2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Modeling improvements for leak detection in pipelines of LPG

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***Abstract*— The fluid behavior in a pipeline of Liquefied Petroleum Gas (LPG)—according to the principles of conservation of mass and momentum—is governed by a set of Partial Differential Equations (PDE). In this work, such equations are modified with the inclusion of an unsteady friction model and approximated using the Finite Difference Method. As a result of this approximation, a model is obtained and subsequently used for the design of a high gain observer to locate leaks in a pipeline. Some results based on real data are exposed to demonstrate the repercussions of the proposed modifications in tasks of leak detection.**

I. Introduction

Risks in pipelines associated with accidental releases of petroleum products are still high. These risks have forced to the scientific community to develop more efficient methods for Leak Detection and Isolation (LDI) in pipelines. The purpose of such LDI methods is to obtain information on the location of leaks in time, avoiding undesirable consequences such as economical losses, damages to the environment and damages to the population, [1]. The two major difficulties to locate leaks are (*i*) the reduced number of sensors along the extended pipelines and (*ii*) the requirement of involved models for complex networks.

The recent paper [2] presents a pretty well-documented overview on LDI techniques, distinguishing *direct transient* methods, *inverse transient* approaches, and *frequency-based* analysis. The first ones are conceived to give an interpretation of signal changes so as to detect leak effects in them (see e.g. [3], [4], [5], [6]); the second type of these approaches refers to the problem of recovering parameters of a time-domain model from a set of actual measurements, allowing then to compare current measurements with a leak-free situation and monitor leaks in this way (e.g. as in [7], [8], [9], [10], [11], [12], [13]); while the third family of techniques gathers all works related to frequency response analysis with regard to the effect of leaks (cf. [14], [15], [16], [17], [18]).

Most of the LDI algorithms are based on the assumption of a constant friction coefficient, neglecting that this coefficient depends on the dynamics of the flow. As consequence, the algorithms with this assumption are only valid in a particular operation point and any change in the operation conditions can produce errors in the diagnostic. For this reason, several unsteady friction models have been investigated, allowing to calculate this parameter in function of the flow. For instance in [19], a friction equation is formulated, which relates the wall shear stress in transient laminar pipe flow to the instantaneous mean velocity and to the weighted past velocity changes.

In [20], two one-dimensional unsteady friction models are considered, an instantaneous acceleration-based model and a convolution-based model. The first one is shown to fail for certain transient event types and the convolution-based model is successful for all transient event types. In [21], a study of the effects of unsteadiness on flow characteristics is performed, showing that in fast transients, both local inertia and friction forces must be evaluated in a specific manner. Since these two effects are correlated and seem to depend on the same quantities. A single expression in the form of an addition to the steady friction term is proposed for both. In [22], a pipeline model that incorporates the Brunone's unsteady friction model is proposed, but in contrast to the standard treatment of the unsteady friction term as a source term, the authors propose a nonconservative formulation.

Considering these studies, this article intends to show how some improvements in the friction modeling can benefit to LDI tasks. Then, a high gain observer is designed to locate single leaks in LPG pipelines. The observer is based on a model that includes the unsteady friction term presented in [21] and a flow-dependent friction coefficient. To compute the friction coefficient, the *Swamee-Jain* equation—an approximation of the implicit *Colebrook* equation– is used. The modifications of the model have been effectuated in order to improve the LDI observer presented in [23]. The idea of improving the observer emerged from the analysis carried out in [24] that show the critical effects of friction in LDI tasks.

Section II presents the PDE equations–with some improvements in the computation of the friction–describing the dynamic behavior of a pipeline of LPG, as well as an approximation of such equations using the Finite Difference Method. Section III exposes the conception of a nonlinear observer based on the approximated pipeline model and considering the input dynamics. Section IV, on the one hand presents a comparison between the response of the improved model and real data, and on the other hand expose some results about the performance of the high gain observer. Finally, Section V provides some conclusions.

II. Pipeline dynamics

Under some assumptions presented in [25], [26], the fluid behavior in the j-th section ($\forall j = 1, ..., N$) of a closed pipeline is governed by the following momentum and continuity equations:

$$\frac{\partial H(z_j,t)}{\partial t} + \frac{b^2}{gA_r}\frac{\partial Q(z_j,t)}{\partial z_j} = 0 \qquad (1)$$

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$$\frac{1}{A_r}\frac{\partial Q(z_j,t)}{\partial t}+g\frac{\partial H(z_j,t)}{\partial z_j}+J(Q(z_j,t))+g\sin\alpha_j=0 \quad (2)$$

where $(z_j,t)\in(O_j,l_{j-1})\times(0,\infty)$ are the time $[s]$ and space $[m]$ coordinates respectively, with O_j as the origin of each j-th section and l_j as its length $[m]$. Hence, a whole pipeline is composed by different j-sections with spatial subdomains z_j forming the spatial domain $z\in(0,L)$, i.e, $z=\bigcup_j z_j$, where $L=\sum_{j=1}^{N} l_j$ is the length of the entire pipeline and N the total number of j-sections. See Fig. 1.

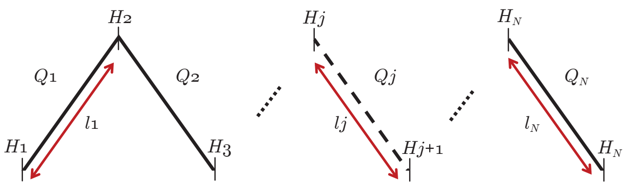

Fig. 1. Pipeline with sections $j=1,\dots,N$

$H(z_j,t)$ is the head pressure $[m]$, $Q(z_j,t)$ the flow rate $[m^3/s]$, b the wave speed in the fluid $[m/s]$, g the gravitational acceleration $[m/s^2]$, A_r the cross-sectional area $[m^2]$ assumed to be constant along the entire pipeline, α_j the elevation angle of the j-th section, and $J(Q(z_j,t))$ represents te friction losses affecting the fluid dynamics of the pipe.

Classically, $J(Q(z_j,t))$ is expressed in the *steady* form given by

$$J(Q(z_j,t))=J_S(Q(z_z,t))=\frac{\mathfrak{f}Q(z_j,t)|Q(z_j,t)|}{2DA_r^2} \quad (3)$$

where $\mathfrak{f}$ is the *Darcy-Weisbach* friction coefficient. In most of model-based LDI approaches, this coefficient is considered to be constant, even if it is sometimes updated when a leak is detected, [27], [28]. But this coefficient actually depends on the so-called Reynolds number ($\mathbf{Re}$) and the roughness coefficient of the pipe (e). This dependence is expressed by the *Colebrook* equation, but this is given in an implicit way. Optionally, an approximated explicit formulation of $\mathfrak{f}$ is given by the *Swamee-Jain* equation expressed as

$$\mathfrak{f}=1.325\left\{\ln\left[0.27\left(\frac{e}{\phi}\right)+5.74\left(\frac{1}{\mathbf{Re}}\right)^{0.9}\right]\right\}^{-2} \quad (4)$$

where the Reynolds number can be calculated with:

$$\mathbf{Re}=\frac{\rho V\phi}{\mu}=\frac{4\rho Q(z_j,t)}{\pi\phi\mu} \quad (5)$$

This equation is valid for $10^{-8}<e/\phi<0.01$ and $5000<\mathbf{Re}<10^8$. Where μ is the viscosity of the fluid.

Notice that $\mathfrak{f}$ is calculated from the flow in the j-th section according to Eq. (4), such that it can be defined as $f|_{z_j}=f_j$. However, to keep the possibility of assuming this coefficient to be constant along the pipeline, the variable $\mathfrak{f}$ is kept in the notation.

In order to obtain a more complete friction modeling, some *unsteady* friction losses $J_U(Q(z_j,t))$ can also be added (namely $J(Q(z_j,t))=J_SQ(z_j,t)+J_UQ(z_j,t)$) in Eq. (1)-(2), according to the model below:

$$J_U(Q(z_j,t))=\frac{\mathbf{k}}{2A_r}\left(\frac{\partial Q(z_j,t)}{\partial t}+b\Phi_A\left|\frac{\partial Q(z_j,t)}{\partial z_j}\right|\right) \quad (6)$$

where $\Phi_A=\mathtt{sgn}(Q(z_j,t))$ and $\mathbf{k}$ denotes the *Brunone* coefficient given by $\mathbf{k}=\sqrt{0.0476}/2$ for laminar flow and by

$$\mathbf{k}=\left(\sqrt{\frac{7.41}{\mathbf{Re}^{\log(14.3/\mathbf{Re}^{0.05})}}}\right)/2$$

for turbulent flow.

A closed-form solution of Eq. (1)-(2) is not available. In the present paper, the Finite Difference Method is chosen because of its simplicity and because the structure of the resulting models—roughly triangular—is considered to be suitable for the conception of high gain observers.

In order to discretize in space Eq. (1)-(2), initial conditions are required expressing the profiles of $Q(z_j,t)$ and $H(z_j,t)$ at the initial instant $t=0$, in this work they are defined as follows:

$$Q(z_j,0):=Q_j^0(z_j),H(z_j,0):=H_j^0(z_j)$$

Moreover, temporal profiles of $Q(z_j,t)$ and $H(z_j,t)$ at the boundaries of the spatial subdomains of each section j are required, i.e., at $z_j=O_j$ and $z_j=l_j$ respectively. These boundary conditions can be functions denoted as follows:

$$H(z_j=O_j,t):=H_j^{in};\ H(z_j=l_j,t):=H_j^{out} \quad (7)$$

$$Q(z_j=O_j,t):=Q_j^{in};\ Q(z_j=l_j,t):=Q_j^{out} \quad (8)$$

Additionally, if the set of N pairs of equations (1)-(2) for a whole pipeline has to be solved, boundary conditions are also required, then let denote

$$H(z=0,t):=H^{in};\ H(z=L,t):=H^{out} \quad (9)$$

$$Q(z=0,t):=Q^{in};\ Q(z=L,t):=Q^{out} \quad (10)$$

Taking as boundary conditions, H_j^{in} and H_j^{out}, the finite-dimensional model for the j-th section of a pipeline is given by the following equations:

$$\dot{Q}_i=\beta\left[a_1\frac{H_i-H_{i+1}}{\Delta z_{ji}}+\mu_0Q_i|Q_i|+\mu_j+\bar{\mu}\left|\frac{Q_i}{\Delta z_{ji}-Q_{ji+1}}\right|\right] \quad (11)$$

$$\dot{H}_{i+1}=a_2\left(\frac{Q_i-Q_{i+1}}{\Delta z_{ji}}\right);\quad \forall i=1,\dots,n_s \quad (12)$$

where i is the index of the spatial discretization and n_s is the total number of spatial discretization segments of the j-th section.

The physical parameters associated to model (11)-(12) are condensed in

$$\beta=\frac{2}{2+\mathbf{k}},a_1=gA_r;a_2=\frac{b^2}{gA_r};\mu_0=-\frac{\mathfrak{f}}{2\phi A_r}$$

and

$$\mu_j=-gA_r\sin\alpha_j,\bar{\mu}=\frac{b\Phi_A\mathbf{k}}{2}$$

while the boundary are set as follows:

$$H_1=H_j^{in},H_{n_s}=H_j^{out}$$

Note that in order to simplify the notation, the time is omitted as an argument of the functions Q_i and H_i.

If a single leak occurs in the j-th section, this section will have a new boundary condition Q_f corresponding to the flow rate of the leak at the point $z_j = z_f$, such that the section will be partitioned into two segments with a boundary condition in between (see Fig. 2). As a consequence, the pressure in the leak point is given by

$$\dot{H}_f = a_2 \left(\frac{Q_i - Q_{i+1} - Q_f}{\Delta z_{ji}} \right) \tag{13}$$

III. Observer design

In [23], an algorithm based on structured residuals has been presented to identify a section of the pipeline with a leak, as well as a nonlinear observer to locate the leak position accurately—once the region was identified.

Here, a high gain observer based on the model (11)-(13) is proposed considering (*i*) a friction coefficient as a function of the flow, and (*ii*) an unsteady friction term. The proposed observer is conceived to be employed after the identification of a section j with a leak, using for instance the approach proposed in [23].

In order to design the observer, let consider system (11)-(12) with $n_s = 2$ and the Eq. (13). The choice of n_s is due to the requirement of representing a flow rate before the leak point and a flow rate after it, i.e., a model with at least two discretization segments is needed. Hence, the minimal model to represent a j-th section with a leak is expressed as follows:

$$\begin{aligned}
\dot{Q}_1 &= \beta \left[a_1 \frac{H_1 - H_f}{\Delta z_{j1}} + \mu_0 Q_1 |Q_1| + \mu_j + \bar{\mu} \left| \frac{Q_1 - Q_2}{\Delta z_1} \right| \right] \\
\dot{H}_f &= a_2 \left(\frac{Q_1 - Q_2 - Q_f}{\Delta z_{j1}} \right) \\
\dot{Q}_2 &= \beta \left[a_1 \frac{H_f - H_3}{l_j - \Delta z_{j1}} + \mu_0 Q_2 |Q_2| + \mu_j + \bar{\mu} \left| \frac{Q_2}{l_j - \Delta z_{j1}} \right| \right] \\
\dot{\Delta} z_{j1} &= 0 \\
\dot{Q}_f &= 0
\end{aligned} \tag{14}$$

where the adjoint states Δz_{j1} and Q_f represent the position of the leak z_f and its respective flow rate, both to be estimated. Notice that $\Delta z_{j2} = l_j - \Delta z_{j1}$, where l_j is the length of the j-th section with a leak. See Fig. 2.

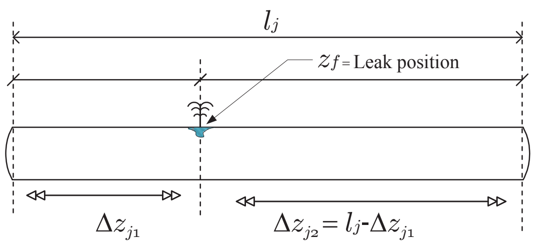

Fig. 2. Segmentation of the j-th section as a consequence of a leak

Then, the state vector of system (14) is:

$$x(t) = [\ Q_1 \quad H_f \quad Q_2 \quad \Delta z_{j1} \quad Q_f\ ]^T \tag{15}$$

where x is composed by the elements x_i with $i = 1, ..., 5$.

The flow rates provided by the system (14) are used here as their outputs, i.e., $y(t) = [Q_1\ Q_2]^T$. Such that, from these outputs and the flow measurements at the extremes of the whole pipeline $y_m(t) = [Q_{in}\ Q_{out}]^T$, the observation error can be calculated:

$$e(t) = [Q_{in}\ Q_{out}]^T - [Q_1\ Q_2]^T \tag{16}$$

Additionally, the boundary conditions H_1 and H_3—pressure measurements corresponding to the j-th section with a leak—are considered as inputs, i.e.:

$$u(t) = [H_1\ H_3]^T = [H_j^{in}\ H_j^{out}]^T \tag{17}$$

Hence, the whole model takes the form of a nonlinear state-space representation as follows:

$$\begin{aligned}
\dot{x}(t) &= f(x(t)) + g_1(x(t))u(t) + g_2(y(t)) \\
y(t) &= [h_1(x), h_2(x)]^T = h(x(t))
\end{aligned} \tag{18}$$

for functions f, g_1, g_2, h_1, h_2, h resulting from (14). The vector $g_2(y(t))$ gathers the steady friction terms.

With the following notations:

$$\Phi : x \mapsto \begin{cases} \xi^1 = [h_2, \mathcal{L}_{f+g_1} h_2]^T \\ \xi^2 = [h_1, \mathcal{L}_{f+g_1} h_1, \mathcal{L}^2_{f+g_1} h_1]^T \end{cases} \tag{19}$$

where $\mathcal{L}$ denotes de Lie derivative, and ξ^1, ξ^2 are composed by the elements ξ_{1i} and ξ_{2j} respectively, with $i = 1, 2$ and $j = 1, 2, 3$, the system in ξ coordinates becomes:

$$\begin{aligned}
\dot{\xi}_{11} &= \xi_{12} & \dot{\xi}_{21} &= \xi_{22} \\
\dot{\xi}_{12} &= \varphi_{12}(\xi_1, \xi_2, u, \dot{u}) & \dot{\xi}_{22} &= \xi_{23} + \varphi_{22}(\xi_1, \xi_2, u, \dot{u}) \\
y_1 &= \xi_{11} & \dot{\xi}_{23} &= \varphi_{23}(\xi, u) \\
& & y_2 &= \xi_{21}
\end{aligned} \tag{20}$$

This system is under the form of a *uniformly observable* system, for which some high gain observer can be designed, [29].

By considering two subsystems of the form:

$$\dot{\xi}^i = A_i \xi^i + \varphi_i(\xi, u, \dot{u}),\ y_i = C_i \xi^i,$$

for $i = 1, 2$ and appropriate A_i, C_i, φ_i resulting from (20), the observer design can for instance be done on the basis of two separate single-output designs as follows:

$$\dot{\hat{\xi}}^i = A_i \hat{\xi}^i + \varphi_i(\hat{\xi}, u, \dot{u}) - S_i^{-1} C_i^T (C_i \hat{\xi}^i - y_i),\ i = 1, 2 \tag{21}$$

where $S_i = [S_i(l, k)]_{1 \leq l, k \leq n_i}$ is classically calculated as follows:

$$S_i(l, k) = \frac{(-1)^{l+k} C_{l+k-2}^{k-1}}{\theta_i^{l+k-1}} \tag{22}$$

where the binomial coefficient is defined: $C_n^k = \frac{n!}{(n-k)!k!}$ and $\theta_1, \theta_2 > 0$.

Notice that the principal difference between the observer proposed here and the observer presented in [23] is the assumption of constant inputs in the last one. Thus, to consider the dynamic of the inputs, observer (21) has been implemented with the new coordinates, while the observer of [23] has been implemented with the original states.

IV. EXPERIMENTAL RESULTS

In this section, some results related to the suggested observer are presented. Real data–provided by sensors installed in a real gas pipeline–were used to validate its performance experimentally.

The whole pipeline—with length $L = 60003$ $[m]$—has gas stations at: $z_A = 0$ $[m]$, $z_B = 10326$ $[m]$, $z_C = 28886$ $[m]$, $z_D = 43866$ $[m]$, $z_E = 47412$ $[m]$ and $z_F = 60003$ $[m]$. The elevation data corresponding to each of these coordinates are $\mathfrak{z}_A = 2247$ $[m]$, $\mathfrak{z}_B = 2245$ $[m]$, $\mathfrak{z}_C = 2252$ $[m]$, $\mathfrak{z}_D = 2334$ $[m]$, $\mathfrak{z}_E = 2500$ $[m]$ and $\mathfrak{z}_F = 2208$ $[m]$. The pipeline is schematized in Fig. 3, whereas the corresponding physical parameters are given in Table I:

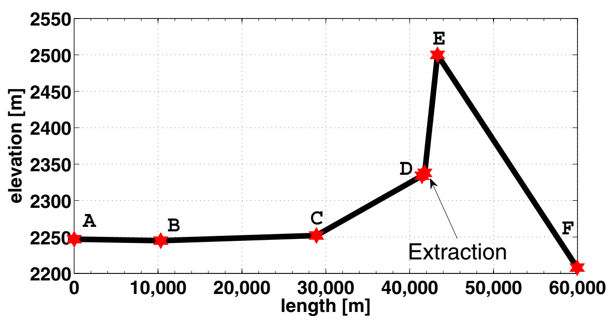

Fig. 3. LPG pipeline under study

TABLE I
PHYSICAL PARAMETERS

Parameter	Value	Units
A_r	0.1939	$[m^2]$
ρ	530	$[kg/m^3]$
b	830	$[m/s^2]$
$\mathfrak{f}$	0.01255	-
k	1.0909	-
e	$14.8e-3$	$[mm]$
μ	$1e-6$	$[N.s/m2]$

The constant friction coefficient given in Table I has been calculated using the Darcy-Weisbach equation [26] to be used by the observer presented in [23].

The pipeline is instrumented with pressure sensors at each station (A to F) and flow rate sensors at the ends (A and F). In this pipeline is possible to extract LPG at 390 $[m]$ downstream the station D. Hence, taking advantage of this situation, the known extraction is considered as a leak to validate the proposed observer. The data used for the validation correspond to 8 hours of LPG extraction approximately.

Fig. 4 exposes a comparison between real data and the response of the model given by the Eq. (11)-(12) with $n_s = 5$. The good agreement between them can be corroborated in Fig. 5, where percentage errors are plotted.

Now, to validate the proposed observer, it is supposed that the section j with a leak has already been identified using structured residuals as in [23]. This section is obviously

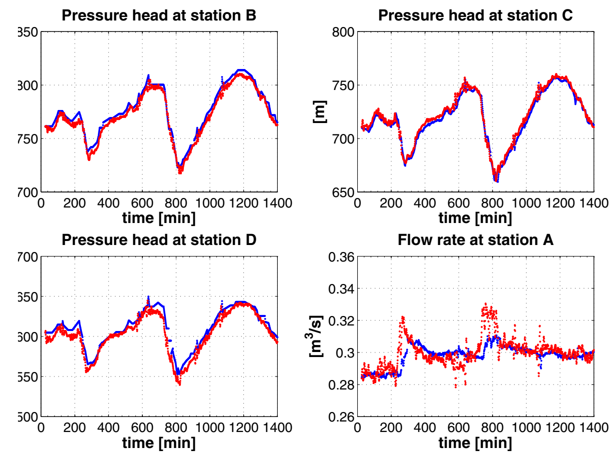

Fig. 4. Comparison between real data and model given by Eq. (12)

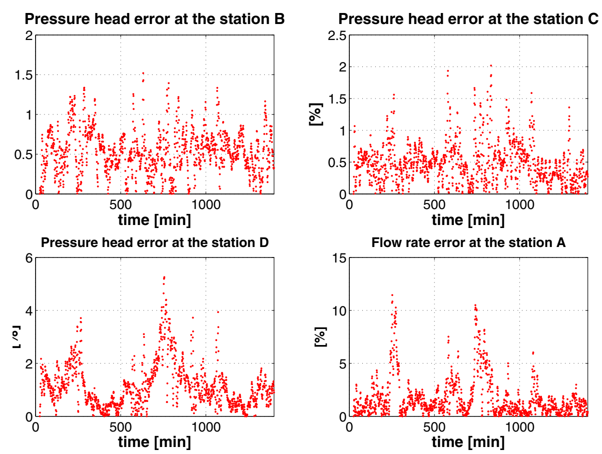

Fig. 5. Modeling error

where the known extraction is located, delimited by the station D and E with a length $l_j = 3546$ $[m]$.

For the observer implementation, the inputs were the pressure measurements at the ends of the section with the leak, as indicated in Eq. (17), i.e. the pressure heads at the stations D and E. Whereas, the observation error was calculated using Eq. (16).

Figure 6 exhibits the position estimation performed by the observer (14), where is noticeable that the estimation is very close to the real value. This figure shows too the estimation realized by the observer proposed in [23]. In Fig. 7, the estimation errors of both observers are plotted.

Both observers do a good estimation until $t = 240$ $[min]$, but after this time, the dynamics of the pressures and flows rates change remarkably (see Fig. 4). Consequently, the estimation performance of both observers is affected. The dynamic changes did not affect so much the good estimation performed by the observer (21), thanks to the inclusion of the unsteady friction model and the friction coefficient calculated from the flow rates. However, the performance of the observer proposed in [23] is not so good with respect to the observer (21), its disadvantage is the use of a constant friction coefficient which does not consider drastic variations in the pipeline dynamics. Since the constant friction coefficient—presented in Table I—is calculated from the pressure heads and flows in an operation point, this is not more valid when there are significant variations in the dynamics.

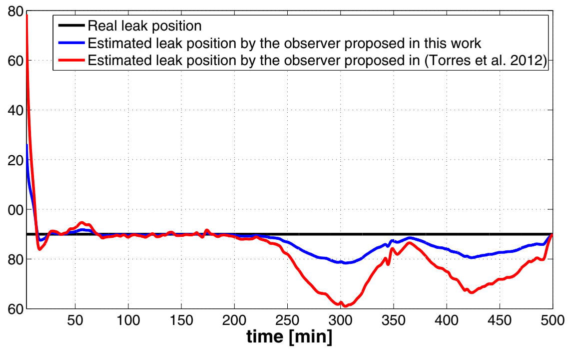

Fig. 6. Position Estimation

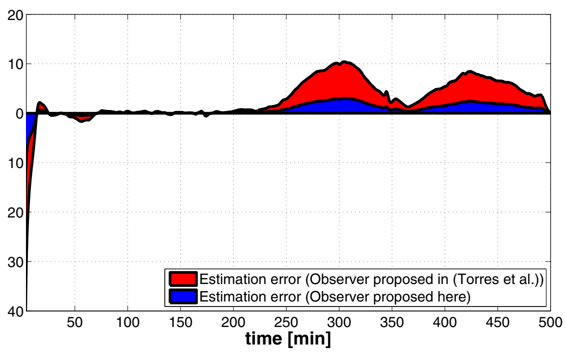

Fig. 7. Error Estimation

V. Conclusion

In this article was presented a nonlinear observer based on an approximated model of the momentum and continuity equations governing the behavior of a pipeline. The principal contribution of this work is the modification of the dynamical model—for leak detection and isolation purposes—with a unsteady friction term and a friction coefficient calculated from the flow. The improved model was used to design a nonlinear observer—tested with real data—showing a good performance with respect to another observer already presented in the literature.

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