as a convex combination of Φ_{ji}, i.e.

$$\Phi_j(\alpha(k)) = \sum_{i=1}^{s} \alpha_i(k)\Phi_{ji} \tag{7}$$

where $\Phi_{ji} = A_i + BK_j$.

Using equations (5), (6), one gets

$$\begin{aligned} x(k+1) &= A(\alpha(k))x(k) + Bu(k) \\ &= \sum_{j=1}^{r} \Phi_j(\alpha(k))x_j(k) = \sum_{j=1}^{r} x_j(k+1) \end{aligned}$$

where

$$x_j(k+1) = \Phi_j(\alpha(k))x_j(k) \tag{8}$$

Since $x_1(k) = x(k) - \sum\limits_{j=2}^{r} x_j(k)$, one has

$$x(k+1) = \Phi_1(\alpha)x(k) + \sum_{j=2}^{r} \left(\Phi_j(\alpha) - \Phi_1(\alpha)\right) x_j(k) \tag{9}$$

Define

$$z = \begin{bmatrix} x^T & x_2^T & \ldots & x_r^T \end{bmatrix}^T \tag{10}$$

Combining equations (8), (9), one obtains

$$z(k+1) = \Gamma(\alpha(k))z(k) \tag{11}$$

with

$$\Gamma = \begin{bmatrix} \Phi_1(\alpha) & (\Phi_2(\alpha) - \Phi_1(\alpha)) & \ldots & (\Phi_r(\alpha) - \Phi_1(\alpha)) \\ \mathbf{0} & \Phi_2(\alpha) & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \Phi_r(\alpha) \end{bmatrix} \tag{12}$$

Based on equation (2), it is clear that matrix $\Gamma(\alpha(k))$ belongs to the convex hull of Γ_i, i.e.

$$\Gamma(\alpha(k)) = \sum_{i=1}^{s} \alpha_i(k)\Gamma_i \tag{13}$$

with

$$\Gamma_i = \begin{bmatrix} \Phi_{1i} & (\Phi_{2i} - \Phi_{1i}) & \ldots & (\Phi_{ri} - \Phi_{1i}) \\ \mathbf{0} & \Phi_{2i} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \Phi_{ri} \end{bmatrix}$$

The constraints on the augmented state $z(k)$ are

$$\begin{bmatrix} F_x & \mathbf{0} & \ldots & \mathbf{0} \\ F_u K_{11} & F_u(K_{21} - K_{11}) & \ldots & F_u(K_{r1} - K_{11}) \\ \vdots & \vdots & \ddots & \vdots \\ F_u K_{1s} & F_u(K_{2s} - K_{1s}) & \ldots & F_u(K_{rs} - K_{1s}) \end{bmatrix} z \le \begin{bmatrix} g_x \\ g_u \\ \vdots \\ g_u \end{bmatrix} \tag{14}$$

For system (11) with constraints (14), using procedures in [15], the maximal positively invariant set $\Psi_a \subset \mathbb{R}^{rn}$ can be found in polyhedral form,

$$\Psi_a = \{z \in \mathbb{R}^{rn} : F_a z \le g_a\} \tag{15}$$

such that for all $z(k) \in \Psi_a$, it follows that $z(k+1) \in \Psi_a$ and $u = K_1(\alpha)x + \sum\limits_{j=2}^{r}(K_j(\alpha) - K_1(\alpha))x_j \in U$. Define

$\Psi \subset \mathbb{R}^n$ as a set obtained by projecting the polyhedral set Ψ_a onto the state space x. The following theorem holds.

Theorem 1: For system (1), the polyhedral set Ψ is controlled positively invariant and constraint-admissible with respect to the constraints (3).

Proof: Clearly, for all $x(k) \in \Psi$, there exists $x_j(k)$ with $j = 2, 3, \ldots, r$ such that

- The augmented state $z(k)$ is in Ψ_a.
- The control action $u = K_1(\alpha)x + \sum_{j=2}^{r}(K_j(\alpha) - K_1(\alpha))x_j$ is in U.
- The successor augmented state $z(k+1)$ is in Ψ_a.

Since $z(k+1) \in \Psi_a$, it follows that $x(k+1) \in \Psi$. Hence Ψ is a controlled positively invariant set. □

B. Cost function determination

For the given control and state weighting matrices $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{m\times m}$, consider the following quadratic function

$$V(k,z) = z(k)^T P(\alpha(k)) z(k) \tag{16}$$

with

$$P(\alpha(k)) = \sum_{i=1}^{s} \alpha_i(k) P_i \tag{17}$$

It will be proved below after equation (23) that there exist matrices P_i with $P_i \in \mathbb{R}^{rn\times rn}$, $P_i \succ 0$ such that

$$\begin{aligned} &V(k+1, z(k+1)) - V(k, z(k)) \\ &\quad \le -x(k)^T Q x(k) - u(k)^T R u(k) \end{aligned} \tag{18}$$

Using equation (11), the left hand side of inequality (18) can be written as

$$\begin{aligned} &V(k+1, z(k+1)) - V(k, z(k)) = \\ &= z(k)^T \Gamma(\alpha(k))^T P(\alpha(k+1)) \Gamma(\alpha(k)) z(k) \\ &\quad - z(k)^T P(\alpha(k)) z(k) \end{aligned} \tag{19}$$

And the right hand side

$$-x(k)^T Q x(k) - u(k)^T R u(k) = -z(k)^T (\mathcal{Q} + R_1(\alpha)) z(k) \tag{20}$$

with

$$\mathcal{Q} = \begin{bmatrix} \mathbb{I} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} Q \begin{bmatrix} \mathbb{I} & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix}$$
$$R_1(\alpha) = \mathcal{R}^T(\alpha) R \mathcal{R}(\alpha)$$

where

$$\mathcal{R}(\alpha) = \begin{bmatrix} K_1(\alpha) & (K_2(\alpha) - K_1(\alpha)) & \ldots & (K_r(\alpha) - K_1(\alpha)) \end{bmatrix}$$

From equation (2), it is clear that $\mathcal{R}(\alpha)$ belongs to the convex hull of $\mathcal{R}_i$, i.e.

$$\mathcal{R}(\alpha(k)) = \sum_{i=1}^{s} \alpha_i(k) \mathcal{R}_i \tag{21}$$

where

$$\mathcal{R}_i = \begin{bmatrix} K_{1i} & (K_{2i} - K_{1i}) & \ldots & (K_{ri} - K_{1i}) \end{bmatrix}$$

Combining equations (18), (19), (20), one gets

$$\begin{aligned} &\Gamma(\alpha(k))^T P(\alpha(k+1)) \Gamma(\alpha(k)) - P(\alpha(k)) \\ &\quad \preceq -\mathcal{Q} - R_1(\alpha(k)) \end{aligned}$$

or equivalently

$$\begin{bmatrix} P(\alpha(k)) - \mathcal{Q} - R_1(\alpha(k)) & * \\ P(\alpha(k+1))\Gamma(\alpha(k)) & P(\alpha(k+1)) \end{bmatrix} \succeq 0$$

By using the Schur complements, the last inequality can be brought into

$$\begin{bmatrix} P(\alpha(k)) & * & * & * \\ P(\alpha(k+1))\Gamma(\alpha(k)) & P(\alpha(k+1)) & * & * \\ \mathcal{Q}^{\frac{1}{2}} & \mathbf{0} & \mathbb{I} & * \\ R^{\frac{1}{2}}\mathcal{R}(\alpha(k)) & \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix} \succeq 0 \tag{22}$$

The left hand side of inequality (22) is linear with respect to the parameters $\alpha(k+1)$. Therefore, it reaches its minimum if and only if $\alpha(k+1) = 1$ or $\alpha(k+1) = 0$. Hence, the matrices $P(\alpha(k+1)) = \sum_{t=1}^{s} \alpha_t(k+1) P_t$ have to be chosen such that the following set of LMI conditions holds

$$\begin{bmatrix} P(\alpha(k)) & * & * & * \\ P_t\Gamma(\alpha(k)) & P_t & * & * \\ \mathcal{Q}^{\frac{1}{2}} & \mathbf{0} & \mathbb{I} & * \\ R^{\frac{1}{2}}\mathcal{R}(\alpha(k)) & \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix} \succeq 0 \tag{23}$$

for $t = 1, 2, \ldots, s$. It is clear that the problem (23) is feasible if and only if the matrix $\Gamma(\alpha(k))$ is asymptotically stable. By denoting $W_t = P_t^{-1}$ and by applying the congruence transformation $diag(G^T, W_t, \mathbb{I}, \mathbb{I})$ to inequality (23) [16], one obtains

$$\begin{bmatrix} G^T P(\alpha(k)) G & * & * & * \\ \Gamma(\alpha(k)) G & W_t & * & * \\ \mathcal{Q}^{\frac{1}{2}} G & \mathbf{0} & \mathbb{I} & * \\ R^{\frac{1}{2}}\mathcal{R}(\alpha(k)) G & \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix} \succeq 0 \tag{24}$$

for $t = 1, 2, \ldots, s$. Here the matrix $G \in \mathbb{R}^{rn\times rn}$ is a slack variable.

Inequality (24) holds true if and only if

$$\begin{bmatrix} G^T P_i G & * & * & * \\ \Gamma_i G & E_t & * & * \\ \mathcal{Q}^{\frac{1}{2}} G & \mathbf{0} & \mathbb{I} & * \\ R^{\frac{1}{2}}\mathcal{R}_i G & \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix} \succeq 0 \tag{25}$$

for all $i = 1, 2, \ldots, s$ and $t = 1, 2, \ldots, s$.

By using the dilation inequality [17], [18], namely

$$G^T \mathcal{T}^{-1} G \succeq G + G^T - \mathcal{T}$$

to the block (1,1) of inequality (24) and recalling that $W_i = P_i^{-1}$, one obtains

$$\begin{bmatrix} G^T + G - W_i & * & * & * \\ \Gamma_i G & W_t & * & * \\ \mathcal{Q}^{\frac{1}{2}} G & \mathbf{0} & \mathbb{I} & * \\ R^{\frac{1}{2}}\mathcal{R}_i G & \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix} \succeq 0 \tag{26}$$

for all $i = 1, 2, \ldots, s$ and $t = 1, 2, \ldots, s$.

Apparently, the problem (26) is linear with respect to the matrix variables W_i, G. One way to obtain the matrices W_i and G is to solve the following LMI problem

$$\min_{W_i,G}\{-\sum_{i=1}^{s} trace(W_i)\} \tag{27}$$

subject to constraints (26). This LMI problem has always a solution from which the matrices P_i can be computed as $P_i = W_i^{-1}$.

C. Linear decomposition via quadratic programing

Once the matrix $P_i = W_i^{-1}$ is computed off-line as a solution of the optimization problem (27), it can be used in practice for real-time control based on the following algorithm, which can be formulated as an efficient optimization problem. The resulting control law can be seen as a predictive control type of construction if (16) is interpreted as an upper bound for a receding horizon cost function.

At each time instant, for a given current state $x(k)$, minimize on-line the following quadratic cost function subject to linear constraints

$$J = \min_{z}\left\{z(k)^T P(\alpha(k))z(k)\right\} \tag{28}$$

such that (10) holds and

$$F_a z(k) \leq g_a$$

The control input is in the form

$$u = K_1(\alpha)x + \sum_{j=2}^{r}(K_j(\alpha) - K_1(\alpha))x_j \tag{29}$$

Theorem 2: The control law (29), where x_j for $j = 2, 3, \ldots, r$ are the solution of the quadratic programming problem (28) guarantees recursive feasibility and closed-loop asymptotic stability for all intial states $x(0) \in \Psi$.

Proof: Theorem 2 stands on two important claims, namely recursive feasibility and asymptotic stability. These can be treated sequentially.

Recursive feasibility: It has to be proved that $F_u u(k) \leq g_u$ and $x(k+1) \in \Psi$ for all $x(k) \in \Psi$.

Since for all $x(k) \in \Psi$, there exists $x_j(k)$ with $j = 2, 3, \ldots, r$ such that $z(k) \in \Psi_a$. Hence the optimization problem (28) is always feasible. From equation (29), it follows that

$$u = K_1(\alpha)x + \sum_{j=2}^{r}(K_j(\alpha) - K_1(\alpha))x_j \in U$$

With this control input, it holds that $z(k+1) \in \Psi_a$. Therefore $x(k+1) \in \Psi$.

Asymptotic stability: From the feasibility proof, it follows that if $x_j^o(k)$ for $k = 2, 3, \ldots, r$ are the solution of the optimization problem (28) at time instant k, then

$$x_j(k+1) = \Phi_j(\alpha(k))x_j^o(k)$$

are a feasible solution of the optimization problem (28) at time instant $k+1$. By solving the quadratic programming problem (28), one obtains

$$V(k+1, z^o(k+1)) \leq V(k+1, z(k+1))$$

and by using the inequality (18), it follows that

$$\begin{aligned} &V(k+1, z^o(k+1)) - V(k, z^o(k)) \\ &\quad \leq V(k+1, z(k+1)) - V(k, z^o(k)) \\ &\quad \leq -x(k)^T Q x(k) - u(k)^T R u(k) \end{aligned}$$

Therefore $V(k, z(k))$ is a Lyapunov function for system (11) [19], [20]. It follows that the closed loop system with the linear decomposition based controller is asymptotically stable. □

D. Choices of K_j

In our approach, K_j for $j = 1, 2, \ldots, r$ are chosen and ordered complying with the following principles.

- The first controller $u(k) = K_1(\alpha(k))x(k)$ is chosen as a priority controller and is used for the performance. As will be proved later, with a block diagonal matrix $P(\alpha(k))$ the linear decomposition based controller eventually reaches $K_1(\alpha(k))x(k)$ as time involves.
- The remain controllers $u(k) = K_j(\alpha(k))x(k)$ with $j = 2, 3, \ldots, r$ are used for enlarging the domain of attraction. For these controllers, there are no other specific requirements than asymptotic stability. For the stability, it is required that the closed - loop systems

  $$x(k+1) = (A(\alpha(k)) + BK_j(\alpha(k)))x(k)$$

  are asymptotic stable.

E. Properties of the solution

Consider the case, when the matrices P_i for all $i = 1, 2, \ldots, s$ are chosen in a block diagonal form

$$P_i = \begin{bmatrix} P_{i1} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & P_{i2} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & P_{ir} \end{bmatrix} \tag{30}$$

In this case, the cost function (28) can be written as

$$J = x^T\left(\sum_{i=1}^{s}\alpha_i P_{i1}\right)x + \sum_{j=2}^{r} x_j^T\left(\sum_{i=1}^{s}\alpha_i P_{ij}\right)x_j$$

The constraints of the optimization problem (28) is rewritten as

$$F_{a1}x + F_{a2}x_2 + \ldots + F_{ar}x_r \leq g_a$$

where F_{aj} are matrices of appropriate dimensions such that

$$F_a = [F_{a1}\ F_{a2}\ \ldots\ F_{ar}]$$

Define the set Ω_s as follows

$$\Omega_s = \{x \in \mathbb{R}^n : F_{a1}x \leq g_a\}$$

It is clear that for all $x \in \Omega_s$, the optimization problem (28) with a block diagonal choice of the matrices P_i has a trivial solution, namely

$$x_j = 0, \ \forall j = 2, 3, \dots, r$$

and thus $x_1 = x$. In this case, the linear decomposition based controller turns out to be the optimal unconstrained controller $u(k) = K_1(\alpha(k))x(k)$.

IV. Example

This example is taken from [3]. Consider the following LPV system

$$x(k+1) = A(\alpha(k))x(k) + Bu(k) \qquad (31)$$

where

$$A(\alpha(k)) = \alpha(k)A_1 + (1-\alpha(k))A_2$$

$$A_1 = \begin{bmatrix} 1 & 0.1 \\ 0.5 & 1 \end{bmatrix}, \ A_2 = \begin{bmatrix} 1 & 0.1 \\ 2.5 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

At each time instant $\alpha(k) \in [0,\ 1]$ is an uniformly distributed pseudo-random number. Recall that the numerical value of $\alpha(k)$ is available for the control action. The constraints are

$$\begin{cases} -6 \leq x_1 \leq 6, \\ -20 \leq x_2 \leq 20, \\ -1 \leq u \leq 1 \end{cases}$$

Two gain scheduled controllers are chosen as

$$\begin{cases} K_1 = \alpha(k)K_{11} + (1-\alpha(k))K_{12} \\ K_2 = \alpha(k)K_{21} + (1-\alpha(k))K_{22} \end{cases}$$

where

$$\begin{cases} K_{11} = [-1.1405 \quad -0.3809], \\ K_{12} = [-1.7202 \quad -0.3886] \end{cases}$$

and

$$\begin{cases} K_{21} = [-1.0447 \quad -0.1944], \\ K_{22} = [-1.2192 \quad -0.1947] \end{cases}$$

With the state and control weighting matrices

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad R = 0.01$$

by solving the LMI problem (27) with a block diagonal matrix P, one obtains

$$P = \alpha(k)P_1 + (1-\alpha(k))P_2$$

with

$$P_1 = \begin{bmatrix} P_{11} & \mathbf{0} \\ \mathbf{0} & P_{12} \end{bmatrix}, \quad P_2 = \begin{bmatrix} P_{21} & \mathbf{0} \\ \mathbf{0} & P_{22} \end{bmatrix},$$

where

$$P_{11} = \begin{bmatrix} 1.5908 & 0.4298 \\ 0.4298 & 0.5016 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 2.8948 & 0.7195 \\ 0.7195 & 0.7339 \end{bmatrix}$$

and

$$P_{21} = \begin{bmatrix} 4.1641 & 1.1585 \\ 1.1585 & 0.6675 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 5.1924 & 1.5430 \\ 1.5430 & 0.7797 \end{bmatrix}$$

Figure 1 shows the controlled invariant set Ψ for the linear decomposition based controller. This figure also shows the maximal invariant set Ω_1 and Ω_2, associated with the gain scheduled controller $u(k) = K_1(\alpha(k))x(k)$ and $u(k) = K_2(\alpha(k))x(k)$, respectively. From Figure 1, it is clear that the set Ψ is larger than the convex hull of the sets Ω_1 and Ω_2, which is the feasible region for the approach in [12].

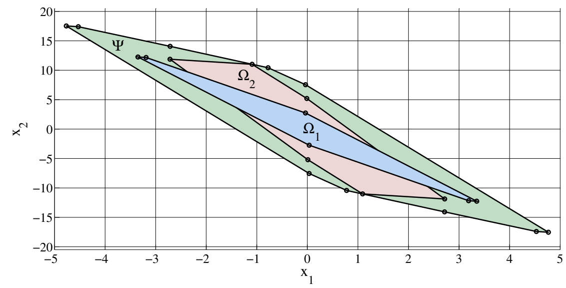

Fig. 1. Feasible sets.

The state trajectories for different initial conditions and different realizations of $\alpha(k)$ are depicted in Figure 2.

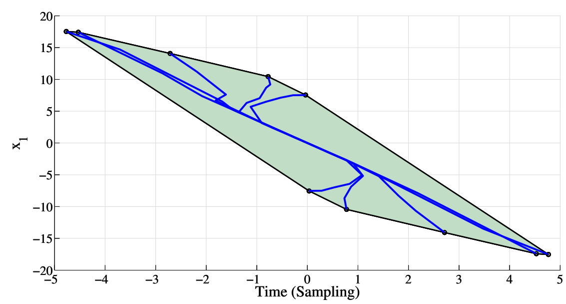

Fig. 2. State trajectories.

For the initial condition $x(0) = [-0.0358 \quad 7.5283]^T$, Figure 3 and Figure 4 present the state and input trajectories of the closed loop system as a function of time. The solid blue line is obtained by using the linear decomposition based control method. For a comparison, Figure 3 and Figure 4 also present the state and input trajectories of the closed loop system obtained by using the method in [3] with the same state and control weighting matrices Q and R. It must be noted that the method in [3] requires a solution of a semidefinte problem.

The objective function J as a Lyapunov function and the realization of $\alpha(k)$ are presented in Figure 5 and Figure 6.

V. Conclusion

In this paper, a novel linear decomposition based control strategy is proposed for linear time-varying systems. The control strategy uses several predefined gain-scheduled controllers. Among them, one controller is chosen as a priority controller and plays the role of a performance candidate, while the remaining are used for extending the domain of attraction. A parameter dependent Lyapunov function

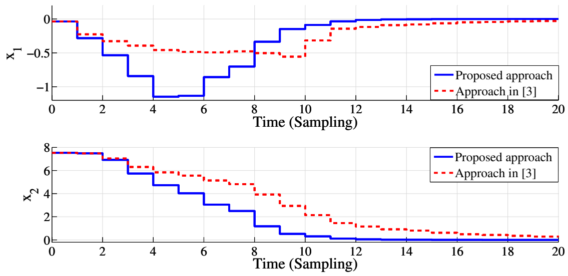

Fig. 3. State trajectories as a function of time. The solid blue line is obtained by using the linear decomposition based control method, and the dashed red line is obtained by using the method in [3].

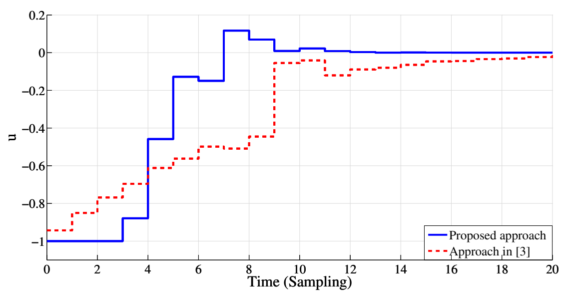

Fig. 4. Input trajectories as a function of time. The solid blue line is obtained by using the linear decomposition based control method, and the dashed red line is obtained by using the method in [3].

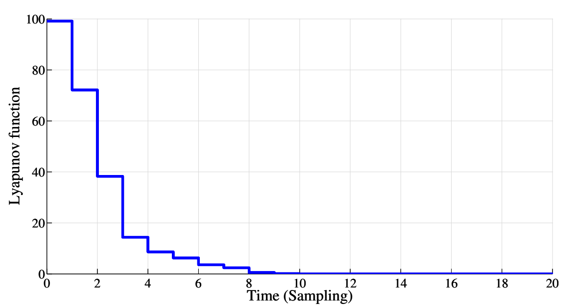

Fig. 5. The objective function J as a function of time.

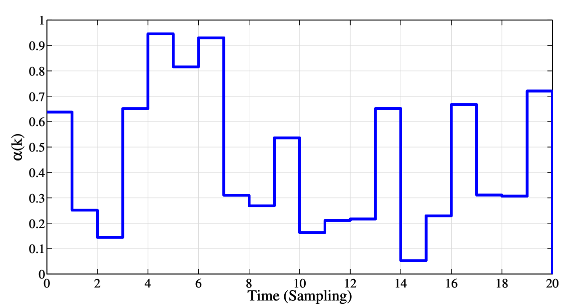

Fig. 6. The realization of $\alpha(k)$ as a function of time.

technique is exploited for reducing conservativeness of the proposed approach.

At each time instant, only one quadratic programming problem is solved on-line. Proofs of recursive feasibility and asymptotic stability are given. A numerical experiment has shown the benefits of the proposed approach.

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