2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

High order integral sliding mode control with gain adaptation

M. Taleb, F. Plestan, and B. Bououlid

Abstract— **In this paper, an adaptive integral sliding mode is proposed. The main interest of gain adaptation is the reduction of the chattering and the possibility to control uncertain nonlinear systems whose the uncertainties have unknown bounds. The proposed control approach consists in using dynamically adapted control gain that ensure the establishment, in a finite time, of a real high order sliding mode. The control is applied by simulation to an academic example to evaluate its efficiency.**

keywords : High order sliding mode, adaptive control, integral sliding mode

I. Introduction

The objective of this paper is to propose a new control strategy based on high order sliding mode theory. This latter theory emerges at the beginning since 30 years [11], [6], [12], [13], [9], [10], [5], [15] and allows to reduce the chattering phenomenon [6], [23], [8], [3], [4], which is the wellknown drawback of the sliding mode theory, by applying the discontinuous sign-function on high order time derivative of the sliding variable, and eventually on time derivative of the control input. An other and recent way to reduce chattering appears by using adaptive gains in the controller. The gain adaptation strategy has also an other advantage : in fact, the sliding mode control design requires the knowledge of uncertainties/ perturbations bounds which are difficult to find in many practical applications. It yields an over-estimation of control gain with respect to uncertainties bound that amplifies the chattering problem. In order to overcome such a constraint and eliminate the need of a priori knowledge of uncertainties upper bound, controllers with dynamical gains have been developed. The interest is the adaptation of the gain magnitude with respect to uncertainty/perturbation effects. Then, a reduced gain induces lower chattering. In [16], an adaptive version of first order sliding-mode position controller was developed for a pneumatic actuator. The idea is to dynamically adapt the control gain, checking all the time whether a sliding mode is established or lost. It has been proved in [16] that the proposed method allows to adapt the gain magnitude to the uncertainties/perturbations. As a result, the gains are reduced as well as the chattering effect. As the reduction of chattering is the major objective, the solution consists in combining gain adaptation with high-order sliding-mode control. This strategy has been successfully applied for the control of an electropneumatic actuator [17].

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The adaptation strategy has been recently extended to two well-known second order sliding mode controller, i.e. the twisting [1] and the super-twisting algorithms [19]. Note that these both control strategies have been already applied to experimental systems [20], [19], [18]. Furthermore, these controllers do not require full informations on the bounds of uncertainties and perturbations. This last point relaxes the identification process.

In this paper, the objective is to extend the previous results of gain adaptation to high order sliding mode control. The controller is based on the integral sliding mode concept [22], this concept being already used for high order sliding mode controller [5], [10], [14]. In [5], the authors are using an homogeneous control law as the so-called "nominal" control whereas, in [10], an optimal controller has been chosen for this "nominal" part. The main feature of these both "nominal" control laws is the finite time convergence. The basic idea is that this "nominal" control can be viewed as a desired trajectories generator : the closed-loop system is forced, thanks to the discontinuous control, to track these trajectories. Given that these trajectories converge to the origin in a finite time, a high order sliding mode can be established.

Section II recall some definitions and states the problem of high order sliding mode control of an uncertain nonlinear system as the stabilization of a perturbed chain of integrators. Section III recall some results on homogeneous controller and a constant-gain high order integral sliding mode [5] : this latter result is modified in order to give an adaptive version, which is the main result of the paper. This adaptive version of the integral sliding mode controller is applied, by simulation, to an academic example in Section IV.

II. Problem formulation

Consider the following nonlinear uncertain system

$$\begin{array}{rcl} \dot{x} & = & f(x,t) + g(x,t)u \\ y & = & \sigma(x,t) \end{array} \tag{1}$$

with $x \in I\!\!R^n$ the state vector, $u \in I\!\!R$ the control input, and $\sigma(x) \in I\!\!R$ a smooth output function (sliding variable). f and g are uncertain smooth vector fields and are differentiable. The uncertainties in $f(x,t)$ and $g(x,t)$ are due to parameter variations, unmodelled dynamics or external disturbances.

Assumption 1. The relative degree r of system (1) with respect to σ is constant and known, and the associated zero dynamics are stable. ■

The r^{th}order sliding mode is defined through the following definition

Definition 1 ([12]): Consider the nonlinear system (1) and the sliding variable σ. Assume that the time derivative $\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}$ are continuous functions. The manifold defined as

$$\Sigma^r = \{x \mid \sigma(x,t) = \ldots = \sigma^{(r-1)}(x,t) = 0\}$$

is called "r^{th}-order sliding mode set", which is non-empty and is locally an integral set in the Fillipov sens [7]. The motion on Σ^r is called 'r^{th}-order sliding mode" with respect to the sliding variable σ. ■

One can introduce also the notion of "real" sliding mode which is directly connected to real systems, in particular due to the presence of sampling period for the computation of control law.

Definition 2 ([12]): Consider the nonlinear system (1) and the sliding variable σ. Assume that the time derivative $\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}$ are continuous functions. The manifold defined as

$$\Sigma_*^r = \{x||\sigma| \leq \mu_0\tau^{r-1}, |\dot{\sigma}| \leq \mu_1\tau^{r-1}, \cdots, |\sigma^{(r-1)}| \leq \mu_{r-1}\tau\}$$

with $\mu_i \geq 0$ (with $0 \leq i \leq r-1$), is called "real r^{th}-order sliding mode set", which is non-empty and is locally an integral set in the Fillipov sens [7]. The motion on Σ_*^r is called "real r^{th}-order sliding mode" with respect to the sliding variable σ. ■

Given the form of system (1), the r^{th}-order sliding mode control (SMC) approach allows the finite time stabilization to zero of the sliding variable σ and its $(r-1)$ first time derivatives by defining a suitable discontinuous control function. The r^{th} time derivative of σ satisfies the equation [1]

$$\sigma^{(r)} = \psi(x,t) + \varphi(x,t)u \tag{2}$$

Assumption 2. Solutions of equation (2) with discontinuous right-hand side are defined in the sense of Fillipov [7]. ■

Assumption 3. Functions ψ and φ are smooth uncertain but bounded functions; furthermore, they can be partitioned into a well-known nominal part (respectively $\bar{\psi}$ and $\bar{\varphi}$) and an uncertain bounded one (respectively $\Delta\psi$ and $\Delta\varphi$), *i.e.*

$$\begin{array}{rcl} \psi(x,t) & = & \bar{\psi}(x,t) + \Delta\psi(x,t) \\ \varphi(x,t) & = & \bar{\varphi}(x,t) + \Delta\varphi(x,t) \end{array} \tag{3}$$

Functions φ and $\bar{\varphi}$ are such that $\varphi > 0$ and $\bar{\varphi} > 0$. There are an upper bound constant ξ and *a priori known* constant $0 < \gamma \leq 1$ such that the uncertain functions satisfy the following inequalities

$$\left|\frac{\Delta\varphi(x,t)}{\bar{\varphi}(x,t)}\right| \leq 1-\gamma, \qquad |\Delta\psi(x,t)| \leq \xi \tag{4}$$

■

1. All over this paper, $\sigma(\cdot)^{(k)}$ ($k \in I\!N$) denotes the k^{th} time derivative of the function $\sigma(\cdot)$. This notation is also applied for every function.

To summarize, the design of a r^{th}-order SMC of (1) with respect to the sliding variable σ is equivalent to the finite time stabilization of the uncertain system $(1 \leq i \leq r-1)$

$$\begin{array}{rcl} \dot{z}_i & = & z_{i+1} \\ \dot{z}_r & = & \psi(x,t) + \varphi(x,t)u \end{array} \tag{5}$$

with $z = [z_1 \; \ldots \; z_r]^T = [\sigma \; \ldots \; \sigma^{(r-1)}]^T$. Consider the following state feedback control

$$u = \frac{1}{\bar{\varphi}}\left(-\bar{\psi} + v\right) \tag{6}$$

with v the auxiliary control input. Note that this state feedback control linearizes (by an input-output point-of-view) the nominal system, *i.e.* system (5) with no uncertainties. Applying (6) to system (5), one gets

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 \\ & \vdots & \\ \dot{z}_{r-1} & = & z_r \\ \dot{z}_r & = & \Delta\psi - \dfrac{\Delta\varphi}{\bar{\varphi}}\bar{\psi} + \left(1 + \dfrac{\Delta\varphi}{\bar{\varphi}}\right)v \end{array} \tag{7}$$

The control objective is now the following : how to define a discontinuous control law v ensuring the stabilization of the previous system, in a finite time and in spite of the uncertainties ?

III. Control design

This section proposes two high order sliding mode controllers based on integral sliding mode concept [10] : this first requires knowledge of the uncertainties bounds, whereas, for the second one, no knowledge of the bounds is required. This latter feature is due to an adaptation law for the control gain.

A. Finite time stabilization of an integrator chain system [5]

The following theorem proposes a continuous finite time stabilizing feedback controller for a chain of integrators, by giving an explicit construction involving a small parameter. One gets an asymptotically stable closed-loop system; the system is homogeneous of negative degree with respect to a suitable dilation which implies the finite time stability. Consider the system (7) with no uncertainty ($\Delta\psi = 0$ and $\Delta\varphi = 0$)

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 \\ & \vdots & \\ \dot{z}_{r-1} & = & z_r \\ \dot{z}_r & = & v \end{array} \tag{8}$$

Theorem 1 ([5]): Let $k_1, \ldots, k_r > 0$ be such that the polynomial $\lambda^r + k_r\lambda^{r-1} + \ldots + k_2\lambda + k_1$ is Hurwitz. There exists $\epsilon \in]0,1[$ such that, for every $\alpha \in]1-\epsilon, 1[$, the origin is a globally finite time stable equilibrium point for system(8) under the feedback

$$v = -k_1\text{sign}(z_1)|z_1|^{\alpha_1} - \ldots - k_r\text{sign}(z_r)|z_r|^{\alpha_r} \tag{9}$$

with $\alpha_1, \ldots, \alpha_{r-1}$ satisfy

$$\alpha_{i-1} = \frac{\alpha_i\alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}$$

for $i = 2, \ldots, r$ with $\alpha_r = \alpha$ and $\alpha_{r+1} = 1$. ■

B. Robust finite time controller design based on integral sliding mode

Consider the following function, named "integral sliding variable", defined as (t_0 being the initial time)

$$s(z(t)) = z_r(t) - z_r(t_0) - \int_{t_0}^{t} v_{nom}(\tau)d\tau \qquad (10)$$

with the term v_{nom} defined by (9) in Theorem 1. Note that $s(z(t_0)) = 0$: then, the system is evolving on the sliding manifold early from the initial time.

This latter feature is a key point of the integral sliding mode controller ; in fact, the definition of the integral sliding variable allows to ensure that a sliding mode is established early from the initial time, thanks to the finite time convergence property of v_{nom}. Then, it is necessary to force the system to evolve on the integral sliding surface $s = 0$ in spite of the uncertainties and perturbations : it will be the role of the discontinuous part of the controller. In fact, the therm v_{nom} appearing in s can be viewed as a desired trajectory generator. By supposing that, $\forall t \geq t_0$, $s = 0$, one has

$$\dot{s} \;=\; \dot{z}_r - v_{nom} = 0 \quad \rightarrow \quad \dot{z}_r = v_{nom} \qquad (11)$$

From the previous inequality, it is clear that, if the control v guarantees that $s = 0$, $\forall t \geq t_0$ and given the features of v_{nom}, system (7) is stabilized at the origin in a finite time.

Then, in order to stabilize system (7), the following control law is defined

$$v = v_{nom} - K\text{sign}(s) \qquad (12)$$

This controller has two parts

– the first one v_{nom}, named "ideal control", is continuous and stabilizes (7) at the origin when there are no uncertainties. This controller is also used in order to generate the ideal trajectories of the system ;

– the second part, $-K\text{sign}(s)$, provides the complete compensation of uncertainties and perturbations, and ensures that control objectives are fulfilled, where the gain is satisfying

$$K \;\; > \;\; \frac{(1-\gamma)(|v_{nom}| + |\bar{\psi}|) + \xi + \eta}{\gamma}. \qquad (13)$$

Theorem 2: [24] Consider the nonlinear system (1) and assume that Assumptions 1-3 are fulfilled. Then, if the gain K fulfills the condition (13), the control law

$$u = \bar{\varphi}^{-1}(x,t)(-\bar{\psi}(x,t) + v_{nom} - K\text{sign}(s)) \qquad (14)$$

ensures the establishment of a r^{th} order sliding mode versus the sliding variable σ, *i.e.* the trajectories of system (2) converge to zero in finite time. ■

Proof. Choose the following Lyapunov function

$$V(s) \;\; = \;\; \frac{1}{2}s^2 \qquad (15)$$

It yields

$$\begin{aligned}
\dot{V} &= s\dot{s} \\
&= s\Big[\Big(1 + \tfrac{\Delta\varphi(x,t)}{\bar{\varphi}(x,t)}\Big)v - \tfrac{\Delta\varphi(x,t)}{\bar{\varphi}(x,t)}\bar{\psi}(x,t) \\
&\quad +\Delta\psi(x,t) - v_{nom}\Big] \\
&= s\Big[-\Big(1 + \tfrac{\Delta\varphi(x,t)}{\bar{\varphi}(x,t)}\Big)K\text{sign}(s) - \tfrac{\Delta\varphi(x,t)}{\bar{\varphi}(x,t)}\bar{\psi}(x,t) \\
&\quad +\Delta\psi(x,t) + \tfrac{\Delta\varphi(x,t)}{\bar{\varphi}(x,t)}v_{nom}\Big] \\
&\leq K|s| + (1-\gamma)K|s| + (1-\gamma)|\bar{\psi}(x,t)| + \xi \\
&\quad +(1-\gamma)|v_{nom}| \\
&\leq -\gamma K|s| + \big[(1-\gamma)(|v_{nom}| + |\bar{\psi}(x,t)|) + \xi\big]\,|s| \\
&\leq -\eta|s|
\end{aligned} \qquad (16)$$

Equation (16) implies that, under the condition (13) on the gain K, the manifold $\{x \in \mathbb{R} | s = 0\}$ is attractive ; given that, at the initial time, $s = 0$, it means that system trajectories are evolving on the manifold for $t \geq 0$. Substituting (12) into (7), one gets the equivalent closed-loop dynamics, in sliding mode, similar as the nominal system (8). Given that the control law v ensures the stabilization around the origin of the uncertain system (7), a r^{th} order sliding mode with respect to the sliding variable σ is then established for the system (1).

C. Robust finite time adaptive gain design

The HOSM controller developed in the previous section requires, for its design, the knowledge of the bounds of ξ and γ ; the problem is that these latter are often not easy to precisely determine. Then, these bounds are often overestimated, which engender large gain. A large gain induces a larger chattering phenomenon. Then, there is a real interest to reduce the controller gain, in order to attenuate the chattering.

In the sequel, an adaptive gain law is proposed in order first to reduce chattering problem. This approach has to also allow to guarantee precise tracking. Furthermore, an objective is to avoid the knowledge of the perturbations/uncertainties bounds. Then, the adaptive gain law for controller (12) reads as

$$\dot{K} \;\; = \;\; \begin{cases} \rho|\delta|^{\frac{1}{4}}K\text{sign}(\delta) & \text{if} \quad K > K_m \\ \eta_K & \text{if} \quad K \leq K_m \end{cases} \qquad (17)$$

with $K_m > 0$ constant chosen arbitrarily small, $\rho > 0$ a positive adaptation parameter and δ the adaptation criterion chosen as follows

$$\delta = |\sigma| + \tau|\dot{\sigma}| + \ldots + \tau^{r-1}|\sigma^{(r-1)}| - \mu\tau^r \qquad (18)$$

with some $\mu > 0$, and τ the sampling time. Note that, if $\delta \leq 0$, it yields

$$|\sigma| \leq \mu_0\tau^r, |\dot{\sigma}| \leq \mu_1\tau^{r-1}, \cdots, |\sigma^{(r-1)}| \leq \mu_{r-1}\tau \qquad (19)$$

with $\mu_i \geq 0$ (with $0 \leq i \leq r-1$). If the r previous inequalities are fulfilled, it means that a "real" sliding mode is established with respect to σ. The parameter η_K in gain adaptation law (17) is introduced to guarantee only positive values of the gain. Obviously $\eta_K > 0$.

Theorem 3: Consider the system (1). Given the sliding variable σ chosen to satisfy the control objective, the control law (14) and the adaptation law (17)-(18), then a real r^{th} order sliding mode with respect to the sliding variable σ will be established in finite time, *i.e.*

$$|\sigma| \leq \mu_0 \tau^r, |\dot{\sigma}| \leq \mu_1 \tau^{r-1}, \cdots, |\sigma^{(r-1)}| \leq \mu_{r-1}\tau \quad (20)$$

with $\mu_i \geq 0$ (with $0 \leq i \leq r-1$). ■

Proof. Choose a Lyapunov function candidate as

$$V(s) = \frac{1}{2}s^2 + \frac{1}{2}(K - K^*)^2 \quad (21)$$

where K^* is the upper bound of the gain K. It's obvious that the gain is bounded since the uncertainties are bounded. The first time derivative of V

$$\begin{aligned} \dot{V} &= s\dot{s} + \dot{K}(K - K^*) \\ &= s\dot{s} + \rho|\delta|^{\frac{1}{4}} K \text{sign}(\delta)(K - K^*) \end{aligned} \quad (22)$$

with

$$\begin{aligned} s\dot{s} &= -K(1 + \frac{\Delta\varphi}{\bar{\varphi}})|s| + \left[\frac{\Delta\varphi}{\bar{\varphi}}(v_{nom} - \bar{\psi}) + \Delta\psi\right] s \\ &\leq -\gamma K|s| + \left[(1-\gamma)(|v_{nom}| + |\bar{\psi}(x,t)|) + \xi\right] |s| \end{aligned} \quad (23)$$

Consider now the following two cases

Case 1. Suppose that $\delta > 0$, which means that the r^{th} order sliding mode with respect to σ is not yet established. According to adaptation law (17), the gain K will increase until the condition (13) is verified. Then

$$s\dot{s} \leq -\eta_1 |s| \quad (24)$$

It yields

$$\begin{aligned} \dot{V} &\leq -\eta_1|s| - \rho\delta^{\frac{1}{4}} K_m |K - K^*| \\ &\leq -\eta_1|s| - \eta_2|K - K^*| \\ &\leq -\eta\left(\frac{s^2}{2} + \frac{|K - K^*|^2}{2}\right)^{\frac{1}{2}} \\ &\leq -\eta V^{\frac{1}{2}} \end{aligned} \quad (25)$$

where $\eta_2 = -\rho|\delta|^{\frac{1}{4}} K_m$ and η=min$(\sqrt{2}\eta_1, \sqrt{2}\eta_2)$. Therefore, the finite time convergence to the domain $\delta < 0$ is guaranteed viewed that, in this case, the condition on K of Theorem 2 is fulfilled. Then, of course, the criterion (18) is satisfied in a finite time and $|\sigma| < \mu\tau^r$, $|\dot{\sigma}| < \mu\tau^{r-1}$ … and $|\sigma^{(r-1)}| < \mu\tau$. A "real" sliding mode with respect to σ is established.

Case 2. Suppose now that $\delta < 0$. The gain K will decrease according to adaptation law (17) : then, there can exist a time for which the condition (13) is violated. It comes that $\dot{V}$ would be sign indefinite, and then it becomes not possible to conclude on the closed-loop system stability. Therefore the δ can increase over 0. As soon as δ becomes greater than 0, the gain increases over (13) and $\dot{V} \leq -\eta V^{\frac{1}{2}}$ and so on.

IV. Example

Consider the following nonlinear system

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = \psi(t) + \varphi(t)u \end{cases} \quad (26)$$

with $\psi(t) = 10+2\sin(0.1t)$ and $\varphi(t) = (2+\cos(t))$. Suppose that the sliding variable is defined as $\sigma = z_1$, which gives that the relative degree of (26) versus σ equals 3 (Assumption 1 fulfilled). The objective is to stabilize, in a finite time, z_1, z_2 and z_3 in a vicinity of the origin. One has ψ and φ defined through nominal and uncertain parts (see (3)) such that

$$\begin{aligned} \bar{\psi}(t) &= 10, \qquad \Delta\psi(t) = 2\sin(0.1t) \\ \bar{\varphi}(t) &= 2, \qquad \Delta\varphi(t) = \cos(t) \end{aligned} \quad (27)$$

One gets

$$\frac{|\Delta\varphi(t)|}{\bar{\varphi}(t)} \leq \frac{1}{2} \quad \text{and} \quad |\Delta\psi(t)| \leq 2. \quad (28)$$

which gives that Assumption 3 is fulfilled given that

$$\gamma = \frac{1}{2} \quad \text{and} \quad \xi = 2. \quad (29)$$

Then, the control law is derived from (6) as

$$u = \frac{1}{2}(-10 + v) \quad (30)$$

with v reading as (12)

$$v = v_{nom} - K\text{sign}(s)$$

The "nominal" term of v is defined from (9)

$$v_{nom} = -\text{sign}(z_1)|z_1|^{\frac{1}{2}} - 1.5\text{sign}(z_2)|z_2|^{\frac{3}{5}} - 1.5\text{sign}(z_3)|z_3|^{\frac{3}{4}}$$

whereas the switching sliding variable is defined as (10)

$$s = z_3(t) - z_3(t_0) - \int_{t_0}^{t} v_{nom} d\tau$$

The gain K is tuned according to (17) with adaptation parameters chosen as follows (the tuning has been made in order to get good compromise between accuracy and smooth response)

$$\rho = 0.05, \ \mu = 2.10^6, \ K_m = 2, \ \eta_K = 5,$$

Simulations have been done with two different values of τ : $\tau = 10^{-4}s$ and $\tau = 0.5.10^{-4}s$.

V. Conclusion

In this paper a novel adaptive control based on an homogeneous control and an integral sliding mode was proposed. the homogeneous control drives the nominal part of the system to an r^{th} order sliding mode where the integral sliding mode control try to force the perturbed system to respond as the nominal part. the strategy was proved using a Lyapunov function and was evaluated on an academic example. The next step is to evaluate its efficiency on an electropneumatic actuator.

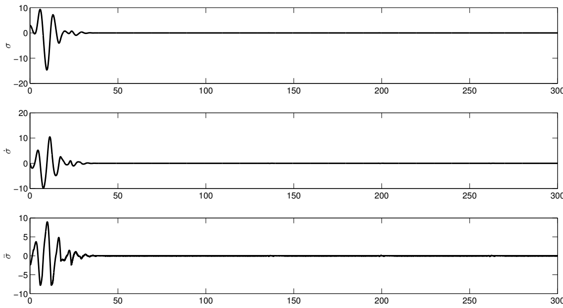

Fig. 1. σ, $\dot{\sigma}$ and $\ddot{\sigma}$ versus time (sec)($\tau = 0.1ms$)

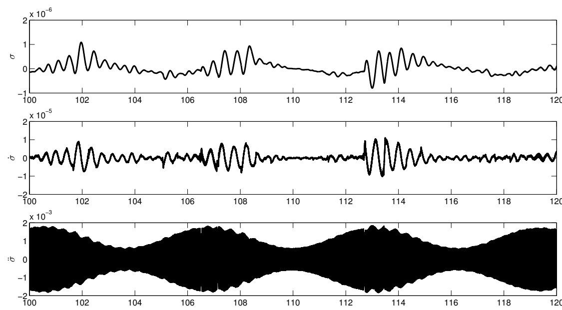

Fig. 2. σ, $\dot{\sigma}$ and $\ddot{\sigma}$ versus time (sec)(Zoom)($\tau = 0.1ms$)

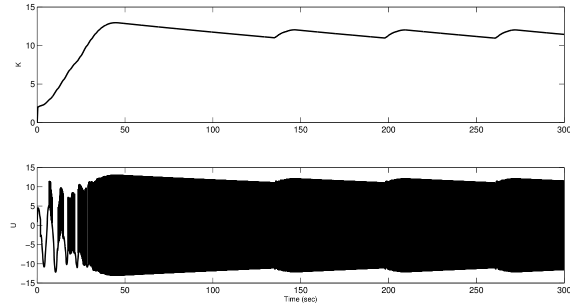

Fig. 3. Gain K and control input versus time (sec)($\tau = 0.1ms$)

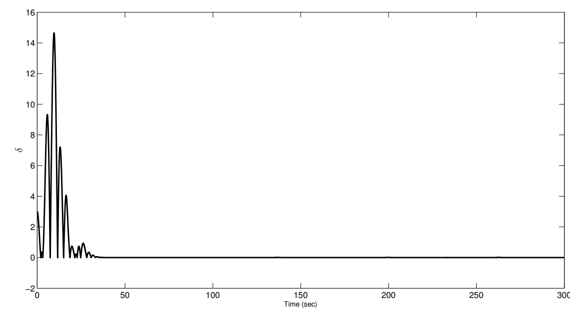

Fig. 4. δ versus time (sec)($\tau = 0.1ms$)

ACKNOWLEDGMENT

The work of Mohammed Taleb has been partially supported by CNRS PEPS grant "MUSCLAIR". Furthermore, the authors acknowledge the support of the ANR grant "ChaSliM" (ANR-11-BS03-0007).

REFERENCES

[1] Bartolini, G., Levant, A., Plestan, F., Punta, E., and M. Taleb,"Adaptation of sliding modes", IMA J. of Mathematical Control and Information, doi : 10.1093/imamci/dns0192012.

[2] Bhat, S.,Bernstein, D.,"Geometric homogeneity with applications to finite time stability", Math. Control, Signals Syst., 2005, 17, pp. 101-127

[3] Boiko, I., and Fridman, L., "Frequency domain analysis of second ordersliding mode", Proc. American Control Conference, Minneapolis, USA, 2006.

[4] Boiko, I."Discontinuous systems", Birkhauser, Boston, USA, 2008.

[5] Defoort, M., Floquet, T., Kokosy, A.,"Finite time control of a class of MIMO nonlinear systems using high order integral sliding mode control" Proc. 9th Int. Conf. on Variable Structure Systems, Alghero, Italy, 2006, pp. 133-138

[6] Edwards, C., and Spurgeon, S., "Sliding mode control, theory and application", Taylor and Francis, Bristol, UK, 1998.

[7] Filippov, A.,"Differential equations with discontinuous right hand side" (Kluwer Academic, Dordrecht, 1988)

[8] Fridman, L."Chattering analysis in sliding mode systems with inertial sensors", Int.J.Control, 2003, 76, pp. 906-912

[9] Laghrouche, S., Smaoui, M., Plestan, F., and Brun, X., "Higher order sliding mode control based on optimal approach of an electropneumatic actuator", International Journal of Control, vol.79, no.2, pp.119-131, 2006.

[10] Laghrouche, S., Plestan, F., Glumineau, A.," Higher order sliding mode control based on integral sliding mode" Automatica, 2007, 43, (3), pp. 531-537

[11] Levant, A. "Sliding order and sliding accuracy in sliding mode control", Int. J. Control,vol.58, no.6, pp.1247-1263, 1993.

[12] Levant, A., "Universal SISO sliding mode controllers with finite time convergence", IEEE Trans. Autom. Control, 2001, 46, (9), pp. 1447-1451

[13] Levant, A.,"Higher order sliding mode, differentiation and output feedback control", International J.Control, vol.76, pp.924-941, 2003.

[14] Levant, A., Alelishvili, L."Integral high-order sliding modes", IEEE Trans. Autom. Control, 2007, 52, (7), pp. 1278-1282

[15] Plestan, F., Glumineau, A., Laghrouche, S." A new algorithm for high order sliding mode control", Int. J. Robust Nonlinear Control, 2007, 18, pp. 441-453

[16] Plestan, F. Shtessel, Y. Brégeault, V. Poznyak, A."New methodologies for adaptive sliding mode control", International Journal of Control, vol.83, no.9, pp.1907-1919, 2010.

[17] Plestan, F., Shtessel, Y., Brégeault, V., and Poznyak, A., "Adaptive sliding mode control for a class of MIMO nonlinear systems : Applicationto an electropneumatic actuator", Control Engineering Practice, http ://dx.doi.org/10.1016/j.conengprac.2012.04.012, 2012

[18] Plestan, F., and Chriette, A., "A robust autopilot based on adaptive super-twisting algorithm for a 3DOF helicopter", Proc. IEEE Conference on Decision and Control, Maui, Hawaii, USA, 2012.

[19] Shtessel, Y., Taleb, M., and Plestan, F., "A novel adaptive-gain supertwisting sliding mode controller : methodology and application", Automatica, vol.48, no.5, pp.759-769, 2012.

[20] Taleb, M., Levant, A., and Plestan, F., "Electropneumatic actuator control : solutions based on adaptive twisting algorithm and experimentation", Control Engineering Practice, http ://dx.doi.org/10.1016/j.conengprac.2012.06.011, 2012.

[21] Utkin, V. I."Variable structure systems with sliding modes", IEEE Trans. Automat. Control, AC-22, 212-222

[22] Utkin, V., and Shi, J.,"Integral sliding mode in systems operating under uncertainty", Proc. IEEE Conference on Decision and Control, Kobe, Japan, 1996.

[23] Utkin, V.I., Guldner, J., Shi, J."Sliding modes in electromechanlical systems", Taylor and Francis, London, UK, 1999

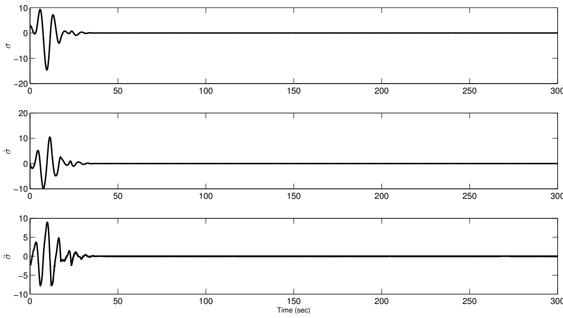

Fig. 5. σ, $\dot{\sigma}$ and $\ddot{\sigma}$ versus time (sec)($\tau = 0.05ms$)

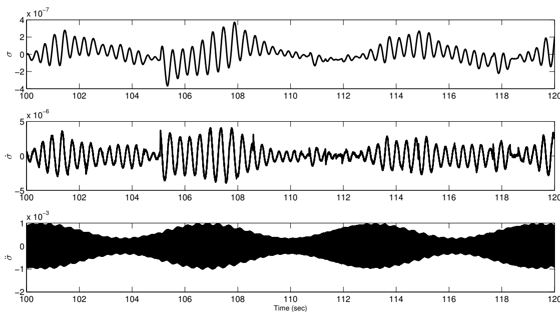

Fig. 6. σ, $\dot{\sigma}$ and $\ddot{\sigma}$ versus time (sec)(Zoom)($\tau = 0.05ms$)

[24] Zong, Q., Zhao, Z.-S., Zhang, J."Higher order sliding mode control with self-tuning law based on integral sliding mode", IET Control Theory Appl., 2010, Vol. 4, Iss. 7, pp. 1282-1289.

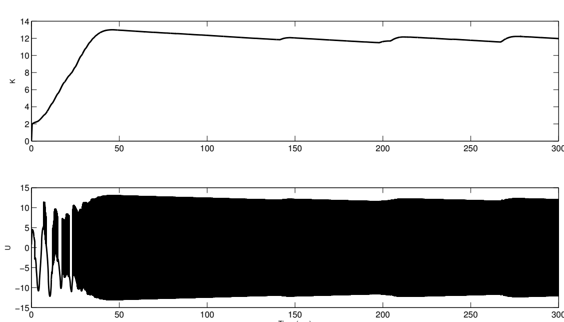

Fig. 7. Gain K and control input versus time (sec)($\tau = 0.05ms$)

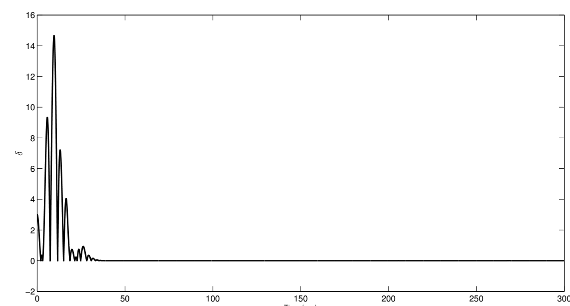

Fig. 8. δ versus time (sec)($\tau = 0.05ms$)