

Analysis of Fractional-order Telegraph Model for Neutron Transport in Nuclear Reactor with Slab Geometry

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Abstract— This paper deals with the analysis of fractional-order (FO) telegraph equation which models the transport of thermal neutrons inside a nuclear reactor with slab geometry. The FO model, previously developed by the authors, models the neutron transport as anomalous diffusion, precisely a subdiffusion. This model removes the lacunae of the conventional integer-order model of neutron movements. Here, the FO model is solved using the well known technique of separation of variables and the spatial distribution and time evolution of the neutron flux in the slab reactor are computed. This exercise is probably being performed for the first time for the fractional-order model of neutron transport. It clearly depicts the wavelike nature of the neutron flux. Also, the convergence of neutron flux for FO telegraph and subdiffusion models for asymptotic time establishes the long-time subdiffusive behaviour of the FO telegraph equation. The analysis carried out in this paper is thus forms a crucial step in the process of development of fractional-order model for a nuclear reactor.

I. INTRODUCTION

The main part of a nuclear reactor is the core inside which a controlled chain reaction of neutrons is carried out (see [1]–[3]). In this, the thermal neutrons collide with the nuclei of fissile materials like Uranium. This collision results into the fission of these nuclei and release of more fast neutrons and energy. The mathematical model of a nuclear reactor originates from the fundamental model of this neutron transport. The most popular transport model is the neutron diffusion equation, the one-dimensional space version of which is given in (1).

$$\frac{1}{v} \frac{\partial \phi(x, t)}{\partial t} + (\Sigma_a - \nu \Sigma_f) \phi(x, t) = D \frac{\partial^2 \phi(x, t)}{\partial x^2}, \quad (1)$$

where, v is the neutron speed, $\phi(x, t)$ is the neutron flux at location x at time t , Σ_a is the macroscopic absorption cross-section area, Σ_f is the fission cross-section area, ν is the average number of neutrons emitted per fission reaction, D is the diffusion coefficient. So the conventional way of modeling of the movements of neutrons is using Fick's law, resulting into a parabolic partial differential equation (PDE). This model has various drawbacks, namely, predicting infinite speed of neutrons and a limited spatial applicability, that is, it is not valid near the strong absorbers like fuel bundles and control rods.

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The remedies suggested for these in our earlier paper [4] are as follows.

- 1) As suggested by [5] and [6], use the Cattaneo's constitutive law and obtain a Neutron Telegraph Equation. This will remove the paradox of infinite neutron speeds.
- 2) Model neutron transport as anomalous diffusion, especially subdiffusion (see [7], [8]) using a Neutron Fractional-order Telegraph Equation.

This analysis was used to model neutron transport in a nuclear reactor with slab geometry as subdiffusion in [4] and the linear fractional-order neutron telegraph equation (FNTE) model was derived (see (2)) using the continuous-time random walk (CTRW) technique as given in [6].

$$\frac{\tau^\alpha}{v^\alpha} \frac{\partial^{2\alpha} \phi(x, t)}{\partial t^{2\alpha}} + M_1 \frac{\partial^\alpha \phi(x, t)}{\partial t^\alpha} + M_2 \phi(x, t) = D \frac{\partial^2 \phi(x, t)}{\partial x^2}, \quad (2)$$

where the characteristic time constant is given by

$$\tau = \frac{1}{v \Sigma_{tr}}. \quad (3)$$

Note that the unit of τ is seconds. Also, $0 < \alpha < 1$, and $M_1 = \tau^\alpha (\Sigma_a - \nu \Sigma_f) + 1/v^\alpha$ and $M_2 = \Sigma_a - \nu \Sigma_f$. The terms like $\frac{\partial^\alpha}{\partial t^\alpha}$ denote the Caputo fractional time-derivatives of order α (see section II). This model was developed for a slab reactor (of finite width) with uniformly distributed fissile material characterized by the cross-sections Σ_a , Σ_{tr} , and Σ_f (see Fig. 1). The neutron source considered was the fission reactions only. Further, comments were made on the short-time and long-time behaviours of the derived FNTE model. In the short-time range, the FNTE model (2) behaves as the telegraph equation with the finite speed of neutron wave propagation, whereas, for $t \gg \tau$, it can be replaced by a much simple subdiffusion model as:

$$\frac{1}{v^\alpha} \frac{\partial^\alpha \phi(x, t)}{\partial t^\alpha} + (\Sigma_a - \nu \Sigma_f) \phi(x, t) = D \frac{\partial^2 \phi(x, t)}{\partial x^2}. \quad (4)$$

Also, note that with $\alpha \rightarrow 1$, the integer-order neutron telegraph equation (INTE)

$$\frac{1}{v} \frac{\partial^2 \phi(x, t)}{\partial t^2} + N_1 \frac{\partial \phi(x, t)}{\partial t} + N_2 \phi(x, t) = D \frac{\partial^2 \phi(x, t)}{\partial x^2}, \quad (5)$$

is recovered from the FNTE (2). Coefficient N_1 is obtained by substituting $\alpha = 1$ in M_1 and $N_2 = M_2$. Fractional-order models for the neutron transport have also been proposed in [9] and [10].

In this paper, the detailed analysis of the FO model (2) is presented. The model is solved using the well known method

of separation of variables considering a slab geometry and the time evolution and spatial distribution of the neutron flux are computed and studied. The paper is organized as follows. A brief introduction to fractional calculus, especially the Caputo fractional derivatives is given in section II. Section III advocates the use of the simplistic slab geometry used in this work. Analysis of the FNTE model is carried out in section IV by solving the PDE models using the separation of variables method. Discussion on the flux profile predicted by the FNTE model is given in section V. Conclusions are given in section VI. The details about the computational work carried out in this paper are given in appendix.

II. FRACTIONAL CALCULUS

Fractional calculus deals with the derivatives and integrals of arbitrary non-integer order (see [11]), the conventional integer-order (IO) calculus being the special case when the differentiation and integration orders are integers. The ordinary and partial differential equations involving fractional derivatives are called fractional differential equations (FDEs) (see [12]). Recently, these have found a plethora of applications in modeling and control (see [13], [14]). We give here only the Caputo definition used in the model. Caputo fractional derivative of order $\alpha \in \mathbb{R}^+$ of a function $f(t)$ is defined as

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha - n)} \int_a^t \frac{f^{(n)}(\tau)}{(t - \tau)^{\alpha - n + 1}} d\tau, \quad (6)$$

for $(n - 1 < \alpha < n)$, where $f^{(n)}(\tau)$ is the n^{th} -order derivative of the function $f(t)$. The Caputo definition is preferred by the engineers and physicists because the FDEs with Caputo derivatives have the same initial conditions as that for the Integer-order Differential Equations (IODEs) [12].

III. SLAB REACTOR

In this work, we consider a simple slab geometry for the nuclear reactor. The FPRK model is developed for the slab reactor. It is a uniform slab of fissile material with various cross-sections Σ_a , Σ_{tr} , and Σ_f as shown in Fig. 1. The length of the slab is a centimeters. The main advantage of using such simple geometry is that the one-dimensional version of the transport equations can be used without any problem. Such a simplification greatly facilitates the analysis.

Almost all reference books (see [1]–[3], [15], [16], and others) on the nuclear reactor theory have sought to take help of this simplified reactor structure to explain various fundamental and important concepts. Also, there are several research articles available that have used slab reactor to develop/establish various results. For example: the critical dimensions required to achieve the criticality in the slab reactor are estimated in [17]. Slab geometry is also used in the study of spatial discretization schemes for the multigroup discrete-ordinates transport equations in [18]. Multigroup space-time reactor kinetics equations are analytically solved for homogeneous slab and spherical reactors with both bare and reflected slabs in [19]. The F_N approximation for the FO transport equation in slab geometry is given in [20].

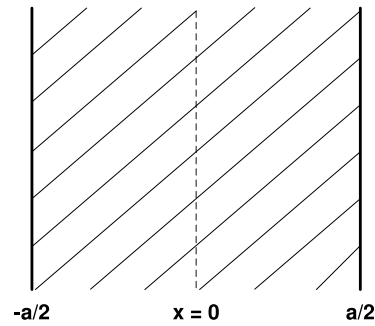


Fig. 1. The slab reactor.

Thus, it is a standard practice to use the simplistic slab geometry to develop new models and test various results related to the reactor modeling.

IV. SOLUTION USING SEPARATION OF VARIABLES METHOD

In this section, we employ the well-known technique of Separation of Variables (SoV) to solve the PDE models (2), (4) and (5). SoV (see [1], [21], [22]) is a very easy to use and simple method for solving a linear PDE whose solution and initial conditions satisfy some smoothness properties. The main principle of the method is to assume that the solution of the PDE (which is a function of space and time) can be expressed as a product of two functions, one is of space and the other of time, only. Thus, the SoV method basically splits the given PDE into two ODEs. Then these two ODEs are solved and the solution of the PDE is expressed as an infinite weighted sum of product of the solutions of these ODEs. Solving the neutron PDEs (both IO and FO) will give us the insights into the movement of neutrons modeled by these equations and understanding the dynamics of the reactor. The reason for choosing this particular method has a couple of advantages: firstly, it is a very straightforward and simple method and secondly, this method is the first step in the process of developing the point-reactor kinetics model for the reactors.

For all the equations, we assume a slab geometry of length ' a ' centimeters. For all models, we make following assumptions.

1) Boundary Condition (BC):

$$\phi\left(\frac{\tilde{a}}{2}, t\right) = \phi\left(-\frac{\tilde{a}}{2}, t\right) = 0, \quad (7)$$

where $\tilde{a} = a + z_0$, with the extrapolation distance, $z_0 = 0.71\lambda_{tr}$. The transport mean free path λ_{tr} is defined as the reciprocal of the macroscopic transport cross section, Σ_{tr} [1].

2) Initial Conditions (IC):

We assume the initial flux to be symmetric. Thus,

$$\phi(x, 0) = \phi_0(x) = \phi_0(-x). \quad (8)$$

In our work, we assume $\phi_0(x)$ to be triangular in shape as shown in Fig. 2. So,

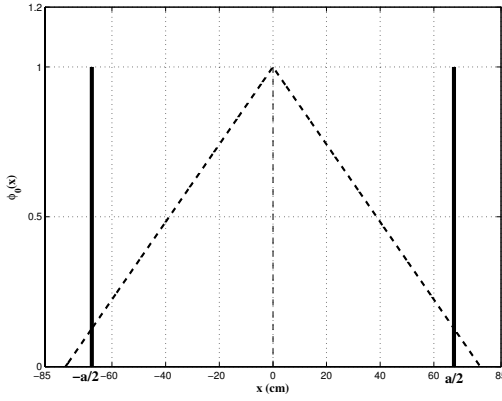


Fig. 2. Initial distribution of flux: $\phi_0(x)$.

$$\begin{aligned} \phi_0(x) &= \frac{2x}{\tilde{a}} + 1 \text{ for } \frac{-\tilde{a}}{2} \leq x < 0, \\ \phi_0(x) &= \frac{-2x}{\tilde{a}} + 1 \text{ for } 0 < x \leq \frac{\tilde{a}}{2}. \end{aligned} \quad (9)$$

The reason to choose this particular initial distribution of flux is to avoid the mismatch between the ICs and BCs at $t = 0$. Note that the choice of symmetric flux $\phi(x, 0)$ will guarantee symmetry for all times, $\phi(x, t) = \phi(-x, t)$ (see [1]).

For the FO telegraph equation, we need to specify one additional IC: the value of time derivative of flux at $t = 0$. Since the telegraph equation asymptotically behaves as diffusion equation, we need to specify this condition in such a way that this transition from telegraph equation to diffusion equation should be smooth [23]. This is possible only if we choose the initial time derivative of the flux equal to zero, that is,

$$\begin{aligned} \left. \frac{\partial \phi(x, t)}{\partial t} \right|_{(-x, 0)} &= \left. \frac{\partial \phi(x, t)}{\partial t} \right|_{(x, 0)} \\ &= \phi_1(-x) = \phi_1(x) = 0. \end{aligned} \quad (10)$$

- 3) For all equations, we use the data (that is the values of parameters) from [1]. This data is for a one speed diffusion model of a bare, homogeneous cylindrical reactor. Its material composition is similar to a modern pressurized water reactor. We use this data for slab geometry. $\Sigma_{tr} = 3.62 \times 10^{-2} \text{ cm}^{-1}$, $\Sigma_a = 0.1532 \text{ cm}^{-1}$, $\nu\Sigma_f = 0.157 \text{ cm}^{-1}$, $v = 3.1 \times 10^5 \text{ cm/sec}$. The diffusion constant is $D = 9.21 \text{ cm}$. Also, $\tilde{a} = 154.58 \text{ cm}$. The extrapolation length is $z_0 = 0.71\lambda_{tr} = 19.61 \text{ cm}$. This gives the width of the slab reactor, $a = 134.97 \text{ cm}$. Also note the relaxation time constant of the neutron current density calculated using (3) as

$$\tau = \frac{1}{v\Sigma_{tr}} = 8.911 \times 10^{-5} \text{ sec}.$$

We follow the same methodology as given in [1] to analyze the neutron flux distribution in the slab reactor

as predicted by the FO neutron telegraph equation model.

To understand the nature of neutron movements modeled using the FO neutron telegraph equation (2), we solve it using the SoV method. The FNTE is rewritten here:

$$\frac{\tau^\alpha}{v^\alpha} \frac{\partial^{2\alpha} \phi(x, t)}{\partial t^{2\alpha}} + M_1 \frac{\partial^\alpha \phi(x, t)}{\partial t^\alpha} + M_2 \phi(x, t) = D \frac{\partial^2 \phi(x, t)}{\partial x^2}, \quad (11)$$

where $M_1 = \tau^\alpha(\Sigma_a - \nu\Sigma_f) + 1/v^\alpha$ and $M_2 = \Sigma_a - \nu\Sigma_f$. We add BC (7) and ICs (9), and (10). We propose to split $\phi(x, t)$ into two functions,

$$\phi(x, t) = \psi(x)T(t). \quad (12)$$

Plugging in this into (11) and simplifying, we extract two ODEs,

- 1) Space ODE

$$\frac{d^2 \psi(x)}{dx^2} + \frac{1}{D} \left[\nu\Sigma_f - \Sigma_a + \frac{\tau^\alpha}{v^\alpha} \lambda \right] \psi(x) = 0. \quad (13)$$

- 2) Time FDE

$$\frac{d^{2\alpha} T(t)}{dt^{2\alpha}} + M_{11} \frac{d^\alpha T(t)}{dt^\alpha} + \lambda T(t) = 0, \quad (14)$$

where λ is a constant and $M_{11} = \frac{v^\alpha}{\tau^\alpha} M_1$.

We first solve the space ODE. The BC (7) becomes:

$$\psi\left(\frac{\tilde{a}}{2}\right) = \psi\left(-\frac{\tilde{a}}{2}\right) = 0. \quad (15)$$

The space ODE (13) is compared with the standard eigenvalue problem (EVP) [21]

$$\frac{d^2 \psi_n(x)}{dx^2} + B_n^2 \psi_n(x) = 0, \quad (16)$$

with the BCs given in (15). Note that we will be using symbols $\psi(x)$ and $\psi_n(x)$ interchangeably (same with the symbols $T(t)$ and $T_n(t)$, and with the symbols λ and λ_n). We wish to have only symmetric solutions for (13) and (15) as $\phi_0(x)$ is symmetric. So the eigenfunctions are

$$\psi_n(x) = \cos(B_n x), \quad (17)$$

and eigenvalues are

$$B_n^2 = \left(\frac{n\pi}{\tilde{a}}\right)^2, \quad (18)$$

where $n = 1, 3, 5, \dots$. Comparing (13) with (16) we find that,

$$B_n^2 = \frac{1}{D} \left[\nu\Sigma_f - \Sigma_a + \frac{\tau^\alpha}{v^\alpha} \lambda_n \right], \quad (19)$$

which gives us the time eigenvalues as

$$\lambda_n = \left(\frac{v}{\tau}\right)^\alpha \left[\Sigma_a - \nu\Sigma_f + DB_n^2 \right]. \quad (20)$$

These are the time eigenvalues since they characterize the time decay of the flux. With $\tau < 1$ and $\alpha < 1$, it can be concluded that, for any n , these will be less than the time EVs of the integer-order neutron telegraph equation.

We now turn our attention to the fractional IVP consisting of the time FDE (14). For its solution, we need two initial

conditions, the value of $T_n(t)$ and its time derivative at $t = 0$. We proceed to evaluate the IC $T_n(0)$. Since $\phi(x, t)$ satisfies IC (8), we can write

$$\phi_0(x) = \phi(x, 0) = \sum_{n \text{ odd}} T_n(0) \cos B_n x, \quad -\frac{\tilde{a}}{2} < x < \frac{\tilde{a}}{2}. \quad (21)$$

But this is just the fourier series representation of $\phi_0(x)$. So,

$$T_n(0) = \frac{2}{\tilde{a}} \int_{-\frac{\tilde{a}}{2}}^{\frac{\tilde{a}}{2}} \phi_0(x) \cos\left(\frac{n\pi x}{\tilde{a}}\right) dx. \quad (22)$$

For our case, the $\phi_0(x)$ is triangular as given in (9). Substituting in (22) and using Mathematica [24] we get,

$$T_n(0) = \frac{16}{n^2\pi^2} \sin^2\left(\frac{n\pi}{4}\right), \quad (23)$$

which is the required expression. The other IC can be deduced from (10) as

$$\left. \frac{dT_n(t)}{dt} \right|_{t=0} = 0. \quad (24)$$

Coming back to the solution of (14), application of the Laplace transformation followed by rearrangement of the terms gives,

$$T_n(s) = T_n(0) \frac{s^{2\alpha-1} + M_{11}s^{\alpha-1}}{s^{2\alpha} + M_{11}s^{\alpha} + \lambda_n}. \quad (25)$$

We need to evaluate the inverse Laplace transform (ILT) for (25), for which no generalized expression exists in time domain [14]. So four values of fractional derivative order are considered, $\alpha = 0.1, 0.3, 0.7,$ and 0.9 . Motivation for choosing these values of the fractional power can be explained as follows: the FO model developed here is applicable everywhere in the heterogeneous reactor core. A value of α close to unity indicates the presence of normal Fickian diffusion implying lesser or negligible fission reactions. Such regions include the moderator and reflectors in a reactor core. On the other hand, a smaller value of α imply a highly subdiffusive environment. This can be interpreted as strong absorbing regions, for example, near the fuel bundles and control rods. For each case, the ILT of (25) is evaluated. A sample calculation is shown for $\alpha = 0.1$. With this value of α ,

$$T_n(s) = T_n(0) \frac{s^{-0.8} + M_{11}s^{-0.9}}{s^{0.2} + M_{11}s^{0.1} + \lambda_n}, \quad (26)$$

Rewriting this as

$$T_n(s) = T_n(0) \frac{1}{s^{0.9}} \tilde{T}_n(s), \quad (27)$$

where

$$\tilde{T}_n(s) = \frac{s^{0.1} + M_{11}}{s^{0.2} + M_{11}s^{0.1} + \lambda_n}. \quad (28)$$

Carrying out the transformation $s^{0.1} = w$,

$$\tilde{T}_n(w) = \frac{w + M_{11}}{w^2 + M_{11}w + \lambda_n}. \quad (29)$$

This is a rational function of polynomials, so we carry out the partial fractions. For all cases, the MATLAB routine

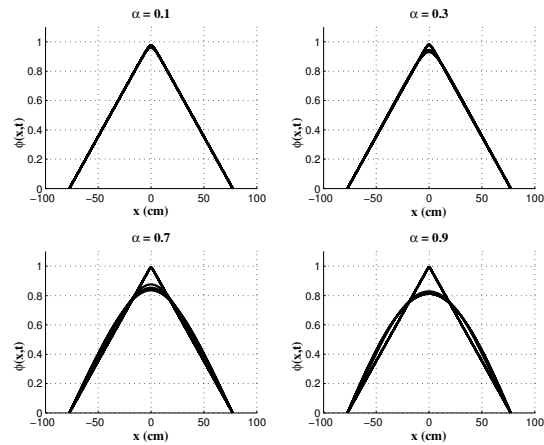


Fig. 3. Spatial distribution of $\phi(x, t)$ for FO neutron telegraph equation.

residue [25] is used. This is followed by back substitution of $w = s^{0.1}$, finally giving (27) as

$$T_n(s) = T_n(0) \sum_{j=1}^2 \frac{R_j}{s^{0.9}(s^{0.1} + P_j)}, \quad (30)$$

where R_j and $P_j, j = 1, 2$ are residues and poles (which can be real or complex, or both) of the partial fractions. Using the Laplace Transform relationship given in [14],

$$\mathcal{L}^{-1}\left(\frac{s^{\mu-1}}{s^{\mu} + p}\right) = E_{\mu}(-pt^{\mu}), \quad \mu > 0, \quad (31)$$

we get from (30),

$$T_n(t) = T_n(0) \sum_{j=1}^2 R_j E_{0.1}(-P_j t^{0.1}). \quad (32)$$

Thus for each n , (32) is solved with the help of (23). As given in the separation of variables method, the neutron flux $\phi(x, t)$ modeled by FNTE is expressed as an infinite summation of product of $T_n(t)$ and $\psi_n(x)$:

$$\phi(x, t) = \sum_{n \text{ odd}} T_n(t) \cos\left(\frac{n\pi x}{\tilde{a}}\right). \quad (33)$$

V. DISCUSSION

The spatial distribution of neutron flux as predicted by the FO telegraph equation is shown in Fig. 3. The initial triangular flux settles in the cosine shape with respect to space. Thus the flux remains symmetrical for all $t > 0$. The effect of subdiffusion is clearly seen. For $\alpha = 0.1, 0.3$, flux takes a much longer time to settle into the final cosine shape. Even after the instant $t = 0.001$ sec., the flux for these models do not assume the spatial cosine shape. On the other hand, for higher values of α , it immediately settles into the cosine shape. This is obviously attributed to the subdiffusive formalism of the neutron transport. The zoomed view of the spatial distribution at the instant $t = 2 \times 10^{-6} < \tau$ sec. is depicted in Fig. 4. We clearly see a wave-like spatial distribution though with a very small amplitude. This

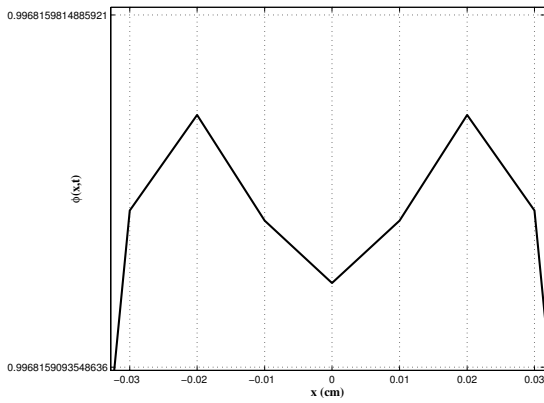


Fig. 4. Wave-like spatial distribution of $\phi(x, t)$ for FO neutron telegraph equation at $t = 2 \times 10^{-6}$ sec. (zoomed view).

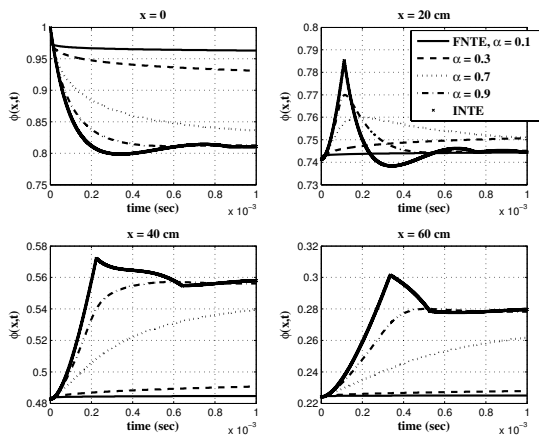


Fig. 5. Time evolution of $\phi(x, t)$ for INTE and FNTE at selected space locations.

confirms the predicted behaviour of the FO neutron telegraph model (2).

Next we plot the time evolution of flux given by FO and IO neutron telegraph models at specific locations in the slab in Fig. 5. The wavelike nature of the flux predicted by the FNTE model is clearly seen at all locations. This behaviour is more predominant for higher values of α . For small values of α which indicate a stronger presence of subdiffusion, obviously the wavelike behaviour will not be dominating. Also, as the value of α approaches unity, the flux plots of FO telegraph model move nearer to that of the IO telegraph model. Next, the time-variation of flux given by the FO telegraph and diffusion model are compared. To gain a clearer understanding, we plot the FNDE and FNTE flux at $x = 0, 20, 40$ and 60 cm for four values of α in Fig. 6 and Fig. 7, Fig. 8 and Fig. 9, respectively. These plots endorse the earlier mentioned fact that the FO neutron telegraph equation (2) behaves as a subdiffusion equation (4) for long times. This behaviour is clearly seen at all the space locations, especially for higher values of the time derivative

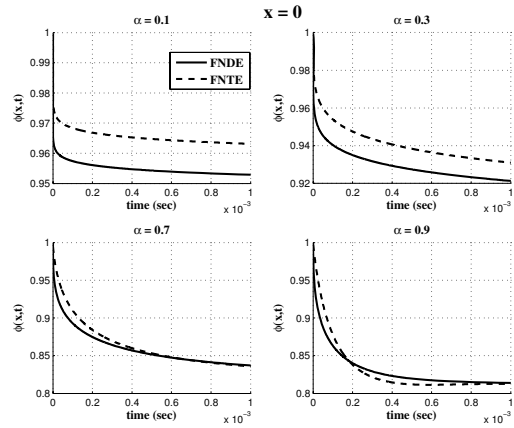


Fig. 6. Time evolution of $\phi(x, t)$ for FNDE and FNTE at $x = 0$ for four values of α .

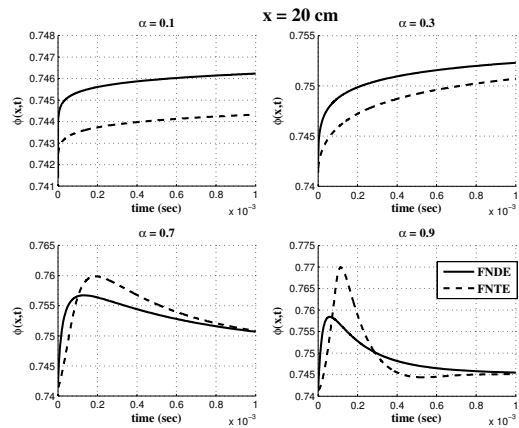


Fig. 7. Time evolution of $\phi(x, t)$ for FNDE and FNTE at $x = 20$ cm for four values of α .

order, $\alpha = 0.7, 0.9$. The plot of the flux predicted by the FNTE model (2) for these cases coincides with that of the FNDE model (4) as time progresses. However, we don't see these flux plots converging for $\alpha = 0.1, 0.3$. This is because these cases represent a strong subdiffusive situation, which might be dominating the wave-like phenomenon. Also, there could be some issues related to the numerical accuracy since these small values of fractional power generally make the convergence of a numerical algorithm difficult [26].

VI. CONCLUSIONS

The spatial distribution and time evolution of the neutron flux in the slab reactor as predicted by the fractional-order neutron telegraph equation is analyzed in detail in this paper. The method of separation of variables is used. The wave-like nature of the neutron flux is clearly visible in case of telegraph models. This exercise also confirms one feature of the FO telegraph model. The time evolution of the FO telegraph model flux coincided with that of the FO subdiffusion equation for $t > \tau$, especially for higher values

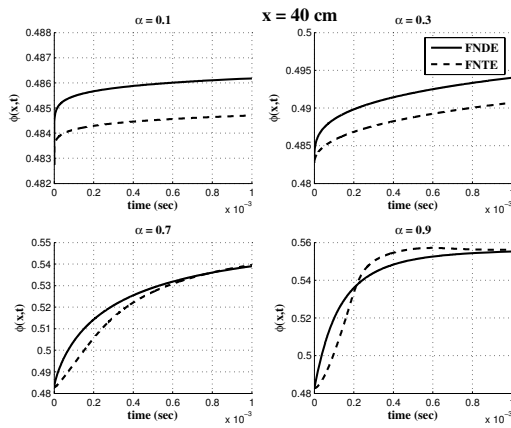


Fig. 8. Time evolution of $\phi(x, t)$ for FNDE and FNTE at $x = 40$ cm for four values of α .

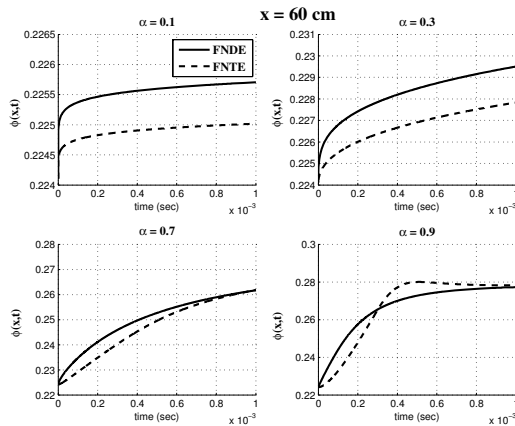


Fig. 9. Time evolution of $\phi(x, t)$ for FNDE and FNTE at $x = 60$ cm for four values of α .

of α . To the best of our knowledge, such type of analysis for the fractional-order model of the neutron transport in the reactor is performed for the first time.

APPENDIX: COMPUTATIONAL DETAILS

The flux computation is carried out using MATLAB on a desktop PC with Core 2 Quad processor with 2 GB RAM. The details regarding the computation are given below.

- 1) The grid for space variable is chosen as, $x = [-77.29 : 0.01 : 77.29]$.
- 2) For time variable, we choose the grid as $t = [0 : 0.000001 : 0.001]$.
- 3) The summation in (33) is carried out up to $n = 1001$.
- 4) As $\phi(x, t)$ is a function of both space and time, we calculate it in two ways:
 - i) $\phi(x, t)$ is calculated at 5 space points, $x = 0, 20, 40, 60, 77.29$ cm, for all the time instants.
 - ii) $\phi(x, t)$ is calculated at 10 time instants, $t = 0, 1 \times 10^{-6}, 2 \times 10^{-6}, 3 \times 10^{-6}, 4 \times 10^{-6}, 5 \times 10^{-6}, 0.00025, 0.0005, 0.00075, 0.001$

seconds, for all the space points in the array of x .

REFERENCES

- [1] J. J. Duderstadt and L. J. Hamilton, *Nuclear Reactor Analysis*. USA: John Wiley & Sons, 1976.
- [2] S. Glasstone and A. Sesonske, *Nuclear Reactor Engineering: Vol. 1*. India: CBS Publishers & Distributors, 2002.
- [3] J. R. Lamarsh, *Introduction to Nuclear Reactor Theory*. USA: Addison-Wesley Publishing Company, 1966.
- [4] V. A. Vyawahare and P. S. V. Nataraj, "Modeling neutron transport in a nuclear reactor as subdiffusion: The neutron fractional-order telegraph equation," in *The 4th IFAC Workshop on Fractional Differentiation and its Applications*, Badajoz, Spain, October 2010.
- [5] R. V. Meghreblian and D. K. Holmes, *Reactor Analysis*. USA: McGraw-Hill Book Company, 1960.
- [6] A. Compte and R. Metzler, "The generalized Cattaneo equation for the description of anomalous transport processes," *Journal of Physics A: Mathematical and General*, vol. 30, pp. 7277–7289, 1997.
- [7] R. Klages, G. Radons, and I. M. Sokolov, Eds., *Anomalous Transport*. WILEY-VCH Verlag GmbH & Co., 2008.
- [8] R. Metzler and J. Klafter, "The random walk's guide to anomalous diffusion: A fractional dynamics approach," *Physics Reports*, vol. 339, pp. 1–77, 2000.
- [9] S. Das and B. B. Biswas, "Fractional divergence for neutron flux profile in nuclear reactor," *International Journal of Nuclear Energy Science and Technology*, vol. 3, no. 2, pp. 139–159, 2007.
- [10] G. Espinosa-Paredes, J. B. Morales-Sandoval, R. Vázquez-Rodríguez, and E.-G. Espinosa-Martínez, "Constitutive laws for the neutron transport current," *Annals of Nuclear Energy*, vol. 35, pp. 1963–1967, 2008.
- [11] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*. Netherlands: Elsevier, 2006.
- [12] I. Podlubny, *Fractional Differential Equations*. USA: Academic Press, 1999.
- [13] R. L. Magin, *Fractional Calculus in Bioengineering*. USA: Begell House Publishers, 2006.
- [14] C. A. Monje, Y. Q. Chen, B. M. Vinagre, D. Xue, and V. Feliu, *Fractional-order Systems and Control: Fundamentals and Applications*. UK: Springer-Verlag London Limited, 2010.
- [15] W. M. Stacey, *Nuclear Reactor Physics*. Germany: WILEY-VCH Verlag GmbH & Co., 2007.
- [16] A. F. Henry, *Nuclear Reactor Analysis*. USA: The MIT Press, 1970.
- [17] T. W. Mullikin, "Estimates of critical dimensions of spherical and slab reactors," *Journal of Mathematical Analysis and Applications*, vol. 5, no. 2, pp. 184–199, 1962.
- [18] R. E. Alcouffe, E. W. Larsen, W. F. Miller-Jr., and B. R. Wienke, "Computational efficiency of numerical methods for the multigroup, discrete-ordinates neutron transport equations: the slab geometry case," *Nuclear Science and Engineering*, vol. 71, no. 2, pp. 111–127, 1979.
- [19] C. E. Lee, "Analytic solutions of the multigroup space-time reactor kinetics equations-I: 1-D multiregion slab and spherical geometry," *Annals of Nuclear Energy*, vol. 13, no. 5, pp. 245–268, 1986.
- [20] A. Kadem and D. Baleanu, " F_N approximation to fractional neutron transport equation in slab geometry," in *New Trends in Nanotechnology and Nonlinear Dynamical Systems*, Ankara, Turkey, July 2010.
- [21] J. D. Logan, *Partial Differential Equations*. USA: Springer-Verlag, 2004.
- [22] S. Farlow, *Partial Differential Equations*. USA: Dover Publishing Company, 2004.
- [23] A. M. Weinberg and E. P. Wigner, *The Physical Theory of Neutron Chain Reactors*. USA: The University of Chicago Press, 1958.
- [24] H. Ruskeepaa, *Mathematica Navigator: Mathematics, Statistics and Graphics*. USA: Academic Press, 2009.
- [25] *MATLAB Manual*. USA: The Mathworks Inc., MATLAB version 7.1 (R14), 2005.
- [26] K. Diethelm, *The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type*. Germany: Springer, 2010.