2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Actuator performance evaluation using LMIs for optimal HVDC placement

Alexander Fuchs (*Member, IEEE*) and Manfred Morari (*Fellow, IEEE*)

Abstract— **This work concerns the closed loop performance evaluation of actuators for constrained linear systems. The main motivation is the optimal placement of High Voltage Direct Current links (HVDC) in a meshed AC power system, but the method is also applicable to other actuator selection problems. The goal is the computation of a performance measure, that can be used to rank different candidate actuators according to their behavior after a disturbance. A measure is proposed, that is not based on simulations, but is computed for general disturbances, using Linear Matrix Inequalities (LMIs). It accounts for global limitations of the system and the actuators. As required for the decision process, it can be chosen to have a physical interpretation. The approach is illustrated by placing HVDC links in a power system with two areas oscillating against each other.**

Index Terms— **LMI, actuator selection, HVDC, power grid planning, transient stability**

I. Introduction

Actuator selection is a critical step for the design of systems since it determines the fundamental properties of the resulting control problem. Choosing and placing an actuator is a multi objective optimization with constraints from different domains, such as physical system limitations, performance specifications and economic requirements. The particular case of power system control involves crucial availability requirements. Therefore, any new actuator device, such as high voltage direct current (HVDC) links, flexible AC transmission devices (FACTS) or power system stabilizers (PSS) have to be carefully evaluated, considering expected disturbance scenarios and system limitations. Problems from power system control involve a large number of dynamic states, at least two to six for every generator modeled in the grid [1]. For the analysis of sufficiently small transients, they can be approximated by linear dynamic equations. It is well known that active power transmission devices such as HVDC links can be used as actuation device to support the power system during transients, for instance by modulating the active and reactive power injections to damp frequency oscillations [2], [3]. They are also subject to constraints that have to be taken into account when designing the control schemes selecting the actuator setpoint. The main topic of this paper is how to assess the performance of these control schemes for different actuator selections and disturbance scenarios.

The traditional approach to analyze the performance of power systems usually involves extensive simulations of different disturbance scenarios [4]. While the simulation model can be very detailed, it is not possible to cover all disturbance situations and critical transients occurring in the system may be missed. Alternatively, small signal analysis of a linearized system model allows to determine the local behavior during small disturbances [1]. This approach works for generic small disturbances but does not evaluate the global behavior where the limitations of the system are met. In linear control theory, the traditional approach to the comparison of different actuators of unconstrained systems involves the computation of the controllability matrix or the controllability gramian and a rank comparison [5]. While the approach can be applied to relatively large systems, it only gives a limited information, e.g. the dimension of the controllable subspace, for systems without nonlinearities and no constraints. For constrained nonlinear systems with disturbances and control inputs from a bounded set, reachability analysis can be used to determine the set of controllable states. Since the set computations involve the solution of partial differential equations [6] or polyhedral algebra [7], the approach is limited to low dimensional systems. Furthermore, it is desired to compute a performance metric, that typically involves the maximization of a scalar function over the controllable set.

In this work, an approach is presented, that is suitable for large linear systems, subject to symmetric constraints on the states, the input and the initial state, restricted to ellipsoidal sets. Using results from [8], the problem can then be reformulated as semidefinite program (SDP), including a scalar value that measures the achievable actuator performance. The approach is tractable and represents an alternative between the simple computation of the controllability matrix and a full reachability analysis.

The paper is structured as follows. Section II summarizes the problem formulation and its background for a power system with HVDC links as actuators. In Section III, the problem is converted to an LMI formulation. Section IV summarizes the LMI based placement approach. It is also shown how to apply the placement algorithm recursively for combinations of multiple actuators. Section V first illustrates the LMI approach using a small example system. Finally, the scheme is applied to place multiple HVDC links in a power system with multiple generators oscillating against one another.

II. Performance evaluation of actuators

This section defines the performance measure used to evaluate different actuators and gives the interpretation for power grids controlled using HVDC links.

A. Fuchs and M. Morari are with the Automatic Control Laboratory, Swiss Federal Institute of Technology (ETH Zürich), Physikstrasse 3, CH - 8092, Zürich, Switzerland. E-mail: fuchs@control.ee.ethz.ch

The authors thank their funding partners swisselectric research and ABB corporate research, Switzerland.

A. Problem statement

The performance of a constrained linear system with a given actuator is the solution of the min-max optimization problem

$$J = \min_{K} \max_{x(0)\in\mathbb{X}_0} \int_0^\infty z(t)^T M z(t) dt \quad (1)$$

$$\text{s.t.} \quad \dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

$$z(t) = Cx(t) \quad (3)$$

$$u(t) = Kx(t) \quad (4)$$

$$\forall x(0) \in \mathbb{X}_0 \ , \forall t : \quad (5)$$

$$0 = H_{\text{eq},x} x(t) + H_{\text{eq},u} u(t) \quad (6)$$

$$u(t) \in \mathbb{U} \quad (7)$$

where

$$\mathbb{X}_0 = \{x \in \mathbb{R}_{n_x} : x^T E_x x \leq 1\} \quad (8)$$

$$\mathbb{U} = \{u \in \mathbb{R}_{n_u} : u^T E_u u \leq 1\} \quad . \quad (9)$$

The objective is the integral performance of the signals of interest, z, weighted with the symmetric positive semidefinite matrix $M = \hat{M}\hat{M}^T \geq 0$. To obtain a tractable problem formulation, the control input cannot be chosen freely as a function of time but is restricted to a linear feedback policy (4). Using the feedback gain K, the goal is to minimize the worst case performance over the set of initial states $\mathbb{X}_0$. The system equations for the given actuator are defined with the matrices A, B, C of appropriate dimensions. As a constraint, the input and the state have to satisfy the equality constraints defined by the matrices $H_{\text{eq},x}$ and $H_{\text{eq},u}$. To leave a degree of freedom for the control input, $H_{\text{eq},u}$ must have less rows than columns. Furthermore, it is required that all possible input trajectories remain in the set $\mathbb{U}$. The ellipsoidal sets are defined using the symmetric positive definite matrices $E_x > 0$ and $E_u > 0$.

B. Power system background

The dynamic equations of the power system model (2) include the generator dynamics as well as the dynamics of local generator controls, linearized around the current operating condition [9]. For the purpose of this paper, the nonlinearities of the true power system dynamics are not modeled and the estimator of the systems dynamic state is assumed to be perfect, so that the state $x(t)$ is available for the feedback controller (4).

The dynamic state $x(t) \in \mathbb{R}_{n_x}$ consists of the states vectors of the n_{gen} generators, including the rotor angle δ and the deviation of the frequencies from the reference value, $\Delta\omega = \omega - \omega_{\text{ref}}$. The vector $z(t)$ contains the signals of interest. In this work, C is chosen so that

$$z(t) = \Delta\omega(t) \in \mathbb{R}_{n_{\text{gen}}} \quad . \quad (10)$$

The matrix

$$M = \text{diag}(H_1, H_2, ..., H_{n_{\text{gen}}}) \quad (11)$$

weighs each of the frequency deviations with the corresponding generator inertia H_i. The manipulated variables $u(t) \in \mathbb{R}_{n_u}$ are the reference values of the actively controlled power system components. In the case of a single HVDC link manipulated to enhance the power system performance,

$$u(t) = [I_{\text{l},\alpha}, I_{\text{r},\alpha}, I_{\text{l},\beta}, I_{\text{r},\beta}]^T \quad , \quad (12)$$

which contains the $\alpha\beta$-phasor representation of the current injections at the left and the right HVDC terminal. The set of constraints (7) originates for instance from limitations of the magnitude of the HVDC links current injections. The set of initial states (8) is formed from bounds on the frequency deviation of each of the generators, $|\Delta\omega| \leq \overline{\omega}$. Bounds on the other system states can be derived from additional security margins on the angular deviation or the terminal voltage level. The set $\mathbb{X}_0$ can be assumed to be bounded, since no state of the power system can deviate infinitely far from the nominal operating point. For example, a generator would be disconnected from the grid if the angular or frequency deviation exceeds a certain security margin. The equality constraints (6) include algebraic constraints of the DC link, i.e. the balance of the active power injections.

Different placements of the HVDC links result in different system matrices (A, B) and a different performance J. The common goal of the grid stabilizing controller K and the selection of the HVDC link's placement is, to minimize the worst case integral performance, as specified in (1).

III. Problem formulation using LMIs

This section presents an LMI formulation for the computation of the performance measure J, defined in (1)-(9). Throughout the section, results for linear matrix inequalities [8] are applied. The same symbol is used to denote scalar inequalities and semidefiniteness constraints. For matrices, $A \geq B$ states that the matrix $A - B$ is positive semidefinite, with no eigenvalues in the open left half of the complex plane.

A. Worst case performance formulation

It is clear that the value of the integral in (1) depends on the initial dynamic state of the power system, $x(0)$. More precisely, if the grid controller is chosen as linear feedback controller (4) and the matrix $A + BK$ is stable, the value of the integral becomes

$$\int_0^\infty z(t)^T M z(t) dt = x(0)^T P x(0) \quad (13)$$

with the symmetric matrix $P > 0$ defined as the solution of the Lyapunov equation

$$P(A + BK) + (A + BK)^T P + C^T M C = 0 \quad . \quad (14)$$

If the choice of the controller K is known, P can be computed and the value of the integral can be calculated for any given initial dynamic state $x(0)$. For the application to the linearized power system, this means that the performance obtained for a specific angular or frequency disturbance can be computed immediately using a simple quadratic function and without any simulation of the system trajectories.

To evaluate the worst possible case that yields the biggest cost (1), it is necessary to compute the value of the integral not only for a specific initial state but instead to determine the largest value that can occur *for all possible initial states* $x(0)$ in the ellipsoid $\mathbb{X}_0$. Since $E_x > 0$ one can compute

$$E_x = \hat{E}\hat{E}^T, \qquad \hat{E} = VD^{1/2} \tag{15}$$

with the columns of V containing the eigenvectors of E_x and $D^{1/2}$ containing the square root of the eigenvalues of E_x on its diagonal. Then the transformation

$$x = \hat{E}^{-T}\tilde{x} \tag{16}$$

converts the ellipsoid $\mathbb{X}_0$ in the x space into a unit ball in the $\tilde{x}$ space and the largest value of the integral can be computed as an eigenvalue problem

$$\begin{aligned}\max_{x\in\mathbb{X}_0} x^T P x &= \max_{\tilde{x}:\tilde{x}^T\tilde{x}\leq 1} \tilde{x}^T\hat{E}^{-1}P\hat{E}^{-T}\tilde{x} \\ &= \lambda_{\max}(\hat{E}^{-1}P\hat{E}^{-T}) \quad .\end{aligned} \tag{17}$$

B. Handling of equality constraints

To incorporate the equality constraints

$$H_{\text{eq},x}x(t) + H_{\text{eq},u}u(t) = 0 \tag{18}$$

into the model, a change of variables

$$u = -\hat{H}_{\text{eq},u}H_{\text{eq},x}x + N_{\text{eq},u}v \tag{19}$$

can be performed using the pseudoinverse and the nullspace of the matrix $H_{\text{eq},u}$,

$$\hat{H}_{\text{eq},u} = \text{pinv}(H_{\text{eq},u}) = H_{\text{eq},u}^T(H_{\text{eq},u}H_{\text{eq},u}^T)^{-1} \tag{20}$$

$$N_{\text{eq},u} = \text{null}(H_{\text{eq},u}) \quad . \tag{21}$$

The columns of $N_{\text{eq},u}$ span the linear space that is mapped to the origin through multiplication with $H_{\text{eq},u}$. Since

$$H_{\text{eq},u}\hat{H}_{\text{eq},u} = I \quad \text{and} \quad H_{\text{eq},u}N_{\text{eq},u} = 0 \quad , \tag{22}$$

the constraint (18) will always be satisfied for inputs of the form (19). Moreover, the system will change to

$$\begin{aligned}\dot{x} &= Ax + Bu & (23)\\ &= (A - B\hat{H}_{\text{eq},u}H_{\text{eq},x})x + BN_{\text{eq},u}v & (24)\\ &= \overline{A}x + \overline{B}v \quad . & (25)\end{aligned}$$

If $H_{\text{eq},u}$ has n_{eq} linearly independent rows, the new input v has $n_v = n_u - n_{\text{eq}}$ elements.

C. Handling of input inequality constraints

The constraints on the control input (7) are stated in ellipsoidal form,

$$\forall x(0) \in \mathbb{X}_0, \forall t : u(t)^T E_u u(t) \leq 1 \quad . \tag{26}$$

where $E_u > 0$. A different interpretation of the matrix P solving (14) is that of an invariant set. Once the dynamic state at time t_1 satisfies the ellipsoidal constraint

$$x(t_1)^T P x(t_1) \leq k \tag{27}$$

for some $k > 0$, it will also satisfy this constraint for all $t > t_1$. Using (17) and (27), one can conclude, that all trajectories $x(t)$ starting in the set $\mathbb{X}_0$ satisfy the inequality

$$x(t)^T P x(t) \leq \lambda_{\max}(\hat{E}^{-1}P\hat{E}^{-T}) \quad . \tag{28}$$

To satisfy (26), the linear feedback $u = Kx$ has to satisfy

$$K^T E_u K \leq \frac{P}{\lambda_{\max}(\hat{E}^{-1}P\hat{E}^{-T})} \tag{29}$$

D. LMI formulation

The LMI formulation to determine the closed loop performance as in (1) incorporates all constraints derived in the previous sections. All equalities have therefore been removed from the problem using the procedure in Section III-B and the matrices $\overline{A}$ and $\overline{B}$ have been computed using (25). The feedback controller of the remaining inputs is denoted by $v = \overline{K}x$. It is useful to formulate the problem in terms of $Q = P^{-1} > 0$. In this case (14) becomes

$$(\overline{A} + \overline{BK})Q + Q(\overline{A} + \overline{BK})^T + QC^TMCQ = 0 \quad . \tag{30}$$

Replacing $M = \hat{M}\hat{M}^T$, relaxing the equality as inequality and using the Schur complement, (30) becomes

$$\begin{pmatrix} (\overline{A}Q + \overline{B}Y) + (\overline{A}Q + \overline{B}Y)^T & QC^T\hat{M} \\ \hat{M}^TCQ & -I \end{pmatrix} \leq 0, \tag{31}$$

where $Y = \overline{K}Q$ makes the constraint linear in the optimization variables Q and Y. To formulate the objective

$$\min_{K} \max_{x(0)\in\mathbb{X}_0} x(0)^T P x(0) \tag{32}$$

using (17), introduce the upper bound

$$\lambda_{\max}(\hat{E}^{-1}P\hat{E}^{-T}) = \lambda_{\max}(\hat{E}^{-1}Q^{-1}\hat{E}^{-T}) \leq \frac{1}{s} \tag{33}$$

with $s \in \mathbb{R}, s > 0$. This bound is equivalent to

$$Q^{-1} \leq \frac{\hat{E}\hat{E}^T}{s} = \frac{E_x}{s} \tag{34}$$

and

$$Q - sE_x^{-1} \geq 0 \quad . \tag{35}$$

Minimizing $\lambda_{\max}$ is achieved by maximizing s in the objective of the semidefinite program.

Finally, (29) can be written as

$$Y^T E_u Y \leq Qs \quad , \tag{36}$$

which is equivalent to

$$\begin{pmatrix} Q & Y^T \\ Y & sE_u^{-1} \end{pmatrix} \geq 0 \tag{37}$$

Hence, the semidefinite program to solve (1)-(7) is

$$\begin{aligned}&\max_{s>0,Q>0,Y} s \\ &\text{s.t.} \quad (31), (35), (37)\end{aligned} \tag{38}$$

The linear control law $u = Kx$ can be extracted as

$$K = -\hat{H}_{\text{eq},u}H_{\text{eq},x} + N_{\text{eq},u}YQ^{-1} \tag{39}$$

with $H_{\text{eq},x}$ specified, $\hat{H}_{\text{eq},u}$ and $N_{\text{eq},u}$ computed as in Section III-B, and Y and Q from the solution of (38). The worst case performance J as in (1) is given by $1/s$.

IV. Optimal placement of HVDC links

A. Placing a single HVDC link

The dynamic performance in a power system during transients can serve as one decision criteria for the planning of newly constructed HVDC links. Other criteria include economic as well as political considerations. In a power network with n_{bus} buses that can be connected to either terminal of the new HVDC link there are

$$n_{\text{cand},1} = \frac{n_{\text{bus}}(n_{\text{bus}}-1)}{2} \tag{40}$$

candidate locations where the new HVDC link could be placed. If there are no further restrictions on the placement, each of these candidate locations can be evaluated regarding the best achievable performance during transients by solving the problem defined in (1)-(7). The solution of the SDP formulation (38) provides the performance measure $J = 1/s$ that can serve as decision criteria to compare different HVDC candidate locations.

Each of the $n_{\text{cand},1}$ candidate locations alters the power flow and the controllable inputs of the power system and corresponds to different system matrices

$$(A_i, B_i) \quad i = 1, ..., n_{\text{cand},1} \quad . \tag{41}$$

To choose the best location for the addition of a single HVDC link, it it suffices to solve (38) for each pair (A_i, B_i) and to compute the resulting performance values of J_i. The smallest value

$$i^* = \arg\min_i J_i \tag{42}$$

yields the optimal HVDC placement with respect to the dynamic performance measure (1).

B. Placing multiple HVDC link

In principal, the evaluation of the effect from multiple HVDC links on the closed loop performance of the power system can be carried out as outlined in Section IV-A. However, the number of candidate topologies n_{cand} is a combinatorial number of the number of buses, n_{bus}, and HVDC links, n_{hvdc}. If repeated HVDC links are permitted,

$$n_{\text{cand}} = \frac{(n_{\text{cand},1} + n_{\text{hvdc}} - 1)!}{(n_{\text{cand},1} - 1)! n_{\text{hvdc}}!} \tag{43}$$

where $n_{\text{cand},1}$ denotes the number of candidate location when placing a single HVDC link, as in (40).

In order to keep a tractable problem formulation, a heuristic is required to eliminate the combinatorial complexity. To this end, a recursive placement is presented as pseudo code in Algorithm 1. At each iteration, the algorithm loops through all non-identical pairs of buses (i, j) as candidate locations for a new HVDC link and computes the linearized power system model (A, B) using the function ComputeModel. This function has the location of the left and right HVDC terminals as argument and carries out the linearization of the power system dynamic equations. The algorithm then solves the corresponding SDP (38) and keeps track of the best location computed so far. In the next iteration, the optimal locations from the previous levels is kept in the updated index vector $\mathcal{I}_\text{l}$ and $\mathcal{I}_\text{r}$. The process is repeated until n_{hvdc} links were placed, requiring a total of

$$n_{\text{cand}} = n_{\text{hvdc}} \cdot n_{\text{cand},1} \tag{44}$$

topologies to be evaluated.

Algorithm 1 PlaceHVDCs($\mathcal{B}$, n_{hvdc})

Require: Set of bus numbers $\mathcal{B}$, number of HVDC links to place n_{hvdc}

Ensure: Set of bus numbers for the left terminal $\mathcal{I}_l$, Set of bus numbers for the right terminal $\mathcal{I}_r$

$\mathcal{I}_\text{l} = \emptyset, \quad \mathcal{I}_\text{r} = \emptyset$
while $n_{\text{hvdc}} > 0$ **do**
  $J_{\min} \leftarrow \infty$
  for $i \in \mathcal{B}$ **do**
    for $j \in \mathcal{B}, j \neq i$ **do**
      $(A, B) \leftarrow$ ComputeModel$([\mathcal{I}_\text{l}, i], [\mathcal{I}_\text{r}, j])$
      $J \leftarrow$ SolveLMI(A, B)
      if $J < J_{\min}$ **then**
        $(J_{\min}, i^*, j^*) \leftarrow (J, i, j)$
      end if
    end for
  end for
  $\mathcal{I}_\text{l} \leftarrow [\mathcal{I}_\text{l}, i^*]$
  $\mathcal{I}_\text{r} \leftarrow [\mathcal{I}_\text{r}, j^*]$
  $n_{\text{hvdc}} \leftarrow n_{\text{hvdc}} - 1$
end while

V. Simulation

A. Illustration of the LMI based performance evaluation

To illustrate the LMI based approach to the solution of (1)-(7), consider the simple two dimensional test system

$$\dot{x} = Ax + Bu \tag{45}$$

$$z = Cx \tag{46}$$

with

$$A = \begin{pmatrix} -1 & 2 \\ -3 & -4 \end{pmatrix}, \; B = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad .$$

Furthermore, the matrices

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \; E_x = \begin{pmatrix} 4 & 1 \\ 1 & 1 \end{pmatrix}, \; E_u = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.2 \end{pmatrix},$$

characterize the objective function, the ellipsoid containing the initial states and the ellipsoid of the input constraints. The SDP (38) was encoded using YALMIP [10] and solved using SDPT3 [11]. The solution provides the gain K and the Lyapunov weight P as

$$K = \begin{pmatrix} 4.326 & -1.811 \\ -1.245 & -1.523 \end{pmatrix}, \; P = \begin{pmatrix} 0.224 & -0.002 \\ -0.002 & 0.113 \end{pmatrix},$$

which reduces the performance measure

$$J = \lambda_{\max}(\hat{E}^{-1} P \hat{E}^{-T}) = 1/s = 0.180 \tag{47}$$

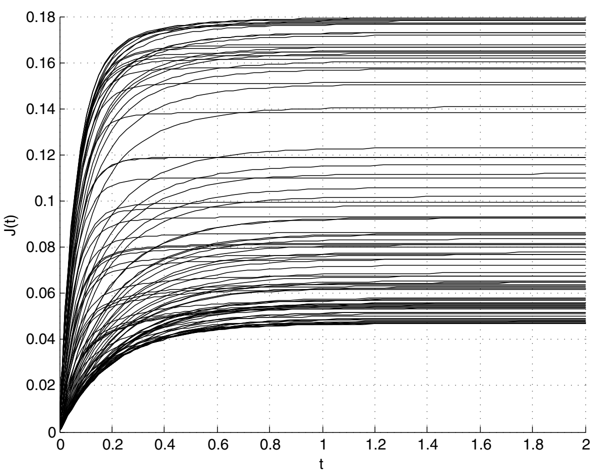

Fig. 1. LMI based control of constrained 2D test system: The integrated cost $J(t)$ approaches the upper bound 0.180 for some of the randomly sampled initial conditions. This bound was determined tightly and minimized in the SDP (38) as $1/s$.

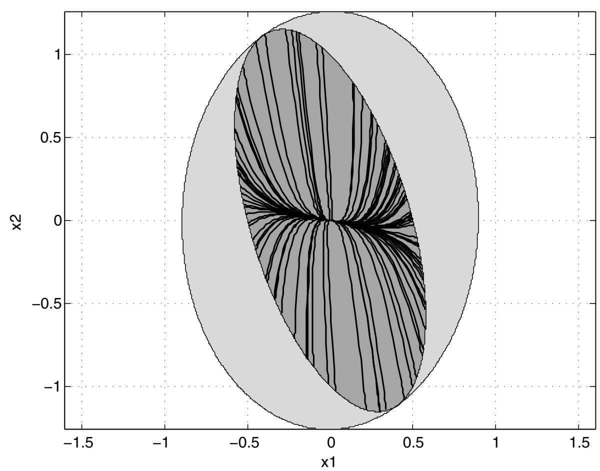

Fig. 2. LMI based control of constrained 2D test system: Specified set of initial states $\mathbb{X}_0$ (dark ellipse), invariant set $x^T Px \leq 1/s$ (light ellipse) and state trajectories for randomly sampled initial conditions (solid lines).

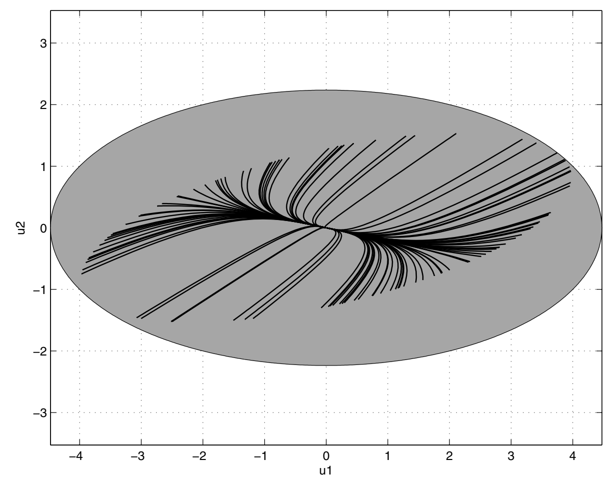

Fig. 3. LMI based control of constrained 2D test system: Specified set of admissible inputs $\mathbb{U}$ (dark ellipse) and input trajectories for randomly sampled initial conditions (solid lines).

as much as possible. Fig. 1 shows that the bound is approached by the integrated cost

$$J(t) = \int_0^t z(\tau)^T M z(\tau) d\tau \tag{48}$$

for some of the randomly sampled initial conditions. The critical initial conditions can be seen in Fig. 2, where the set of initial states $\mathbb{X}_0$ is contained inside the invariant set $x^T Px \leq 1/s$. Also, Fig. 3 illustrates that all input trajectories remain inside the specified boundaries of $\mathbb{U}$.

B. Placement of HVDC links in power systems

The initial motivation of the closed loop performance assessment of constrained linear systems was the selection of the best location for newly constructed HVDC links in a meshed AC power grid. A benchmark case is the two area power system with four generators and 11 buses, connected through a weak AC link, as presented in Section 12.8 of [1] and illustrated in Fig. 4. Since four of the buses are transformer buses, there remains a total of $n_{\text{bus}} = 7$ candidate connections for the terminals of new HVDC links. Without any further constraints on the placement, this results in $n_{\text{cand},1} = 21$ possible locations for the placement of a single HVDC link.

The nonlinear dynamic equations consist of four second order generator models with the rotor angle and the frequency deviations $(\delta_i, \Delta\omega_i)$, $i = 1, ..., 4$ as dynamic states. The model is linearized about the current operating point, using the parameters and operating points given in [1]. The resulting system has eight dynamic states and for every HVDC link three free inputs, one used to satisfy the active power balance constraint. Algorithm 1 is used to repeatedly evaluate the 21 HVDC locations, placing a link of 200 MW active power and 40 MVar reactive power rating. The algorithm imposes a bound on the current injections, $u^T u \leq 2$. The ellipsoidal set of initial states $\mathbb{X}_0$ is chosen within the bounds $|\delta_i| \leq 0.5$ and $|\Delta\omega_i| \leq 0.01$. The performance measure in (1) integrates the equally weighted frequency deviations $\sum_i \Delta\omega_i(t)^2$.

Fig. 5 shows the performance measure that is obtained using a single HVDC link in each of the 21 locations. The square denotes the optimal location that is kept for the second iteration of Algorithm 1. Note that the general level of the performance measure J in Fig. 5 decreases as HVDC links are added to the system. Also, the location of the peaks change since at each level, the optimal HVDC links from the previous levels remain in the system and contribute to the rejection of the most critical disturbance directions. The final result, depicted in Fig. 6, shows the topology of the system after three iterations of Algorithm 1. Fig. 7 and Fig. 8 show the simulation result of the system with three HVDC links for randomly sampled initial conditions in $\mathbb{X}_0$. It can be seen that the trajectories approach the computed performance bound of $J = 3.705$ while respecting the input constraints.

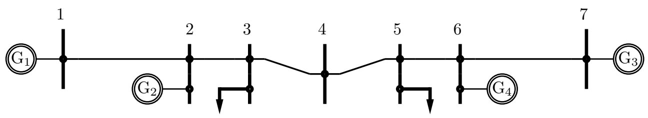

Fig. 4. Test system with four generators and seven buses in two areas [1].

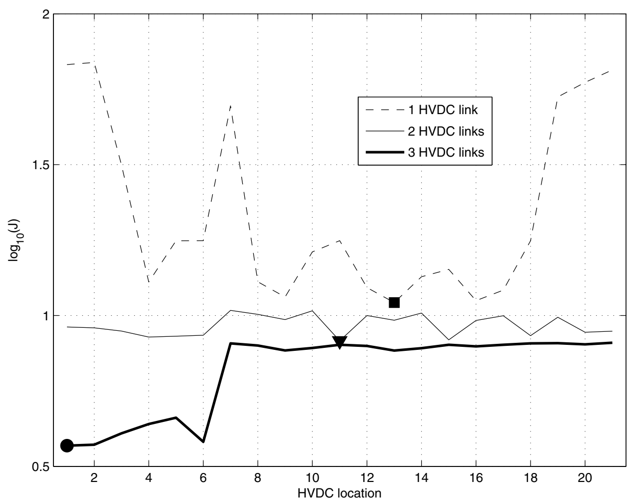

Fig. 5. Recursive placement of three HVDC links in a four generator system: Performance measure J as in (1) for 21 different locations. Optimal location of the first link (square), the second link (triangle) and third link (circle).

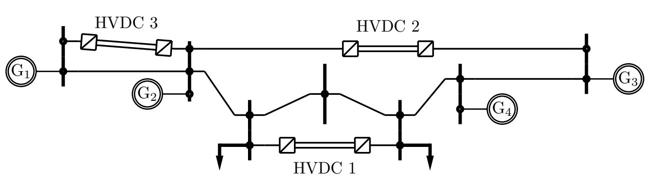

Fig. 6. Recursive placement of three HVDC links in a four generator test system: All bus pairs are candidate links. Each HVDC link added to the system minimizes (42), based on the solution of (1)-(7).

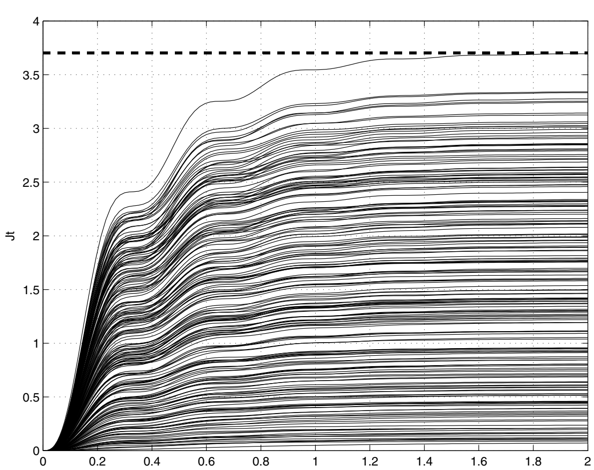

Fig. 7. LMI based control of power system with 3 HVDC links: The integrated cost $J(t)$ approaches the upper bound 3.705 for some of the randomly sampled initial conditions. This bound was determined tightly and minimized in the SDP (38) as $1/s$.

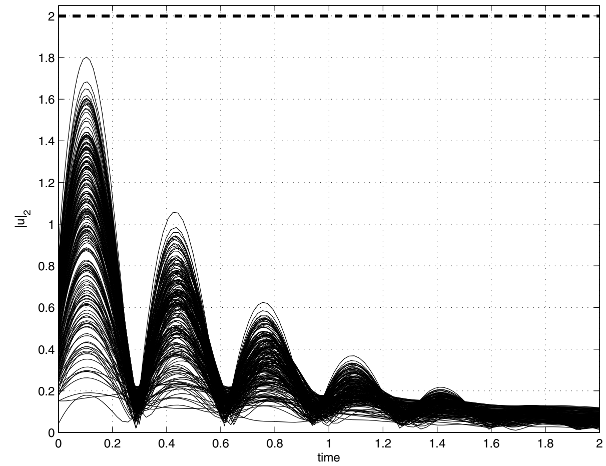

Fig. 8. LMI based control of power system with 3 HVDC links: Specified bound on input norm (dashed line) and input norm trajectories for randomly sampled initial conditions (solid lines).

VI. Conclusion

An efficient approach for the closed loop performance evaluation of a constrained linear system with different actuator choices has been presented. The min-max problem that arises from the actuator evaluation is formulated as an LMI, embedding equality constraints and actuator inequality constraints. A heuristic is proposed for the selection of multiple actuators. The approach is illustrated with the problem of placing HVDC links in a two area power system test case.

References

[1] P. Kundur. *Power System Stability and Control*. McGraw-Hill, Inc., 1993.

[2] T. Smed and G. Andersson. Utilizing hvdc to damp power oscillations. *Power Delivery, IEEE Transactions on*, 8(2):620 –627, apr 1993.

[3] A.N. Fuchs, S. Mariéthoz, M. Larsson, and M. Morari. Grid stabilization through VSC-HVDC using wide area measurements. In *IEEE Powertech, Power System Technology*, Trondheim, Norway, June 2011.

[4] G. Andersson T. Demiray and L. Busarello. Evaluation study for the simulation of power system transients using dynamic phasor models. August 2008.

[5] Sigurd Skogestad and Ian Postlethwaite. *Multivariable Feedback Control: Analysis and Design*. John Wiley & Sons, 2005.

[6] I.M. Mitchell, A.M. Bayen, and C.J. Tomlin. A time-dependent hamilton-jacobi formulation of reachable sets for continuous dynamic games. *Automatic Control, IEEE Transactions on*, 50(7):947 – 957, july 2005.

[7] S.V. Rakovic, E.C. Kerrigan, D.Q. Mayne, and J. Lygeros. Reachability analysis of discrete-time systems with disturbances. *Automatic Control, IEEE Transactions on*, 51(4):546 – 561, april 2006.

[8] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*, volume 15 of *Studies in Applied Mathematics*. SIAM, Philadelphia, PA, June 1994.

[9] J.R. Bumby J. Machowski, J.W. Bialek. *Power System Dynamics: Stability and Control*. John Wiley & Sons, Ltd., 2008.

[10] J. Löfberg. Yalmip : A toolbox for modeling and optimization in MATLAB. In *Proceedings of the CACSD*, Taipei, Taiwan, 2004.

[11] M.J. Todd K.C. Toh and R.H. Tutuncu. Sdpt3 – a matlab software package for semidefinite programming, version 1.3. pages 545–581, 1999.