

Piecewise affine modeling of NOx emission produced by a diesel engine

Yana Vereshchaga, Stephan Stadlbauer, Laurent Bako, Luigi del Re

Abstract—Nitrogen oxides (NOx) emission control is a critical task of engine control as far as raw emissions and after treatment is concerned. Hardware NOx sensors are now available, but they still have several limitations, starting from their costs. Against this background, a lot of work inside both companies and in the academy has been done to develop virtual sensors for these emissions. In this paper, we propose a minimum-complexity model in the form of a piecewise affine model, and show that its performance is comparable to more complex models and better than conventional sensors if dynamic response is concerned.

I. INTRODUCTION

Nitrogen oxides (NOx), mainly consisting of nitrogen dioxide (NO₂) and nitrogen mono-oxide (NO), are one of the main pollution concerns in diesel engines. NOx can be reduced in several ways, e.g. at the level of raw emissions by exhaust gas recirculation (EGR), multiple fuel injections and by exhaust after treatment, typically by catalytic reduction (SCR), or NOx traps (LNT). All this has motivated a strong interest for NOx sensors. NOx emission values can be obtained directly from a physical sensor [1] or indirectly from engine control variables by the evaluation of a model, namely a virtual sensor.

A number of various approaches were used so far to determine the corresponding models. The emissions simulation was described by means of 3D computational fluid dynamics employing chemical reaction kinetics [2], phenomenological models [3] as well as hybrid 2-zone models [4] and empirical based ones [5]. Genetic algorithms have been used by [6] to extract both model structure and parameters from data. Simulations via black-box approaches like artificial neural networks [7], and evolutionary fuzzy logic [8] as well as by polynomial mean value [9] were presented. To our knowledge the black box-approaches give more sophisticated predictions compared to gray-box ones such as 3D computational fluid dynamics [2] or thermodynamics calculation [4].

In the current work a virtual sensor for NOx emission based on the identification of a PieceWise Affine (PWA) model [10] is proposed. For practical purposes the method is improved with a switching mechanism which automatically distinguishes between the subsystems in a distance dependent manner.

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Switched models in input-output form can be used to describe hybrid and non-linear phenomena that are frequent in practical situations, e.g. when a system may work in different modes of operation. Due to the great potential of hybrid methods to solve a pattern recognition problem in computer vision, system biology, electromechanical and automotive fields several intense efforts in a development of identification methods for switching models, described in [11] and [12], were done. The methods perform different problem formulations to piecewise affine systems: algebraic-geometric methods [13], a Bayesian [14], a bounded error approach [15] and clustering-based methods [16], [17].

Although the dynamic relation between the amount of NOx emission and its influence quantities is intrinsically nonlinear, it can be approximated by a PWA model. In comparison to the general nonlinear modeling approaches [9], [18], [19] which yield to a single but complex model, the PWA model enjoys the attractive feature of being locally simple and probably easier to embed in a control strategy. To estimate the model from the available experimental measurements, a new clustering-based identification technique is introduced and discussed. The motivation for such a method is mainly the computational requirement of the existing algorithms devised for PWA models identification. Indeed most of the existing algorithms which are surveyed in [11] and [12], although generally efficient, turn out to be computationally heavy for processing a dataset with more than 25000 samples.

To design the identification algorithm inspiration from some recent works on sparsity-inducing identification methods has been taken (see e.g., [20]–[23]). A partition of the regression space is obtained in the form of a Voronoi diagram (parameterized set of points called centers) where each cell is associated with one affine submodel described by a parameter vector. The pairs of centers and parameter vectors are, similarly as in [20], identified one after another by solving a non-smooth optimization problem and removing a certain amount of data.

The proposed method shares some similarities with those presented in [10], [14]–[17] since it relies on data clustering. Considering the incremental way in which the algorithm computes the parameters, it is more comparable to the MAX FS procedure used in [15]. However, there is a crucial difference in the principles of the proposed methods. The former formulates and solves a non-smooth optimization problem, the latter solves a feasibility problem for a pre-defined threshold on the acceptable model error.

The report is organized as follows. Section II is concerned with the identification method developed for computing a

piecewise affine model. In Section III the NOx emission identification problem is formulated and the experimental setup used for data generation is depicted. The modeling results are exposed and discussed in Section IV. Finally, the study is concluded in Section V.

II. THE IDENTIFICATION METHOD

A. The system

The system is defined to embody the causality relationship between the NOx emission and 7 chosen inputs n , $m_{f,tot}$, MAF , MAP , φ_{MI} , p_{rail} and O_{2exh} (see Figure 2). This inputs were selected because they contain the most significant influences on the combustion process itself and therefore also the formation of the NOx emissions. We assume that this relationship (system) can be mathematically described with a nonlinear structure of the form

$$y_t = f(x_t) + e_t \quad (1)$$

where f is an unknown nonlinear function mapping the regressor

$$x_t = [y_{t-1} \ \cdots \ y_{t-n_a} \ u_{t-n_k}^\top \ \cdots \ u_{t-n_b}^\top]^\top \quad (2)$$

to the output at time t . Here, $u_t \in \mathbb{R}^{n_u}$ and $y_t \in \mathbb{R}$ denote the input and the output vector at time t respectively. e_t is a scalar sequence accounting for possible measurement noise or model uncertainties. The integers n_a , n_b are the output and input lags respectively and n_k describes the delay. It is assumed that the dynamics of the system x_t can be assigned to a bounded set $\mathcal{X} \subset \mathbb{R}^d$, $d = n_a + (n_b - n_k + 1)n_u$, which will be referred as the operating space.

For the NOx process, u_t is formed as a concatenation of the 7 ECU inputs depicted in Figure 2 and y_t corresponds to the NOx emission. The orders n_a , n_b , n_k , are not known a priori and will therefore be determined by means of iterative trials on the data.

B. A piecewise affine model (PWA)

Trying to capture the global behavior of a nonlinear system such as (1) in a single mathematical model is likely to result in a highly complicated model structure. As a consequence, such a model may be difficult to exploit in practice, for control for instance. With respect to this goal, a more appealing approach is to decompose the operating space of the system into a finite number of regions to approximate the system behavior with an affine model. The main advantage of this approach is that linear models are simple and there exists an abundant theory for controlling and analyzing them.

The PWA model is defined as follows

$$\hat{f}_{PWA}(x) = \begin{cases} \theta_1^\top \begin{bmatrix} x \\ 1 \end{bmatrix} & \text{if } x \in \mathcal{X}_1 \\ \vdots & \vdots \\ \theta_s^\top \begin{bmatrix} x \\ 1 \end{bmatrix} & \text{if } x \in \mathcal{X}_s, \end{cases} \quad (3)$$

where $\{\mathcal{X}_i\}_{i=1}^s$ constitutes a complete partition of the system operating space $\mathcal{X} \subset \mathbb{R}^d$. That is, $\cup_{i=1}^s \mathcal{X}_i = \mathcal{X}$

and $\mathcal{X}_i \cap \mathcal{X}_j = \emptyset \forall i \neq j$. The regions are parameterized by a set of points $\{c_i\}_{i=1}^s$ in \mathbb{R}^d as follows

$$\mathcal{X}_i = \left\{ x \in \mathcal{X} : \|x - c_i\|_2 < \|x - c_j\|_2 \forall j = 1, \dots, s, j \neq i \right. \\ \left. \text{or } \|x - c_i\|_2 = \|x - c_j\|_2, i < j \right\}. \quad (4)$$

Thus each affine submodel i can be viewed as the affine model obtained by linearizing the nonlinear function f in the neighborhood of c_i . From the definition (4), it can be seen that if a point $x \in \mathcal{X}$ lies on the common boundaries of any two different sets \mathcal{X}_i and \mathcal{X}_j , it is arbitrarily assigned to the region with the smallest index. Note that the so-defined set of regions $\{\mathcal{X}_i\}_{i=1}^s$ corresponds to the Voronoi diagram (also called Voronoi tessellation) whose generators are the c_i 's.

C. The identification algorithm

In this section we derive an algorithm for estimating the PVs $\{\theta_i\}_{i=1}^s$ and the associated centers $\{c_i\}_{i=1}^s$ from the experimental measurements $\{(x_t, y_t)\}_{t=1}^N$. We can do so by minimizing the sum of squared errors between the output of the model \hat{f}_{PWA} defined in (3) and the measured one. That is, we can solve the following optimization problem

$$\min_{\substack{c_1, \dots, c_s \\ \theta_1, \dots, \theta_s}} \sum_{t=1}^N (y_t - \hat{f}_{PWA}(x_t))^2. \quad (5)$$

Let σ be the discrete mode of (3) defined by $\sigma(t) = i \in \{1, \dots, s\}$ iff $x_t \in \mathcal{X}_i$. Then the identification problem can be recast in the form

$$\min_{\substack{c_1, \dots, c_s \\ \theta_1, \dots, \theta_s \\ \sigma(1), \dots, \sigma(N)}} \sum_{t=1}^N (y_t - \theta_{\sigma(t)}^\top \begin{bmatrix} x_t \\ 1 \end{bmatrix})^2 \\ \text{s.t. } \|c_{\sigma(t)} - x_t\|_2 \leq \|c_i - x_t\|_2, \\ i = 1, \dots, s, t = 1, \dots, N. \quad (6)$$

Because of the non convex inequality constraints and the combinatorial nature of the objective function, the problem (6) is a hard non convex optimization problem. An exact solution is intractable at a reasonable computational cost and time. For this reason, we turn here to a sub-optimal solution that we describe next.

In the estimation process, it is perhaps practical to view any local model i as arising from a linearization of the nonlinear map f at c_i . However, this holds only if f is continuous in which case euclidean closeness of x and c_i implies that $|f(x) - f(c_i)|$ is small. This is no longer true if f is discontinuous. In order to design an algorithm that can work for both situations, it is advisable to introduce the information about the output fitting error in the data to be clustered [10]. Define an extended regressor vector as $\bar{x}_t = [y_t \ x_t^\top]^\top \in \mathbb{R}^{d+1}$. Consider the vector sequence $\{\phi_t(\theta, \bar{c})\}_{t=1}^N$, defined by

$$\phi_t(\theta, \bar{c}) = \begin{bmatrix} y_t - [x_t^\top \ 1]\theta \\ \bar{c} - \bar{x}_t \end{bmatrix} \in \mathbb{R}^{d+2} \quad (7)$$

for any parameter vector candidate $\theta \in \mathbb{R}^{d+1}$ and center $\bar{c} \in \mathbb{R}^{d+1}$. $\phi_t(\theta, \bar{c})$ concatenates the difference between the extended regressor \bar{x}_t and the center \bar{c} and the model error. Then our identification method hinges on the fundamental observation that points in the set $\{\phi_t(\theta_i, \bar{c}_i) : \sigma(t) = i\}$, must roughly form a cloud of points around 0. This follows indeed from the definition of the map \hat{f}_{PWA} and the validity regions \mathcal{X}_i defined in (3) and (4). Motivated by this observation, we propose to search for one (arbitrary) pair (θ_i, \bar{c}_i) by solving the following optimization problem for (θ, \bar{c}) ,

$$\min_{\theta, \bar{c}} \sum_{t=1}^N \|W \phi_t(\theta, \bar{c})\|_2 \quad (8)$$

where $W = \text{diag}(w_1, \dots, w_{d+2})$ is a diagonal weighting matrix with non negative entries. The matrix W allows, when necessary, for minimizing the entries of $\phi_t(\theta, \bar{c})$ in a discriminative manner. In particular, it might sometimes be desirable to give more importance to the first component, that is, to minimize the model error more than the distances to \bar{c} . The objective function (8) is a non smooth sum-of-norms criterion which is not differentiable at the points where each term takes zero value. Compared to the more customary sum-of-squared-norms cost, the objective in (8) has the appealing property of being less sensitive to the large terms. In fact it treats the large distances as outliers and automatically alleviate their effect. As a consequence, although all the data samples are involved in the optimization problem (8), we can hope to recover only a single pair (θ_i, \bar{c}_i) .

The parameters are estimated one after another, similarly as in [20]. Solving (8) is expected to yield one pair $(\hat{\theta}_1, \hat{c}_1)$ of estimates that we arbitrarily index with one. The hat symbol is used here to emphasize that the outcomes of the algorithm are just estimates, not necessarily the ideal values. Given $(\hat{\theta}_1, \hat{c}_1)$, we can sort the samples in an increasing order of the distance $\|\phi_t(\hat{\theta}_1, \hat{c}_1)\|_2$ and remove the r first samples, r being a user-specified parameter. We repeat the procedure on the remaining data to identify the second pair and continue this way until all s pairs of parameters are identified. Some additional refinements steps can be applied.

In more details, the identification algorithm operates in three steps as follows.

- 1) **Estimation of the centers.** Define $\mathcal{S}_1 = \{1, \dots, N\}$. For $i = 1, \dots, s$, compute

$$(\hat{\theta}_i, \hat{c}_i) = \arg \min_{\theta, \bar{c}} \sum_{t \in \mathcal{S}_i} \|\phi_t(\theta, \bar{c})\|_2, \quad (9)$$

$$\mathcal{S}_{i+1} = \mathcal{S}_i \setminus I_r(\hat{\theta}_i, \hat{c}_i), \quad (10)$$

with $I_r(\hat{\theta}_i, \hat{c}_i)$ consisting of the r first samples when the data are sorted in increasing order of their distance to \hat{c}_i .

- 2) **Data classification.** In this step, the data are classified into s groups, each of which is included in one region \mathcal{X}_i . The classification proceeds from the estimated \hat{c}_i 's and applies the definition of the regions (4).

- 3) **Refinement of the parameter vector estimates.** The final step consists in recomputing the parameter vectors as

$$\hat{\theta}_i = \left(\sum_{x_k \in \mathcal{X}_i} \begin{bmatrix} x_k \\ 1 \end{bmatrix} \begin{bmatrix} x_k \\ 1 \end{bmatrix}^\top \right)^{-1} \sum_{x_k \in \mathcal{X}_i} \begin{bmatrix} x_k \\ 1 \end{bmatrix} y_k. \quad (11)$$

III. DESIGN OF EXPERIMENT FOR IDENTIFICATION

A. System setup

The measurements for model parameterization as well as for validation were carried out on a highly dynamical engine test bench at the Johannes Kepler University in Linz see Fig. 1. The considered test candidate was a 4 cylinder 7 liter heavy duty off-road Diesel engine. This engine was equipped with a common rail injection system, an external exhaust gas recirculation and a waste gate turbocharger. For the stationary NOx analyses a Horiba Mexa 7100 was used and for the dynamical analyses a Cambustion fNOx400 (with a response time in the range of 10ms). Hence it was possible to detect also transient NOx peaks. Several tests also included variations of the engine air system control inputs, which were done with an engine control unit (ECU) bypass and the use of a dSpace rapid prototyping system. The data acquisition was done with the dSpace system, too.

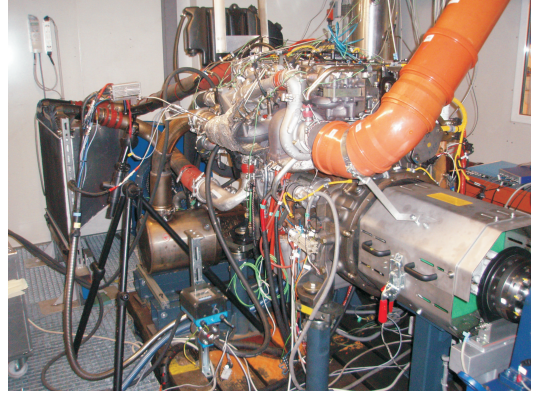


Fig. 1. Dynamical Engine test bench at the Johannes Kepler University Linz

B. Experiments for the data generation

In order to avoid large amounts of measurement data a Design of Experiment (DoE) was designed to excite all fundamental engine inputs which mainly influences the NOx raw emissions of the Diesel engine. Hence, this DoE shown in [19] was used to guarantee the required optimal setup of the engine inputs by minimizing the determinant of the covariance matrix in a D-optimal [19] way.

In principle, the applied set points (Table I) could be used directly to determine a data based model of the target quantities. However, it was more reliable to use actual quantities for the modeling. As each set point affect different physical quantities, we have the choice among a large set of regressors for the control structure. For the actual case it is also highly advisable to use quantities which are available in a conventional vehicle control, defined by the ECU.

Based on the data generated by DoE, seven ECU signals were selected for the identification of the NOx model, consisting of the engine speed (n), the injected fuel amount ($m_{f,tot}$), the air mass flow (MAF), the intake manifold pressure (MAP), the exhaust oxygen concentration ($O_{2,exh}$), the angle of the main injection (φ_{MI}) and the rail pressure (p_{rail}) (as depicted in Fig. 2).

TABLE I
INPUTS FOR DoE AND THE SIMULATION MODEL.

DoE input/Set points	Engine Output/NOx Model Input
Engine speed (n)	Engine speed (n)
Accelerator pedal position (α)	Injected fuel amount ($m_{f,tot}$)
EGR valve position (X_{EGR})	Mass air flow (MAF)
Waste gate position (X_{WG})	Intake manifold absolute pressure (MAP)
Angle of the main injection (φ_{MI})	Angle of the main injection (φ_{MI})
Rail pressure (p_{rail})	Rail pressure (p_{rail})
	Oxygen concentration ($O_{2,exh}$)

Fig. 2 shows the structure which is used for the identification of the NOx model. First, the DoE excitation with 6 inputs is created and then these signals are used as inputs for the engine via a bypass system which finally results in the measured 7 ECU signals which are used for the identification of the NOx emissions. Additionally to the already mentioned inputs, y specifies the measured NOx emissions and θ the estimated parameters of the NOx model.

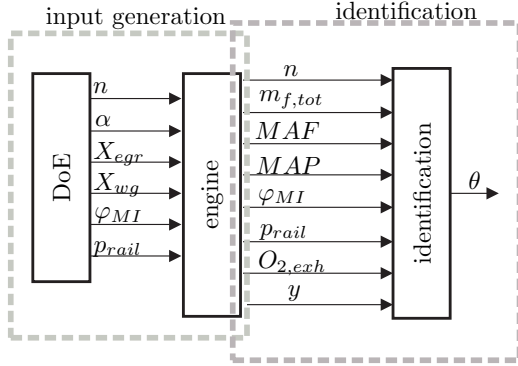


Fig. 2. **Identification structure.** DoE excitation with 6 inputs and 7 measured ECU outputs of the engine used for identification.

In Table II the limits for the DoE excitation specified according to the limits of the test bench are shown.

TABLE II
VARIATION OF THE INPUTS DELIVERED BY THE DOE.

u1: α [10-100%]	u4: WG[0-10%]
u2: speed [1250 – 1950rpm]	u5: P_{rail} [nominal \pm 200bar]
u3: EGR [1.2 \times nominal – 0%]	u6: φ_{MI} [nominal \pm 2deg]

In total 500 points each with a length of 10s have been

specified as set points for the engine which is necessary to make sure that all measured variables reach their steady state behavior.

Additionally to the steady state behavior also the dynamic responses are used to identify the model because they excite the engine in a different way than the steady state operation points and improve the tracking quality of high dynamical NOx peaks during transient driving situations.

IV. RESULTS

In this section the results obtained by applying the proposed identification method to the NOx estimation task are shown. To measure the performance of the identified model, three well known criteria, namely the Variance-Accounted-For (VAF), the FIT value and the integral error

$$\begin{aligned}
 \text{FIT} &= \left(1 - \frac{\|\hat{\mathbf{y}} - \mathbf{y}\|_2}{\|\mathbf{y} - \bar{y}\mathbf{1}_N\|_2} \right) \times 100\% \\
 \text{VAF} &= \left(1 - \frac{\text{var}(\mathbf{y} - \hat{\mathbf{y}})}{\text{var}(\mathbf{y})} \right) \times 100\% \quad (12) \\
 e_{int}(k) &= \frac{\sum_{k=1}^T (y_k - \hat{y}_k)}{\sum_{k=1}^T y_k}
 \end{aligned}$$

are used, where \mathbf{y} and $\hat{\mathbf{y}}$ are vectors formed by the measured output and the model output respectively. \bar{y} is an average of \mathbf{y} , $\mathbf{1}_N$ is a N-dimensional vector with all entries equal to one, T is the duration of the test cycle.

Besides FIT and VAF also the integral error is used for the NOx value estimation, because it is a sensible value for the failure after one complete driving cycle, which is also the reference for the diesel emission legislation, and thus a good prediction value to characterize the quality of the approach.

A. Estimation

For identification purpose, around 25000 experimental data samples are collected from the Diesel engine. As mentioned earlier, the input sequences are generated by the DoE method. The estimated PWA model consists of 2 affine submodels, each of them has the orders $n_a = 2$, $n_b = 1$, $n_k = 0$. The choice of the amount and the orders of the submodels were carried out based on the minimal model error between the estimation and the measurement. Therefore also the weighting factor W was chosen accordingly with a 100 times higher rating for the model error than for the error between the actual center and the regression vector, shown in (7).

Fig. 3 shows a comparison between the estimated $\hat{\mathbf{y}}$ and the measured output \mathbf{y} . As illustrated in the first subplot of Fig. 3 the NOx values are scaled to the maximum occurring value, which is consequently done for all additional figures. The second plot of Fig. 3 shows the integral error of the identification. If this error is analyzed in more detail it turned out that an adequate approximation with errors less than 2% are achievable. Also the two other specified fitting values with a VAF of 75% and a FIT of 58% show promising results in relation to the already in literature proposed models [9]. In addition to that also a part of the imprecision is

system-inherent, as the engine behavior is strongly affected by operating conditions like temperature, humidity etc. which cannot be kept completely constant during the whole test cycle.

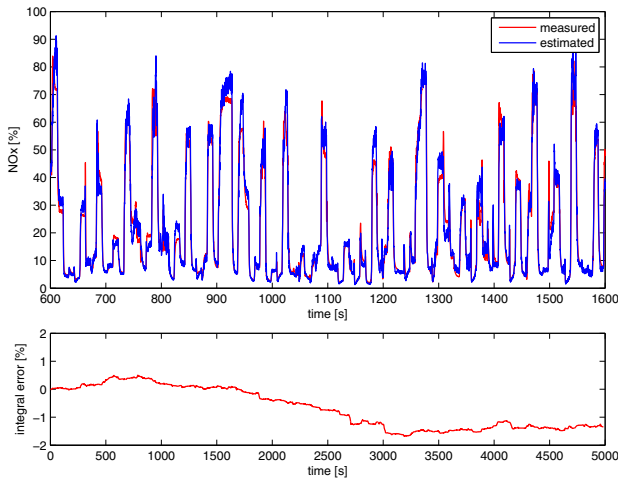


Fig. 3. Estimation characteristics of NOx emission identification for the DoE

B. Validation

The PWA model is finally validated on a test cycle of a wheel loader which represents a very challenging application for an off-road HD engine. The “wheel-loader cycle“ consists, as depicted in Figure 4, of a highly dynamic part during the first 70 seconds and a quasi-stationary part between 70 and 140 seconds. Thus, it is challenging to predict the dynamical as well as the static behaviour of NOx emission. Moreover, Fig. 4 illustrates the comparison between the estimated and measured NOx values. The evaluated PWA model describes the quasi-stationary behavior well and shows an adequate dynamical behavior even if not all of the peaks are reproduced correctly. Hence, this kind of virtual sensor can be already used for after treatment systems, because peaks of the NOx emissions are due to the SCR storage volume less important than the cumulative amount of the NOx emissions which can be represented by the integral error. The corresponding values for the validation of the previous defined criteria are a VAF of 69%, a FIT of 55% and the integral error of about 2%, which is shown in the second subplot of Fig. 4.

C. Model Parameters

An interesting comparison of the two used PWA models is shown in Table III. In this table the different centers \bar{c}_i as well as the corresponding parameter vectors θ_i are shown. Although some similarities for the different parameters can be detected (i.e. $u_{t-1}(\varphi_{MI})$) also large varieties (i.e. $u_{t-1}(MAF)$) appear, which may lead to an intuitive guess which input has the biggest impact on the NOx emissions and the centers as well. Nevertheless, due to the different variation ranges of the inputs and the complex physical

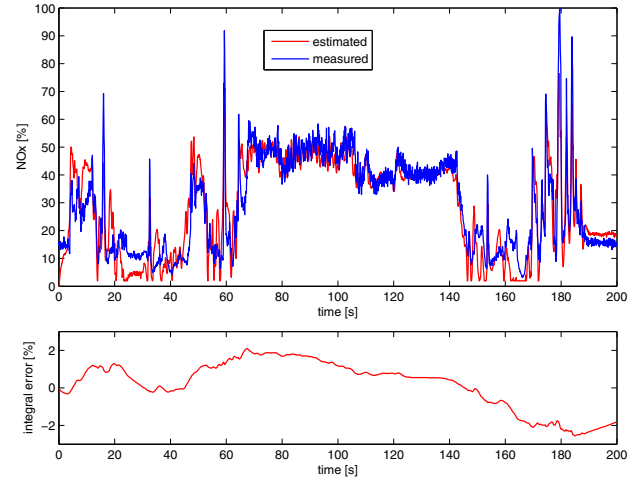


Fig. 4. Validation characteristics of NOx emission for the wheel-loader cycle

process behind the NOx formation in the Diesel engine, neither the real effects of the different parameters nor a real physical connection can be evaluated based on these models.

TABLE III
PARAMETERS OF THE PWA MODELS SUBMODEL 1: $\{ \bar{c}_1, \theta_1 \}$,
SUBMODEL 2 $\{ \bar{c}_2, \theta_2 \}$

Regressors	\bar{c}_1	θ_1	\bar{c}_2	θ_2
y_t	210.79	-	246.23	-
y_{t-1}	211.02	1.03	246.00	0.79
y_{t-2}	211.22	-0.089	245.66	0.090
$u_t(n)$	2613.1	-0.06	1596.50	0.01
$u_t(m_{f,tot})$	97.10	1.43	107.25	2.49
$u_t(MAF)$	2.14	-194.26	2.19	-660.73
$u_t(MAP)$	10.80	18.47	10.32	23.42
$u_t(\varphi_{MI})$	1495.40	0.03	1494.40	0.06
$u_t(p_{rail})$	5.51	13.99	5.68	8.98
$u_t(O_{2,rail})$	0.92	376.2	0.96	865.54
$u_{t-1}(n)$	1612.80	0.059	1596.80	-0.04
$u_{t-1}(m_{f,tot})$	97.89	-1.25	107.31	-2.21
$u_{t-1}(MAF)$	2.14	184.95	2.19	596.55
$u_{t-1}(MAP)$	10.82	-17.08	10.30	-19.46
$u_{t-1}(\varphi_{MI})$	1495.00	-0.03	1494.50	-0.03
$u_{t-1}(p_{rail})$	5.51	-13.90	5.69	-6.61
$u_{t-1}(O_{2,rail})$	0.92	-364.89	0.96	-687.08
1	-	-23.23	-	-88.58

D. Virtual versus conventional sensors

Another advantage of such an approach is the relative speed of the virtual sensor compared to the rather slow commercially available sensor (the Smart NOx sensor) and the fast Cambustion 400 measurement system. Indeed, Fig.5 shows the different behavior after a sudden change of the engine inputs. Whilst all three measurement systems have comparable steady state behavior, the SmartNOx sensor has a very slow dynamical response, the Cambustion is significantly better than the NOx sensor, but only the virtual sensor captures the whole dynamics of the change.

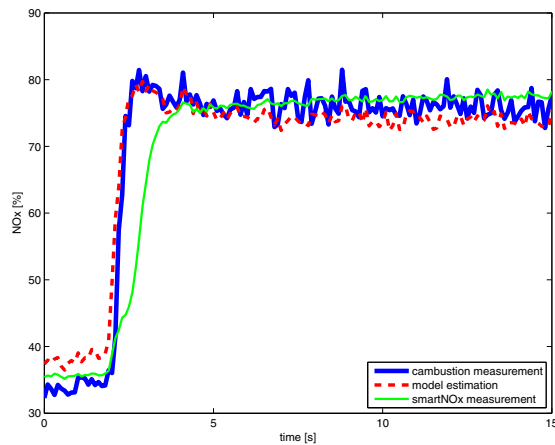


Fig. 5. Comparison of the Smart NOx measurement with the combustion measurement and the virtual NOx sensor estimation.

V. CONCLUSION

In this work the mathematical modeling of the NOx emissions produced by a Diesel engine process has been studied. For this purpose, a piecewise affine model have been proposed. The obtained algorithm relies essentially on a partitioning of the regression space into a finite number of subsets each associated with a single affine submodel.

Considering the fact that an identification of NOx emission is still a challenging practical problem, new identification algorithms to solve the assignment can be beneficial. In particular, the proposed PWA model has a number of properties such as simplicity, linearity and fast reaction times compared to accessible sensor concepts. This quality assessment compared to other approaches suggests the interest in further pursuing the PWA approach. Despite the usability of the proposed model for the after treatment control further improvement of the approach is required.

As next step, further investigations of the prediction quality but also at the control of an exhaust after treatment system based on the presented virtual NOx sensor concept will be carried out. As far as the prediction quality is considered, examinations on the choice of the model structure and the clusterisation algorithm will be executed to achieve higher FIT values. In addition to that the design of the experiments (e.g. including humidity variations and relevant temperatures) will be improved to minimize uncertainties and attain a better NOx emission model.

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