

Formation control design for car-like nonholonomic robots using the backstepping approach

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Abstract— In this paper we study the formation control problem for car-like robots which may be viewed as a more general type of robots than the usually considered unicycle type robots. We develop a controller using the backstepping approach and give conditions solving the formation control problem as well as the coordination control problem. We also present simulation results to demonstrate the applicability of the proposed controller.

I. INTRODUCTION

There has been considerable research done in the field of formation control of multiple unicycle mobile robots, see e.g. [2], [8], [13], [16], [18]. However, to our knowledge, only little work regarding formations of car-like nonholonomic mobile robots has been performed. Nonetheless, this is an important subject to study because it covers a more general type of robots than the unicycle-type robots and therefore, further research is essential. Arriving at this conclusion has instigated our research in this field.

In the scope of tracking control of a single car-like mobile robot, multiple results have been proposed in the literature, including using the backstepping approach [9], [10], dynamic feedback linearization [19] or control algorithm designed for general nonholonomic systems in chained form, see eg. [1], [12]. Similarly, existing algorithms to solve the formation control problem for a group of car-like mobile robots also utilize these techniques. For example, the control algorithms proposed in [3] apply to formation control of general nonholonomic systems; hence they can also be used for control of a formation consisting of car-like mobile robots. However, the disadvantage of this approach is that only constant formation shapes are allowed which arguably poses a considerable limitation. Therefore, in our work we allow for time-varying formation shapes. Other results on formation control of car-like mobile robots include [14] in which a control scheme based on the leader-follower strategy was proposed. In particular, the idea of the follower maintaining a desired distance and angle between itself and the leader was studied on the level of the robots' dynamics, as opposed to solely kinematics of the robots. Also [4] exploited the leader-follower scheme but in addition it also included some results in the realm of the behavioral approach to formation control. The strategy, as per principles of behavioral robotics, is such that individual behaviors like goal seeking

or obstacle avoidance are combined to create the overall formation control. However, clearly both approaches bear the usual disadvantages of the leader-follower and behavioral approach. These include the lack of explicit mathematical equations of motion in the behavioral approach which makes it difficult to analyze the behavior of the formation in a mathematical fashion. Furthermore, although leader-follower strategies are commonly straightforward mathematically, arguably they may be fault-prone due to the existence of group leaders. Thus, the whole formation may fail to execute its task if the leaders fail.

The formation control problem considered in this paper may be stated in short as the requirements for robots in the formation to create a given desired formation shape and then for the formation as a whole to track a given desired trajectory. We use in this paper the virtual structure approach [11] to fulfill this task. Following [11], the virtual structure is a geometric structure whose vertices are formed by the robots in the formation. The virtual structure is then supposed to track the given desired trajectory for the formation control problem to be solved. Since we allow for the formation shape to be time-varying, the virtual structure can also vary its shape in time.

Motivated by the rather insufficient attention being paid to the formation control problem for car-like mobile robots, the main contribution of the paper is the formation control algorithm employing the virtual structure approach, adapting the results in [15] for car-like robot formations. The controller design in this paper uses the backstepping technique to develop the control actions for robots in the formation and is based on a distributed inter-robot communication network hence limiting unnecessary burden on communication effort. Distributed communication networks are particularly desirable in practical applications where sensors have limited strength and thus communicating over a large area with many other agents may be infeasible.

The outline of the rest of the paper is as follows. In Section II we formulate the control problem studied in this paper. Then, in Section III the formation control algorithm for car-like mobile robots is designed. It is followed by a simulation study in Section IV and our concluding remarks in Section V.

II. PROBLEM FORMULATION

In what follows we consider the formation control problem in more depth. We consider a formation consisting of N car-like mobile robots with indices $i \in \mathcal{I}$ where $\mathcal{I} = \{1, \dots, N\}$. The kinematics of a car-like mobile robot with rear-wheel

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drive is assumed to be given by (cf. [1], [17]):

$$\dot{x}_i = v_i \cos \theta_i, \quad \dot{y}_i = v_i \sin \theta_i, \quad \dot{\theta}_i = \frac{v_i}{l} \tan \varphi_i, \quad \dot{\varphi}_i = \omega_i, \quad (1)$$

where the state vector is $q_i(t) = (x_i(t), y_i(t), \theta_i(t), \varphi_i(t))^T$ denoting Cartesian position of a midpoint of the rear axis $p_i(t) = (x_i(t), y_i(t))^T$, the heading angle of the robot $\theta_i(t)$ and the steering angle of the front wheel $\varphi_i(t)$. The control inputs are the forward velocity of the robot $v_i(t)$ and angular velocity of the front wheel $\omega_i(t)$, and l is the length of the robot. Having robot trajectories $q_i(t)$ at hand, the inputs $v_i(t)$ and $\omega_i(t)$ can be calculated as follows

$$v_i = \sqrt{(\dot{x}_i)^2 + (\dot{y}_i)^2},$$

$$\omega_i = l \frac{(\ddot{y}_i \dot{x}_i - \ddot{x}_i \dot{y}_i)((\dot{x}_i)^2 + (\dot{y}_i)^2) - 3(\dot{x}_i \ddot{x}_i + \dot{y}_i \ddot{y}_i)(\dot{y}_i \dot{x}_i - \dot{x}_i \dot{y}_i)}{((\dot{x}_i)^2 + (\dot{y}_i)^2)^{-1/2} \left(((\dot{x}_i)^2 + (\dot{y}_i)^2)^3 + l^2 (\dot{y}_i \dot{x}_i - \dot{x}_i \dot{y}_i)^2 \right)}. \quad (2)$$

The formation control problem relies on robots creating a desired, possibly time-varying, formation shape and tracking a given trajectory as a group. To this end, we follow the virtual structure approach. Therefore, we define the so-called virtual center as a certain central point for the formation, chosen according to the specifications of a particular application. We prescribe a desired trajectory for the virtual center to track to be $q_{vc}^d(t) = \text{col}(p_{vc}^d(t), \theta_{vc}^d(t), \varphi_{vc}^d(t)) = \text{col}(x_{vc}^d(t), y_{vc}^d(t), \theta_{vc}^d(t), \varphi_{vc}^d(t))$ such that the corresponding forward velocity v_{vc}^d and angular velocity ω_{vc}^d are bounded. The latter two may be calculated using expressions analogous to (2). Note that the desired trajectory needs to satisfy $\dot{x}_{vc}^d \sin \theta_{vc}^d - \dot{y}_{vc}^d \cos \theta_{vc}^d = 0$. We also define the desired time-varying formation shape with the aid of vectors $l_i^d(t) = \text{col}(l_{ix}^d(t), l_{iy}^d(t))$ such that $\frac{d}{dt}(l_i^d(t))$ are bounded, $\forall i \in \mathcal{I}$, that give desired Cartesian positions of each robot in reference to the virtual center. Consequently, we define desired trajectories for all individual robots in the formation as

$$p_i^d = p_{vc}^d + R(\theta_{vc}^d) l_i^d, \quad (3)$$

in which $p_i^d = (x_i^d, y_i^d)^T$, $\varphi_i^d = \text{atan}\left(l \frac{\dot{\theta}_{vc}^d}{v_{vc}^d}\right)$ and $R(\theta_{vc}^d)$ is a rotation matrix [17]. Moreover, we can calculate the desired forward and angular velocities associated with the desired trajectories (3) again using counterparts of (2). For the sake of completeness we include the resultant expressions for v_i^d and ω_i^d :

$$v_i^d = \sqrt{(\dot{x}_i^d)^2 + (\dot{y}_i^d)^2},$$

$$\omega_i^d = l \frac{(\ddot{y}_i^d \dot{x}_i^d - \ddot{x}_i^d \dot{y}_i^d)((\dot{x}_i^d)^2 + (\dot{y}_i^d)^2) - 3(\dot{x}_i^d \ddot{x}_i^d + \dot{y}_i^d \ddot{y}_i^d)(\dot{y}_i^d \dot{x}_i^d - \dot{x}_i^d \dot{y}_i^d)}{((\dot{x}_i^d)^2 + (\dot{y}_i^d)^2)^{-1/2} \left(((\dot{x}_i^d)^2 + (\dot{y}_i^d)^2)^3 + l^2 (\dot{y}_i^d \dot{x}_i^d - \dot{x}_i^d \dot{y}_i^d)^2 \right)}. \quad (4)$$

To accommodate for the singularity of the car-like mobile robot when $\varphi_i = \pm \frac{\pi}{2}$ for which the dynamics (1) become discontinuous, we state the following condition on desired trajectories of robots in the formation.

Assumption 1: All desired trajectories of robots satisfy $\varphi_i^d \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

We discuss in the sequel of this paper how the condition $\varphi_i \neq \pm \frac{\pi}{2}$ can also be guaranteed for robot trajectories.

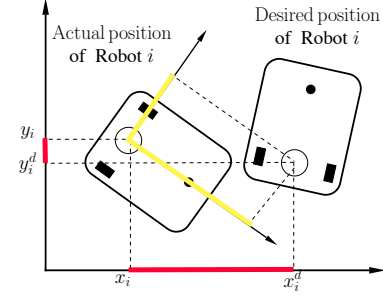


Fig. 1. Global e_i (red line) and local e_i^{xy} (yellow line) error variables of a robot.

In view of the above description of the formation control problem, we now define error variables between actual robot states and the desired states as ([6])

$$e_i^{xy} = R(-\theta_i) e_i, \quad \theta_i^e = \theta_i^d - \theta_i, \quad (i \in \mathcal{I}), \quad (5)$$

where $e_i = p_i^d - p_i$ is the tracking error in the global coordinate frame and e_i^{xy} is the tracking error variable in the robot-fixed coordinate frame, see Figure 1.

Differentiating (5) with respect to time leads to the following error dynamics

$$\dot{e}_i^{xy} = \dot{R}(-\theta_i) e_i + R(-\theta_i) \dot{e}_i$$

$$= \left(-\frac{v_i}{l} \tan \varphi_i \right) S e_i^{xy} + \begin{pmatrix} v_i^d \cos \theta_i^e - v_i \\ v_i^d \sin \theta_i^e \end{pmatrix}, \quad (6)$$

$$\dot{\theta}_i^e = \frac{v_i^d}{l} \tan \varphi_i^d - \frac{v_i}{l} \tan \varphi_i, \quad (7)$$

where S is the skew-symmetric matrix defined as

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \quad (8)$$

Note that we do not calculate the error dynamics of φ_i but instead we consider the dynamics of φ_i itself as in (1). In addition, in view of the earlier conditions on the feedforward velocities of the virtual center $v_{vc}^d(t)$ and $\omega_{vc}^d(t)$ and the desired formation shape coordinate $l_i^d(t)$ we conclude that the desired forward speed $v_i^d(t)$, for all $i \in \mathcal{I}$ is bounded. Assume moreover that $v_i^d(t)$ is bounded away from zero, for all t , i.e. there exists $\underline{v}_i^d > 0$ such that

$$v_i^d(t) \geq \underline{v}_i^d, \quad \forall t. \quad (9)$$

As mentioned above, the control objective is twofold. First, we require the formation as a whole to track a given trajectory. Secondly, we want robots in the formation to create a desired formation shape. Both control objectives are satisfied when robots in the formation track their individual trajectories. We now explicitly define what the condition for the formation shape maintenance is regardless of the trajectory tracking component in the spirit of the developments in [8], [15], [18]. In particular, we consider the formation to be maintained if robots create the given formation geometry, i.e. when $p_i(t) - p_{vc}(t) = R(\theta_{vc}(t)) l_i^d(t)$, for all $i \in \mathcal{I}$ for certain $p_{vc}(t)$ and $\theta_{vc}(t)$, or equivalently if $p_i^d(t) - p_i(t) = p_j^d(t) - p_j(t)$, for all $i, j \in \mathcal{I}$. Note that this allows for possible translations of the formation in the plane. However, no rotation or reflection of the whole formation is permitted.

Now, we define a coordination error between a pair of

robots i, j similarly to the developments in [8], [15], [18] to be

$$\sigma_{ij}(t) = e_i - e_j. \quad (10)$$

We then choose to redefine σ_{ij} so that the coordination error between robot i and its neighbours is expressed in the same coordinate system as the tracking error variables e_i^{xy} in (5). The reason why we have decided to use these transformed errors is that it proved to simplify the stability analysis. Having said that, error variables expressed in the world frame still are partly used in the sequel of this paper.

The coordination errors associated with robot $i \in \mathcal{I}$ expressed in the local coordinate frame of robot i is

$$\varepsilon_{ij}(t) = R^T(\theta_i)\sigma_{ij}(t) = e_i^{xy}(t) - R^T(\theta_i - \theta_j)e_j^{xy}(t). \quad (11)$$

Accordingly, taking into account (11) and (6) it can be demonstrated that ε_{ij} satisfies

$$\begin{aligned} \dot{\varepsilon}_{ij} = & \left(-\frac{v_i}{l} \tan \varphi_i\right) S\varepsilon_{ij} + \begin{pmatrix} v_i^d \cos \theta_i^e - v_i \\ v_i^d \sin \theta_i^e \end{pmatrix} \\ & - R^T(\theta_{ij}) \begin{pmatrix} v_j^d \cos \theta_j^e - v_j \\ v_j^d \sin \theta_j^e \end{pmatrix}, \end{aligned} \quad (12)$$

in which $\theta_{ij} = \theta_i - \theta_j = (\theta_i^d - \theta_j^e) - (\theta_j^d - \theta_j^e)$.

In the light of the introduced error variables, we may proceed with the problem statement in terms of the stability of the origins of the error dynamics (6), (7) and (12). Specifically, the *formation control problem* is solved when the origin of the tracking error dynamics (6, 7) is globally asymptotically stable for all robots in the formation. Of course, this implies that the coordination error variables $\varepsilon_{ij}(t)$ also converge to zero, for all $i, j \in \mathcal{I}$. Moreover, although we do not pose any explicit condition on the convergence of φ_i to φ_i^d , if all other error variables converge to zero, we can conclude that also $\varphi_i \rightarrow \varphi_i^d$ as $t \rightarrow \infty$, $i \in \mathcal{I}$.

In addition, we define the *coordination control problem* as a relaxation of the formation control problem in which it is only required that the coordination error $\varepsilon_{ij}(t)$ converges to zero for all $i, j \in \mathcal{I}$ without posing any conditions on the convergence of the tracking error variables $e_i^{xy}(t)$.

III. FORMATION CONTROL DESIGN

In this section we design the formation control algorithm to solve the formation control problem defined in the previous section by means of backstepping. For details on this technique, the reader is referred to [7].

The control design method is motivated by the developments in [10] in which a control strategy for a single robot is proposed. Our modifications are triggered by the benefits that inter-robot communication provides. Consequently, the control law in [10] was altered to allow for communication of robots with their neighbours to enhance the formation behaviour.

In order to keep φ_i within $(-\frac{\pi}{2}, \frac{\pi}{2})$ as required given that $\varphi_i(0) \in (-\frac{\pi}{2}, \frac{\pi}{2})$, we introduce a new variable $\mu_i = \tan \varphi_i$, $i \in \mathcal{I}$ [19]. This implies that $\varphi_i = \text{atan} \mu_i$ and therefore controlling μ_i results in $\varphi_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Let $\dot{\mu}_i = \xi_i$, where ξ_i is an auxiliary control, $i \in \mathcal{I}$. Once the control input for

ξ_i is derived, the original control input ω_i can be retrieved from

$$\dot{\varphi}_i = \frac{1}{1 + \mu_i^2} \xi_i = \omega_i. \quad (13)$$

To use the backstepping control design method, assume in the first instance that we can control θ_i^e directly through a virtual control $\bar{\mu}_i$, i.e. the dynamics of θ_i^e is given by

$$\dot{\theta}_i^e = \frac{v_i^d}{l} \tan \varphi_i^d - \bar{\mu}_i. \quad (14)$$

To find the control inputs v_i and $\bar{\mu}_i$ stabilizing the origin of the error dynamics (6, 14), consider the Lyapunov function candidate

$$V = \sum_{i=1}^N \left[c_i^e (e_i^{xy})^T e_i^{xy} + \frac{1}{2} \sum_{j \in N_i} c_{ij} \varepsilon_{ij}^T \varepsilon_{ij} + (\theta_i^e)^2 \right], \quad (15)$$

where N_i is a set of indices of robots being in the communication neighborhood of robot i . Calculating its time derivative along the dynamics (6, 14) yields

$$\dot{V} = \sum_{i=1}^N \left[\theta_i^e \left(v_i^d \kappa_i^T \lambda_i + \frac{v_i^d}{l} \tan \varphi_i^d - \bar{\mu}_i \right) + \eta_i (v_i^d - v_i) \right], \quad (16)$$

where $\kappa_i = c_i^e e_i^{xy} + \sum_{j \in N_i} c_{ij} \varepsilon_{ij} \in \mathbb{R}^{2 \times 1}$, $\eta_i = [1 \ 0] \kappa_i = c_i^e x_i^e + \sum_{j \in N_i} c_{ij} \varepsilon_{ij}^x \in \mathbb{R}$ and $\lambda_i = \left(\frac{\cos \theta_i^e - 1}{\theta_i^e}, \frac{\sin \theta_i^e}{\theta_i^e} \right)^T$. Consider the temporary controller to be defined by

$$v_i = v_i^d + \chi_i(\eta_i), \quad (17)$$

$$\bar{\mu}_i = \frac{v_i^d}{l} \tan \varphi_i^d + v_i^d \kappa_i \lambda_i + c_i^\theta \theta_i^e, \quad (18)$$

where the function $\chi_i(\cdot)$ is continuously differentiable and satisfies $\chi_i(x)x > 0$ for $x \neq 0$ and $\chi_i(\cdot) < \underline{v}_i^d$, in which \underline{v}_i^d is defined in (9). The reason for defining $\chi_i(\cdot)$ in this way is to ensure that $v_i \neq 0$ and its importance becomes apparent later in this section. Clearly, with such a choice of $\chi_i(\cdot)$, the resultant control input v_i is bounded with a bound that can be specified off-line beforehand.

The derivative of the Lyapunov function (16) when control inputs (17, 18) are applied becomes

$$\dot{V}(e_i^{xy}, \varepsilon_{ij}, \theta_i^e) = - \sum_{i=1}^N [c_i^\theta (\theta_i^e)^2 + \eta_i \chi_i(\eta_i)] \leq 0. \quad (19)$$

Hence, by [7, Theorem 4.4] we can show that $\lim_{t \rightarrow \infty} \dot{V} = 0$. Therefore, we conclude that as $t \rightarrow \infty$ we have $\sum_{i=1}^N [c_i^\theta (\theta_i^e)^2 + \eta_i \chi_i(\eta_i)] \rightarrow 0$ implying

$$\theta_i^e \rightarrow 0, \quad (20)$$

$$\eta_i \rightarrow 0. \quad (21)$$

Using the dynamic equation (14) for θ_i^e

$$\dot{\theta}_i^e = -v_i^d \kappa_i^T \lambda_i - c_i^\theta \theta_i^e, \quad (22)$$

and [5, Lemma 2], we infer that also

$$[0, 1] \kappa_i \rightarrow 0, \quad (23)$$

which together with (21) implies

$$\kappa_i \rightarrow 0, \quad (24)$$

and consequently

$$\left(c_i^e e_i + \sum_{j \in N_i} c_{ij} (e_i - e_j) \right) \rightarrow 0. \quad (25)$$

Note that the last condition is given with respect to the tracking error e_i in the global coordinate frame as opposed to previously considered e_i^{xy} which is with respect to a robot-attached moving frame. Now, we can write (25) in terms of the horizontal and vertical components of the tracking error $e_i = (e_i^x, e_i^y)^T$ for $t \rightarrow \infty$

$$Ae^\nu \rightarrow 0, \quad (26)$$

where $e^\nu = \text{col}(e_1^\nu, \dots, e_N^\nu)$. Here $\nu \in \{x, y\}$ and A is a matrix in which the diagonal and off-diagonal elements are $a_{ii} = c_i^e + \sum_{j \in N_i} c_{ij}$ and $a_{ij} = -c_{ij}$, $i \neq j$ respectively. From the Geršgorin disc theorem, it is evident that the matrix in (26) is nonsingular and hence $e_i \rightarrow 0$ as $t \rightarrow \infty$. Consequently $e_i^{xy} \rightarrow 0$ and $\varepsilon_{ij} \rightarrow 0$ as $t \rightarrow \infty$, for all $i, j \in \mathcal{I}$.

Note that because of the assumptions on v_i^d and the conditions that we pose on function χ_i , it can be assured that $v_i > 0$.

Clearly, in reality we cannot control θ_i^e directly with $\bar{\mu}_i$ as the dynamics of θ_i^e are given by (7). Therefore, using the backstepping technique as in [10] we define a new error variable for all $i \in \mathcal{I}$

$$z_i = v_i^d \tan \varphi_i^d - v_i \mu_i + v_i^d \kappa_i^T \lambda_i + c_i^\theta \theta_i^e. \quad (27)$$

Then, error dynamics of θ_i^e in (7) can be re-written to be

$$\dot{\theta}_i^e = -v_i^d \kappa_i^T \lambda_i - c_i^\theta \theta_i^e + \frac{1}{l} z_i = \frac{v_i^d}{l} \tan \varphi_i^d - \bar{\mu}_i + \frac{1}{l} z_i, \quad (28)$$

and the dynamics of z_i is assumed to be given by

$$\dot{z}_i = l \dot{\bar{\mu}}_i - \dot{v}_i \mu_i - v_i \xi_i. \quad (29)$$

Let the Lyapunov function candidate be

$$\bar{V} = V + \frac{1}{2} \sum_{i=1}^N z_i^2. \quad (30)$$

Derivative of \bar{V} along system dynamics is given by

$$\dot{\bar{V}} = \dot{V} + \sum_{i=1}^N z_i \left(\theta_i^e \frac{1}{l} + l \dot{\bar{\mu}}_i - \dot{v}_i \mu_i - v_i \xi_i \right). \quad (31)$$

Therefore, allowing $\xi_i = \frac{1}{v_i} \left(\theta_i^e \frac{1}{l} + l \dot{\bar{\mu}}_i - \dot{v}_i \mu_i + c_i^z z_i \right)$, where from (17) it is clear that $v_i \neq 0$, gives that $\dot{\bar{V}} = \dot{V} - \sum_{i=1}^N c_i^z z_i^2$. Consequently, using the same lines of argument as above it can be shown that the origin of the error dynamics of $(e_i^{xy}, \theta_i^e, z_i)$ is globally asymptotically stable.

The actual control ω_i is given by

$$\omega_i = \frac{\cos^2 \varphi_i}{v_i} \left(\theta_i^e \frac{1}{l} + l \dot{\bar{\mu}}_i - \dot{v}_i \mu_i + c_i^z z_i \right). \quad (32)$$

It can be seen that $\varphi_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$ is ensured by noticing that the derivative of the Lyapunov function candidate (30) is negative semi-definite and thus we have that $z_i = 0$ is a stable equilibrium point. Hence, z_i is uniformly bounded. Therefore, since $v_i \neq 0$ from (27) it can be concluded that μ_i is also bounded and consequently $\varphi_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Indeed,

for φ_i approaching $\pm \frac{\pi}{2}$, the control input ω_i (32) tends to 0. Hence, $\varphi_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$ is an invariant set as required.

We formally state the conditions under which the controller in (17, 32) solves the formation control problem in the following theorem.

Theorem 3.1: Consider a group of N car-like mobile robots, each of which is described by the kinematic model (1), a desired trajectory of the virtual center $q_{vc}^d(t)$ such that the associated feedforward velocities v_{vc}^d and ω_{vc}^d are bounded. Let the desired formation shape be defined using bounded vectors $l_i^d(t)$, $i \in \mathcal{I}$, subject to $\frac{dl_i^d(t)}{dt}$ bounded. Moreover, denote by $q_i^d(t)$ the desired individual trajectories of robots with associated desired forward and angular velocities $v_i^d(t)$ and $\omega_i^d(t)$ satisfying Assumption 1 and $v_i^d > 0$ and bounded away from zero (9), and ω_i^d is bounded. Let the formation control law be defined by (17, 32) with $c_i^e > 0$, $c_{ij} = c_{ji} > 0$, $c_i^\theta > 0$ and $c_i^z > 0$. Then, the origin of the closed-loop error dynamics (6, 17, 28, 29, 32) is globally asymptotically stable and for all pairs of robots $i, j \in \mathcal{I}$, $\varepsilon_{ij} \rightarrow 0$ as $t \rightarrow \infty$. Hence, the formation control problem is solved.

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^N \left[c_i^e (e_i^{xy})^T e_i^{xy} + \frac{1}{2} \sum_{j \in N_i} c_{ij} \varepsilon_{ij}^T \varepsilon_{ij} + (\theta_i^e)^2 + \frac{1}{l} z_i^2 \right]. \quad (33)$$

The time derivative of (33) along dynamics (6, 28, 29) with the controller given by (17, 32) is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N \left(-c_i^\theta (\theta_i^e)^2 + \frac{1}{l} \theta_i^e z_i + \frac{c_i^z}{l} z_i^2 - \frac{1}{l} \theta_i^e z_i - \eta_i \chi_i(\eta_i) \right) \\ &= -\sum_{i=1}^N \left(c_i^\theta (\theta_i^e)^2 + \frac{c_i^z}{l} z_i^2 + \eta_i \chi_i(\eta_i) \right). \end{aligned} \quad (34)$$

Therefore, we have $\dot{V} \leq 0$ and using [7, Theorem 4.4] leads to the conclusion that (20) and (21) are satisfied together with $z_i \rightarrow 0$ as $t \rightarrow \infty$. Therefore, applying [5, Lemma 2] leads again to (25) and, in terms of horizontal and vertical components of the tracking error, to (26). As shown earlier, the matrix in (26) is nonsingular and thus $e_i \rightarrow 0$ as $t \rightarrow \infty$ which implies that also $e_i^{xy} \rightarrow 0$ and $\varepsilon_{ij} \rightarrow 0$ as $t \rightarrow \infty$, for all $i, j \in \mathcal{I}$. Hence, the formation control problem is solved. \blacksquare

Expanding the ideas from [15] for car-like robot formation, in the following corollary we give some additional conditions under which we solve the *coordination control problem* for car-like robot formations. We omit the proof here due to space restrictions.

Corollary 3.2: Consider a group of N car-like mobile robots, each of which is described by the kinematic model (1) and consider the settings as described in Theorem 3.1. Let the formation control law be defined by (32) and a modification of (17) with all position tracking gains set to zero $c_i^e = 0$. Moreover, assume $c_{ij} = c_{ji} > 0$, $c_i^\theta > 0$ and $c_i^z > 0$. Then, if the communication graph of the formation is connected, the set $\{e_i, \theta_i^e \mid i \in \mathcal{I}, j \in N_i, (\varepsilon_{ij}, \theta_i^e) = (0, 0)\}$ is a globally attracting invariant set of (6, 28, 29) for all robots and the

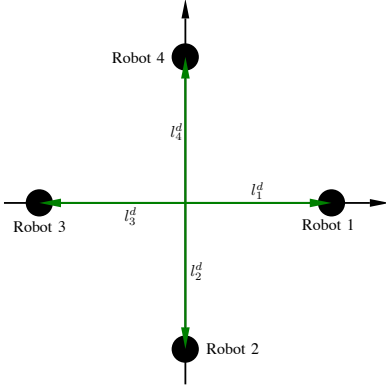


Fig. 2. Desired formation shape used in the simulations.

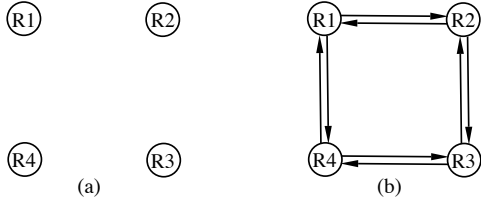


Fig. 3. Communication graph structures used in simulations: (a) a disconnected graph (b) a connected graph.

desired formation shape is created by all robots.

Remark 3.3: The attractive feature of the results presented in Theorem 3.1 is that we only give mild conditions regarding control parameters $c_i^e > 0$, $c_{ij} = c_{ji} > 0$, $c_i^\theta > 0$ and $c_i^z > 0$. Therefore, they can be chosen in a way that is suitable for a particular application. The meaning of these parameters is as follows. Tracking of individual robot trajectories can be influenced by c_i^e while to influence the formation shape keeping the mutual coupling terms c_{ij} should be adjusted. One can also decide if tracking of individual trajectories or formation shape keeping should be more important. In the first case, the position tracking gains c_i^e should dominate the mutual coupling gains c_{ij} and vice versa. The remaining parameters c_i^θ and c_i^z influence directly the convergence of θ_i^e and z_i to zero respectively.

IV. SIMULATION STUDY

In this section we present the simulation results of the formation control algorithm introduced in this paper. As an illustrative example, we use a formation consisting of four car-like robots whose length is $l = 0.1$. The desired formation shape is a square, where the length of a side is equal to $0.15\sqrt{2}$. It is defined by $l_1^d = \text{col}(0.15, 0)$, $l_2^d = \text{col}(0, -0.15)$, $l_3^d = \text{col}(-0.15, 0)$ and $l_4^d = \text{col}(0, 0.15)$, as depicted in Figure 2.

To show the complete view of robots behavior in the formation, we study the influence of a disconnected and a connected communication graphs on the formation performance. The disconnected communication topology is shown in Figure 3(a) and the connected communication topology is presented in Figure 3(b).

The initial conditions of the four robots in the formation are the following: $q_1(0) = \text{col}(0, 0.2, \frac{\pi}{18}, 0)$, $q_2(0) = \text{col}(2.3, -0.05, -\frac{\pi}{3}, 0)$, $q_3(0) = \text{col}(1.5, 0.6, -\frac{\pi}{4}, 0)$ and

$q_4(0) = \text{col}(0.5, -1, \frac{\pi}{4}, 0)$. The control parameters for the connected communication graph are: $c_i^e = 30$, $c_i^\theta = 0.5$, $c_i^z = 44$, $c_{12} = 91$, $c_{14} = 85$, $c_{23} = 77$, $c_{34} = 81$ and $c_{ij} = c_{ji}$. For the disconnected communication graph, all coupling gains c_{ij} are equal zero. Moreover, for all robots the function $\chi_i(s)$ is assumed to be given by $\chi_i(s) = \frac{0.2}{\pi} \text{atan}(s)$.

All simulations in this section are done for $t \in [0, 30]$. During that time, at $t = 21$ we displace Robot 1 along $(\delta x, \delta y) = (0.2, 0.35)$ to observe how robots in the formation behave in face of a perturbation.

Simulation results are given in Figure 4 where robot paths in the plane for a disconnected and a connected communication graph are depicted. It can be seen that robots initially converge to the desired formation shape. Then, the formation shape temporarily ceases to be maintained because of the perturbation. In particular, when the formation is disconnected all robots are in fact completely decoupled so no robot, except for the very perturbed one Robot 1, is aware of the disturbance occurring to any other robot. Therefore, the robots have no means to try to preserve the formation in face of a perturbation, see Figure 4a. In contrast, when the communication graph is connected, after the perturbation all unperturbed robots diverge from their desired trajectories due to the connectivity of the communication graph of the formation. This means that because of the relatively strong coupling gains c_{ij} as compared to the tracking gains c_i^e , robots act primarily towards maintaining formation shape as opposed to purely tracking their individual desired trajectories (irrespective of the behaviour of other robots). For robots to be able to benefit from this mechanism, the communication graph of the formation needs to be connected as in such circumstances robots work explicitly towards coordination of the group. This is despite the lack of such a requirement in Theorem 3.1. Understandably, if robots face a perturbation, all members of the formation need to be aware of it through inter-agent communication to counteract the effect of the perturbation.

V. CONCLUSIONS

In this paper we have studied the formation control problem for a group of car-like mobile robots. We have proposed a distributed formation control algorithm based on the backstepping approach. In addition, we have also examined the coordination problem in which the formation does not track the desired trajectory but it creates the desired formation shape and follows a trajectory that is ultimately translated relative to the desired one. For this strategy to work, all robots in the formation need to communicate with each other, possibly indirectly through other robots. In other words, the communication graph of the formation needs to be connected. In the main algorithm solving the formation control problem proposed in the paper we have observed a beneficial influence of the connectivity of the communication graph on the formation behavior. More specifically, allowing robots to communicate with each other assures that they implicitly act towards maintaining desired formation geometry as opposed to only tracking their individual desired

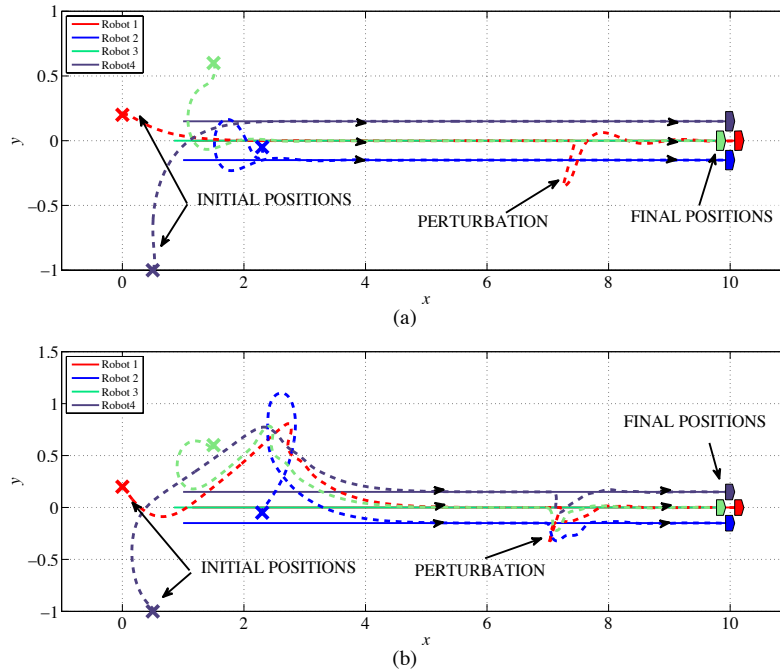


Fig. 4. Desired robot paths (solid line) and actual robots paths (dashed line) in the plane in the case of (a) a disconnected and (b) a connected communication graph.

trajectories. In particular, if the formation shape preservation is of importance, when some of the robots face perturbation, the others can only counteract it when aware of the perturbation through inter-robot communication. To be noted, in the control law as such there is no requirement on the connectivity of the communication graph due to the stabilizing effect of the nonzero tracking gains on the formation error dynamics.

Future work on the topic presented in the paper may include limiting the possible range of control input ω_i , $i \in \mathcal{I}$ so its maximum value can be chosen as a control parameter. That would confirm applicability of these results for real robots where actuators have physical limitations. Also studying formation control problem for heterogenous robots in a formation as well as incorporating an inter-robot collision avoidance scheme to the formation control law might be interesting.

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