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State Estimation of Unknown Input Fuzzy Bilinear Systems: Application to Fault Diagnosis

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Abstract— **This article aims the observer synthesis for a class of nonlinear systems and affected by unknown inputs, represented under the multimodel bilinear formulation. Sufficient conditions to design an unknown input fuzzy bilinear observer are given in Linear Matrix Inequalities (LMIs) terms. The paper studies also the problem of fault detection and isolation. An unknown input fuzzy bilinear fault diagnosis observer design is proposed. Numerical example is given to illustrate the effectiveness of the given results.**

I. INTRODUCTION

Bilinear systems have been of great interest in recent years. This interest arises from the fact that a wide of physical, chemical, biological and nuclear systems can be adequately approximated by a bilinear model [1]. The bilinear system presents the main advantage to be an intermediate structure between linear and nonlinear forms; it preserves the nonlinear character of the system while recalling the simplicity of the linear form.

On the other hand, the approximation of nonlinear systems by bilinear models can be improved using the Takagi-Sugeno fuzzy approach [2], [3], [4], [5]. This technique provides the description of the original nonlinear system by a multimodel bilinear structure based on *if-then* rules. Then a T-S fuzzy bilinear model is obtained by a combination of the local models ponderable by validity coefficients. In many approaches, the transformation of nonlinear systems to bilinear T-S models provides a better approximation than classical T-S models [6], [7]. Motivated by this reasons, we consider in this work bilinear T-S fuzzy models whose consequent parts are bilinear systems with unknown inputs.

Considering the advantages of bilinear systems and fuzzy control, the fuzzy bilinear system (FBS) based on the T-S fuzzy model with bilinear rule consequence has attracted the interest of researchers [2], [3], [5], [6] and [8]. For example, robust stabilization for the T-S FBS has studied in [2] [3] [6], and extension to the T-S FBS with time-delay is given in [8]. An adaptive fuzzy bilinear observer (FBO) based synchronization design for generalized Lorenz system (GLS) was also examined in [9]. In [5] an observer is designed using iterative procedure.

Moreover, the design of unknown input observer plays an essential role in robust model-based fault detection [10], [11], [12] and in robust estimator [13], [14], [15], [16]. The design of unknown input observer has received considerable attention in the case of linear systems [17], bilinear systems [18], or T-S systems [19], [20], [21]. Unfortunately, to the authors knowledge, the design of observer subjected to unknown input has not been treated in the case of T-S fuzzy bilinear model. The aim of this paper is not only to generalize the existing works on the design of unknown input observer to T-S fuzzy bilinear system, but also to apply this new observer in the field of fault diagnosis of multimodel bilinear systems affected by unknown inputs which has not been treated so far.

Thus, in this paper, a novel approach for designing a fuzzy bilinear observer subjected to unknown input of a class of nonlinear system is proposed. This system is modeled as FBS with unknown inputs. This kind of T-S fuzzy model is especially suitable for a nonlinear system with a bilinear term. The stability of FBO is ensured by finding a suitable Lyapunov matrix which is based on the resolution of an LMI problem. The design conditions lead to the resolution of linear constraints easy to solve with existing numerical tools. The given observer is then applied for fault detection.

To the best of our knowledge, the FBO synthesis and fault diagnosis for fuzzy bilinear model subjected to unknown input seem not fully addressed in the past works. Moreover, in contrast with previous works, the proposed design is given in LMI formulation solved simultaneously.

The paper is organized as follows. In section II, the considered structure of the FBS with unknown input is presented. Section III is devoted to the problem of state estimation for the fuzzy bilinear system with unknown input. In section IV, the synthesis of fuzzy bilinear observers with actuator fault is presented. Finally, an illustrative example is provided to show the effectiveness of the proposed approach.

Notations. In the rest of the paper, the following useful notations are used: $\Re$ denotes the set of real numbers, X^T denotes the transpose of the matrix X, $X \succ 0$ denotes symmetric positive definite matrix, X^{-1} denotes the Moore-

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Penrose inverse of X, and X^+ denotes the pseudo inverse of X such that $XX^+X = X$.

II. TAKAGI-SUGENO FUZZY BILINEAR MODEL REPRESENTATION

The T-S fuzzy model is described by *if-then* rules and used to present a fuzzy bilinear system. The i^{th} rule of the FBS for nonlinear systems is represented by the following form:

R^i: if $\xi_1(t)$ is F_{i1} and ... and $\xi_g(t)$ is F_{ig}
then

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + N_i x(t) u(t) + F_i d(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where R^i denotes the i^{th} fuzzy rule $\forall i = \{1, \cdots, r\}$, r is the number of *if-then* rules, $\xi_i(t)$ are the premise variables assumed to be measurable and $F_{ij}(\xi_j(t))$ is the membership degree of $\xi_j(t)$ in the fuzzy set F_{ij}, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re$ is the control input, $d(t) \in \Re^q$ is the unknown inputs vector and $y(t) \in \Re^p$ is the system output. The matrices $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times 1}$, $N_i \in \Re^{n\times n}$, $F_i \in \Re^{n\times q}$, $C \in \Re^{p\times n}$ are known matrices.

Then, the overall FBS can be described as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \left(A_i x(t) + B_i u(t) + N_i x(t) u(t) + F_i d(t)\right) \\ y(t) = Cx(t) \end{cases} \quad (2)$$

where $h_i(\xi(t)) = \frac{\mu_i(\xi(t))}{\sum_{j=1}^{r} \mu_j(\xi(t))}$, $\mu_i(\xi(t)) = \prod_{j=1}^{g} F_{ij}(\xi(t))$ and $h_i(.)$ verify the following properties

$$\begin{cases} \sum_{i=1}^{r} h_i(\xi(t)) = 1 \\ 0 \le h_i(\xi(t)) \le 1 \end{cases} \quad \forall i \in \{1, 2, \cdots, r\} \quad (3)$$

The activation functions $h_i(.)$ depend on the decision vector $\xi(t)$ assumed to depend on measurable variables.

Remark 1 : Matrices A_i, B_i , N_i, F_i and C can be obtained by using the bilinearization around some operating points and using adequate weighting functions. This technique is given by [2] where it is proved that often nonlinear behaviors can be approximated by T-S bilinear multimodel description.

III. DESIGN OF A FUZZY BILINEAR OBSERVER WITH UNKNOWN INPUT

The proposed fuzzy bilinear observer is a full-order observer, which can be described as follows.
R^i: if $\xi_1(t)$ is F_{i1} and ... and $\xi_g(t)$ is F_{ig}
then

$$\begin{cases} \dot{z}(t) = H_i z(t) + L_i y(t) + J_i u(t) + M_i y(t) u(t) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \quad (4)$$

The overall FBO can be represented by:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \left(H_i z(t) + L_i y(t) + J_i u(t) + M_i y(t) u(t)\right) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \quad (5)$$

where $\hat{x}(t) \in \Re^n$ is the estimated state vector and the activation functions are the same as those used in FBS (2). H_i, M_i, L_i, J_i and E are constant matrices with appropriate dimensions.

Then let us define the state estimation error:

$$e(t) = \hat{x}(t) - x(t) = z(t) - Tx(t) \quad (6)$$

where $T = I_n + EC$.

The dynamics of the state estimation error is governed by:

$$\dot{e} = \sum_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} H_i e + (H_i T + L_i C - TA_i) x + \\ (M_i C - TN_i) xu + (J_i - TB_i) u - TF_i d \end{pmatrix} \quad (7)$$

Hence, if the following constraints are satisfied

$$H_i T + L_i C - TA_i = 0 \quad (8)$$

$$M_i C - TN_i = 0 \quad (9)$$

$$J_i - TB_i = 0 \quad (10)$$

$$TF_i = 0 \quad (11)$$

$$T = I_n + EC \quad (12)$$

the equation of the observing error becomes

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\xi(t)) H_i e(t) \quad (13)$$

The problem of designing the fuzzy bilinear observer for the fuzzy bilinear system with unknown inputs is reduced to find the parameter gains H_i, M_i, L_i, J_i, E such that the state estimation error dynamic equation (13) is stable. The following theorem gives sufficient design conditions for the unknown inputs FBS (2).

Theorem 1: If there exist a symmetric definite positive matrix P, and matrices W_i, V_i, S, R_i such that the following linear conditions hold $\forall i = 1 \cdots r$

$$((P+SC)A_i - W_i C)^T + ((P+SC)A_i - W_i C) \prec 0 \quad (14)$$

$$R_i = (P+SC) B_i \quad (15)$$

$$V_i C = (P+SC) N_i \quad (16)$$

$$(P+SC) F_i = 0 \quad (17)$$

then the state estimation of the FBO (5) converges globally and asymptotically to the state of the FBS (2). The observer gains are determined by:

$$E = P^{-1} S \quad (18)$$

$$J_i = P^{-1} R_i \quad (19)$$

$$M_i = P^{-1}V_i \tag{20}$$

$$H_i = (I_n + EC)A_i - P^{-1}W_iC \tag{21}$$

$$L_i = P^{-1}W_i - H_iE \tag{22}$$

Proof: In order to establish the stability of the estimation error $e(t)$, let us consider the following Lyapunov function

$$V(t) = e^T(t)Pe(t), P = P^T > 0 \tag{23}$$

Using (13), the derivative of the Lyapunov function (23) is given by

$$\dot{V}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \left(e^T(t)\left(H_i^T P + PH_i\right)e(t)\right) \tag{24}$$

From (8) and using (12), we get

$$H_i = TA_i - K_iC \tag{25}$$

with

$$K_i = H_iE + L_i \tag{26}$$

Then, the derivative of the Lyapunov function is negative if

$$(TA_i - K_iC)^T P + P(TA_i - K_iC) < 0 \tag{27}$$

Taking into account (12) and considering the variables change:

$$S = PE \tag{28}$$

$$W_i = PK_i \tag{29}$$

we get the LMI (14). Taking into account (12) and (28), equality (17) is derived from (11).

Similarly, using the following variable change

$$R_i = PJ_i \tag{30}$$

$$V_i = PM_i \tag{31}$$

we get equalities (15) and (16) from (10) and (9) respectively. Which ends the proof.

■

Remark 2. Classical numerical tools may be used for solving the LMI problem (14) subject to linear equality constraints (15)-(17). Solving this linear problem allows to deduce the observer parameters from P, W_i, V_i, S, and R_i as mentioned by (18)-(22).

In the following, the proposed observer is used for fault detection, and isolation of actuator fault.

IV. DESIGN OF AN UNKNOWN INPUT FAULT DIAGNOSIS OBSERVER

In this section, an unknown input fuzzy bilinear fault diagnosis observer is synthesized using T-S modeling. Indeed, a more general situation is analyzed since both unknown input and faults are envisaged. Thus, we consider a fuzzy bilinear system affected by an actuator fault vector $f(t) \in \Re^{nf}$. A residual generator is then synthesized such that it is sensitive to fault vector $f(t)$ and insensitive to the unknown inputs $d(t)$.

The considered FBS affected by a fault vector $f(t)$ is descibed by the following equation:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} A_ix(t) + B_iu(t) + N_ix(t)u(t) \\ +F_id(t) + G_if(t) \end{pmatrix} \\ y(t) = Cx(t) \end{cases} \tag{32}$$

where G_i is a matrix with appropriate dimensions.

The proposed unknown input fault diagnosis fuzzy bilinear observer for the system (32) has the following equations:

$$\begin{cases} \dot{z}(t) = \sum\limits_{i=1}^{r} h_i(\xi(t)) \left(H_iz(t) + L_iy(t) + J_iu(t) + M_iy(t)u(t)\right) \\ \hat{x}(t) = z(t) - Ey(t) \\ r(t) = E_1z(t) + E_2y(t) \end{cases} \tag{33}$$

where $z(t)$ represents the estimated vector, and $r(t)$ being the output signal called the residual.

In order to study the dynamic of the unknown input fuzzy bilinear fault diagnosis observer (33), we consider the estimating error:

$$e(t) = \hat{x}(t) - x(t) = z(t) - \bar{T}x(t)$$

where $\bar{T} = I_n + EC$, and the gain matrices $H_i, L_i, J_i, M_i, E, E_1, E_2$ in (33) will be determined such that to ensure the convergence of the errors $e(t)$ into zero. From equations (32) and (33), one has:

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} H_ie(t) + (H_i\bar{T} + L_iC - \bar{T}A_i)x(t) \\ +(M_iC - \bar{T}N_i)x(t)u(t) - \bar{T}F_id(t) \\ +(J_i - \bar{T}B_i)u(t) - \bar{T}G_if(t) \end{pmatrix} \tag{34}$$

and

$$r(t) = E_1e(t) + (E_1\bar{T} + E_2C)x(t) \tag{35}$$

If the following conditions are satisfied:

$$H_i\bar{T} + L_iC - \bar{T}A_i = 0 \tag{36}$$

$$M_iC - \bar{T}N_i = 0 \tag{37}$$

$$J_i - \bar{T}B_i = 0 \tag{38}$$

$$\bar{T}F_i = 0 \tag{39}$$

and

$$E_1\bar{T} + E_2C = 0 \tag{40}$$

one gets:

$$\dot{e}(t)=\sum_{i=1}^{r}h_i(\xi(t))(H_ie(t)-\bar{T}G_if(t)) \quad (41)$$

$$r(t)=E_1e(t) \quad (42)$$

Multiplying (40) by F_i we get:

$$E_1\bar{T}F_i+E_2CF_i=0 \quad (43)$$

Taking into account the constraint (39), equation (43) yields:

$$E_2CF_i=0, i=1,...,r \quad (44)$$

or equivalently

$$E_2CF=0, F=[F_1,F_2,\cdots,F_r] \quad (45)$$

If the condition $rank(CF)=rank(F)$ is satisfied, then we get

$$E_2=\Omega\left(I_p-CF(CF)^{+}\right) \quad (46)$$

where $(CF)^{+}$ is the pseudo inverse of CF, and Ω is an arbitrary matrix.

Substituting equation (46) in (40) leads to

$$E_1T+\Omega C\left(I_n-F(CF)^{+}C\right)=0 \quad (47)$$

A suitable choice of E_1 and $\bar{T}$ satisfying the relation (47) is

$$E_1=-\Omega C \quad (48)$$

and

$$\bar{T}=I_n-F(CF)^{+}C \quad (49)$$

Then, the following theorem 2 states the existence conditions and the gains determination of the unknown input and fault diagnosis fuzzy bilinear observer.

Theorem 2: If there exist a symmetric definite positive matrix P, and matrices Z_i, V_i, U_i such that the following linear conditions hold $\forall i=1\cdots r$

$$Z_i^T+Z_i\prec 0$$

$$Z_i\bar{T}+U_iC-P\bar{T}A_i=0$$

$$V_iC-P\bar{T}N_i=0$$

then the state estimation of the FBO (33) converges globally asymptotically to the state of the FBS (32). The observer gains are determined by:

$$H_i=P^{-1}Z_i \quad (50)$$

$$L_i=P^{-1}U_i \quad (51)$$

$$M_i=P^{-1}V_i \quad (52)$$

$$J_i=\bar{T}B_i \quad (53)$$

where $\bar{T}$, E_1 and E_2 are given in (49), (48), (46) respectively.

Proof: The proof of this theorem is similar to that of theorem 1 by considering the following change of variables: $Z_i=PH_i$, $U_i=PL_i$, and $V_i=PM_i$.

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V. ILLUSTRATIVE EXAMPLE

The proposed approach is illustrated on an academic T-S system (2) described by the following data:

$$A_1=\begin{bmatrix}-3 & -0.5 & 0\\ 2.5 & -0.5 & 0\\ 0 & 0.5 & -1\end{bmatrix}, A_2=\begin{bmatrix}-2 & 0 & -1\\ 2.5 & -2 & 0\\ 0 & 0 & -1\end{bmatrix}$$

$$B_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, B_2=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$N_1=N_2=\begin{bmatrix}-0.5 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -0.5\end{bmatrix}$$

$$F_1=F_2=\begin{bmatrix}1\\0.5\\2\end{bmatrix}, C=\begin{bmatrix}1 & 1 & 0\\ 0 & 0.1 & 1\end{bmatrix}$$

The weighting functions depending on the first component of the state vector are defined by:

$$h_i\left(\xi\left(t\right)\right)=\frac{\mu_i\left(x_1(t)\right)}{\sum_{j=1}^{2}\mu_j\left(x_1\left(t\right)\right)}, i=1,2$$

where

$$\mu_1(x_1(t))=\exp(1/2(\frac{x_1+5}{2})^2)$$

$$\mu_2(x_1(t))=\exp(1/2(\frac{x_1-5}{2})^2)$$

1) Unknown input FBO design: Solving the LMIs in theorem 1 yields the following matrix:

$$P=10^3*\begin{bmatrix}1.2870 & 0.8661 & -0.2629\\ 0.8661 & 1.2368 & -0.0643\\ -0.2629 & -0.0643 & 2.2066\end{bmatrix}$$

Then the observer gains are given from (18)-(22) by:

$$H_1=\begin{bmatrix}-1.7387 & 1.3290 & -0.2442\\ 1.2500 & -1.8255 & 0.1667\\ -0.1596 & 0.1002 & -0.4943\end{bmatrix},$$

$$H_2=\begin{bmatrix}-2.4079 & 2.0291 & -0.3402\\ 1.9186 & -2.5258 & 0.2668\\ -0.2283 & 0.1684 & -0.5040\end{bmatrix}$$

$$L_1=\begin{bmatrix}-2.5024 & 0.5506\\ 2.5024 & -0.5506\\ -0.2502 & 0.0551\end{bmatrix}, L_2=\begin{bmatrix}-2.5650 & 0.9398\\ 2.5650 & -0.9398\\ -0.2565 & 0.0940\end{bmatrix}$$

$$J_1=\begin{bmatrix}-0.0263\\ 0.0263\\ -0.0026\end{bmatrix}, J_2=\begin{bmatrix}0.2632\\ -0.2632\\ 0.0263\end{bmatrix}$$

$$M_1=M_2=\begin{bmatrix}0.0132 & -0.1316\\ -0.0132 & 0.1316\\ 0.0013 & -0.0132\end{bmatrix}$$

$$E=\begin{bmatrix}-1.0263 & 0.2632\\ 0.0263 & -0.2632\\ -0.0026 & -0.9737\end{bmatrix}$$

The FBS with unknown input was simulated by choosing a systems initial condition $x_0 = \begin{bmatrix} -1 & -1 & -1 \end{bmatrix}^T$ and observers initial condition $\hat{x}_0 = 0$. The input signal is defined as a sine wave signal of amplitude 0.5 and frequency 50 rad/s, and the unknown input is defined as a constant signal of amplitude 0.4 applied for $t \succ 0$.

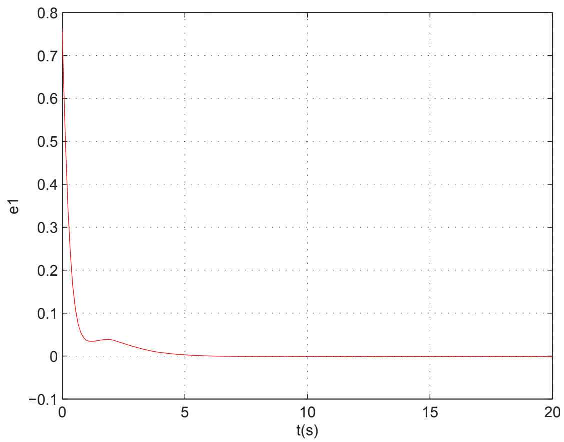

Fig. 1. Trajectories of the estimation error between $x1$ and $\hat{x}1$

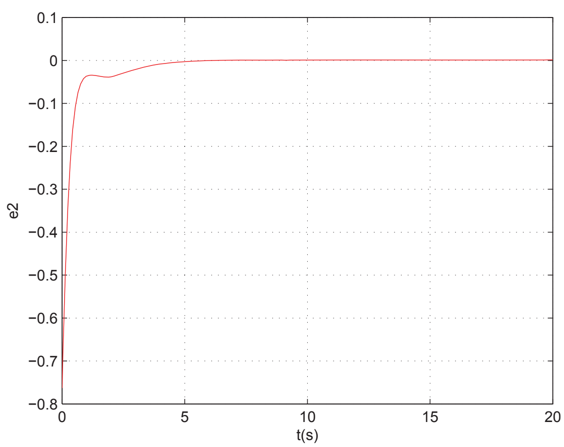

Fig. 2. Trajectories of the estimation error between $x2$ and $\hat{x}2$

Figures (1) (2) and (3) show respectively the evolution of the estimation error between the actual system variables and their corresponding observer ones. It can be seen that the state estimation error converges asymptotically tends to zero in spite of the presence of the unknown input.

2) Unknown input and fault diagnosis observer: We consider now the same previous system subject to actuator fault:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(t) \left(\begin{array}{l} A_i x(t) + B_i u(t) + N_i x(t) u(t) \\ + F_i d(t) + G_i f(t) \end{array} \right)$$
$$y(t) = Cx(t)$$

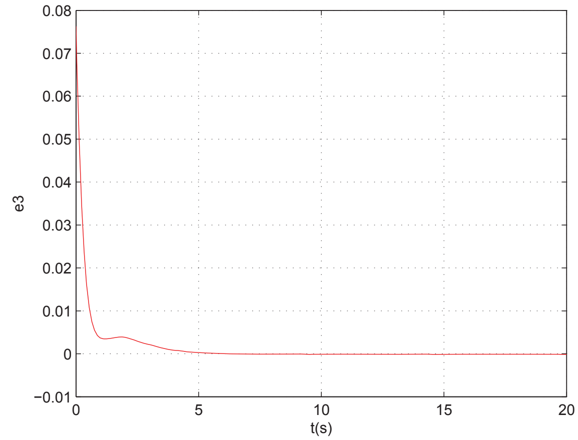

Fig. 3. Trajectories of the estimation error between $x3$ and $\hat{x}3$

with A_i, B_i, N_i, F_i and C are the same previous matrices and

$$G_1 = G_2 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

and

$$f(t) = \begin{cases} 1 & for \quad t \in \begin{bmatrix} 10 & 14 \end{bmatrix} \\ 0 & \qquad\qquad elsewhere \end{cases}$$

We can check that the following condition $rank(CF) = rank(F) = 1$ is satisfied.

We may choose the arbitrary matrix Ω as follows :

$$\Omega = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Then we get the following two gain matrices :

$$E_1 = \begin{bmatrix} -1 & -1.1 & -1 \\ -1 & -1.1 & -1 \\ -1 & -1.1 & -1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0.1747 & -0.1279 \\ 0.1747 & -0.1279 \\ 0.1747 & -0.1279 \end{bmatrix}$$

Figure (4) displays the convergence of the residual corresponding to the actuator fault signal. It shows that the residual $r(t)$ is sensitive to $f(t)$ and insensitive to $d(t)$. Indeed, one can notes that the residual takes the zero value throughout the simulation time except at the interval where the actuator fault occurs $[10 - 14s]$.

VI. CONCLUSION

This paper has addressed the problem of state estimation and diagnosis for dynamic systems that can be described by T-S fuzzy bilinear models subject to unknown input. The state estimation is obtained by an Unknown Inputs Fuzzy Bilinear Observer which has been used here for the

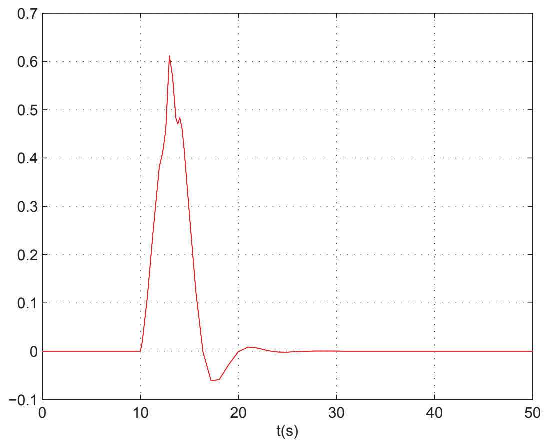

Fig. 4. Residual signal $r(t)$

diagnosis of actuators faults in spite of unknown input. Stability conditions of this unknown input FBO have been formulated and solved within a linear matrix inequality framework. The synthesis conditions lead to the resolution of linear constraints easy to solve with existing numerical tools. The effectiveness of the FBO proposed approach has been illustrated on an example of a dynamical system described by a fuzzy bilinear model. Further developments will concern the case of T-S system with immeasurable premise variables.

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