

An adaptive version of a second order sliding mode output feedback controller

Antonio Estrada, Franck Plestan and Benyamine Allouche

Abstract—This paper proposes a novel adaptive-gain second order sliding mode controller. The proposed controller is based on discontinuous-impulsive action and only uses information of the output, no differentiation is needed. A practical second order sliding mode is ensured in spite of bounded perturbations with unknown bound. The gain adaptation is based on particular conditions attainable on sliding mode which can be verified with the only information of the output. Both, the convergence conditions and adaptation proposal are tested through simulations.

I. INTRODUCTION

In spite of the extensive and successful development of robust adaptive control and backstepping techniques, sliding mode (SM) / higher order sliding mode (HOSM) control remains probably the main choice in handling bounded uncertainties/disturbances and unmodelled dynamics [1], [4]. The main common properties of SM/HOSM control are robustness (insensitivity) to the bounded disturbances matched by control, and finite convergence time. Furthermore, the HOSM control algorithms can handle systems with arbitrary relative degree, achieve any given control smoothness (by artificially increasing the relative degree), stabilize at zero not only the sliding variable, but also its $(k - 1)$ first time derivatives (in case of k -th order SM), and provide sliding variable stabilization with an accuracy proportional to T_e^k , T_e being the sampling period used for the implementation of the controller [6], [4]. The main drawback of HOSM control laws is the requirement of high order time derivatives of the sliding variable [6], [4]. Unfortunately, the use of differentiators yields a performance degradation of the controlled system due to the presence of measurement noise. Then, there is a real interest to propose high order sliding mode controllers with a reduced number of time derivatives of the sliding variable. In the case of second order sliding mode controller, this reduction leads to the use of *only* the sliding variable in the controller, which means that the controller is a (static) output feedback one.

The additional advantages of reducing chattering and relaxing the requirement of perturbation bounds knowledge have played a crucial role in the interest for developing sliding mode controllers with adaptive gain, see [10], [13], [12] for instance. In [12], an adaptation law for the so called *supertwisting* [6] is reported. The adaptation is based on the *equivalent control* and ensures the gain reduction to a

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minimum according to the bound of perturbations. In [13] an adaptive proposal for a twisting controller is proposed using the Lyapunov function reported in [14] for the stability analysis. A practical/ideal second order sliding mode is ensured depending on the possibility of nonoverestimating/overestimating the gain.

In the present paper, a gain adaptive version of the impulsive output feedback (IOF) controller reported in [9], see also [3], is proposed. Unfortunately, neither the approach in [12] nor the one in [13] can be applied to the impulsive aforementioned controller. The formulation of the IOF controller intrinsically considers the existence of a finite sampling time and ensures practical sliding mode establishment. The equivalent control approach is based on the establishment of ideal sliding mode. On the other hand in [13], in order to know if system trajectories are inside some vicinity of the origin, information on the sliding variable and its derivative is required. Besides the lacking of a Lyapunov function for the IOF controller, the use of differentiation is not considered. In the current work, an adaptation law which considers the finite sampling time and maintains the output feedback nature of the IOF controller, by avoiding the use of the sliding variable derivative, is presented. The paper is organized as follows, Section II states the problem of second order sliding mode output feedback control of uncertain nonlinear systems. For selfcontent purposes, a result from [3] is presented in Section III. In Section IV two claims and its respective proofs are introduced before presenting the adaptive proposal as the main result of the section. The results were tested through simulations and presented in Section V. Finally, some conclusions are given in Section VI.

II. PROBLEM STATEMENT

As shown in many previous works [9], second order sliding mode control of uncertain nonlinear system is equivalent to the finite time stabilization of the following system

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= u + \omega\end{aligned}\tag{1}$$

where $z = [z_1 \ z_2]^T$ is the state vector, u the control input and $|\omega| \leq \delta$ ($0 \leq \delta$) a bounded perturbation. The system (1) is viewed as an uncertain nonlinear continuous one, whereas the control law u is supposed to be evaluated at a sampling period T_e which yields

$$u = -K(t)\text{sign}(z_1(kT_e)).\tag{2}$$

One has $K(t) > 0$ and $k \in \mathbb{N}$ (k can be viewed as a time counter). Gain K is constant on the time interval $t \in [k \cdot T_e, (k+1) \cdot T_e]$, and $k(0) = 0$. Note that the control input depends only on the output z_1 . The objective of the paper is then to propose a robust output feedback controller (2) which allows to reach, in a finite time, a vicinity of the origin of (1) in spite of perturbation ω . If it is the case, a second order sliding mode versus z_1 is established.

III. SECOND ORDER SLIDING MODE OUTPUT FEEDBACK CONTROL OF THE UNPERTURBED CASE

The first result displayed in the sequel concerns the finite time stabilization of system (1) when $\omega = 0$. The control objective consists in ensuring the finite time establishment of a real second order sliding mode with respect to z_1 in system (1)-(2) with $\omega = 0$.

Theorem 1 ([3]). *Consider system (1) with $\omega = 0$ and controlled by (2). The gain $K(t)$ is defined as*

$$K(t) = \begin{cases} K_m & \text{if } t \notin \mathcal{T} \\ K_M & \text{if } t \in \mathcal{T} \end{cases} \quad (3)$$

with $\mathcal{T} = \{t \mid (k-1)T_e \leq t < kT_e \text{ and } \text{sign}(z_1((k-2)T_e)) \neq \text{sign}(z_1((k-1)T_e)), k \in \mathbb{N}, k \geq 2\}$ and $K_m > 0$. If K_M is such that

$$K_M = \gamma K_m \quad \text{with } \gamma > 3, \quad (4)$$

then the control law (2)-(3) ensures that there exists a time t_F such that for all $t \geq t_F$

$$|z_1| < \frac{1}{2} K_m [\bar{\eta}(\gamma) - 1]^2 T_e^2, \quad |z_2| < K_m \bar{\eta}(\gamma) T_e, \quad (5)$$

with

$$\bar{\eta} = \frac{\gamma^2 - \gamma - 2}{2(\gamma - 3)}. \quad (6)$$

Remark. This theorem allows to give a constructive solution for the tuning of K_M . The initial result [9] only claims that there always exists a gain K_M such that the establishment of a second order sliding mode w.r.t. z_1 is effective, but without giving any condition on K_M . ■

IV. ADAPTIVE ALGORITHM FOR UNKNOWN BOUNDS OF PERTURBATIONS

As claimed previously, there is a real practical interest to consider the perturbations with unknown bounds. In fact, it allows to reduce their ‘‘overestimations’’, and to have lighter process of identification. A consequence is that, in the case of bounded perturbations with unknown bounds, it is required to have an adaptation mechanism which allows to establish the practical sliding mode and, at the same time, to adjust the gains according to these unknown bounds. In the sequel, an algorithm for such gain adaptation is proposed.

A. A fixed-gain controller for unperturbed system

Denote T_s^i ($i \in \mathbb{N}$ being like a clock-counter, $i > 0$) a time at which the change of z_1 -sign is detected, and T_s^{i-1} the respective previous one. Denote also η^i the number of sampling periods between the time instant for which $z_2 = 0$ (z_1 is maximum), and the z_1 -sign switching detection; as shown in [3], η^i is a time-varying function. Furthermore, in [3], once the system trajectories have converged to (5), one has $\eta^i \leq \bar{\eta}(\gamma)$, $\bar{\eta}$ being defined by (6).

Proposition 1. *Consider system (1)-(2) with $\omega = 0$. If $\gamma \geq 7$, a practical second order sliding mode is established in finite time such that the period of time between two consecutive z_1 -sign commutations is at most $3T_e$, i.e. the next inequality is fulfilled*

$$T_s^i - T_s^{i-1} \leq 3T_e. \quad (7)$$

Proof of Proposition 1. It has been established by Theorem 1 that the impulsive control attracts trajectories to some vicinity of the origin. Then, after convergence, K_m -trajectories cannot spend more than $\bar{\eta}(\gamma)$ sampling times between a maximum value of z_1 and the next detection of z_1 -sign switching. Nevertheless, the claim is conservative given that K_m -trajectories can be further reduced. It is obvious that, for values of γ such that $\bar{\eta}(\gamma) < \gamma$, K_m -trajectories never cross the z_1 -axis. This latter is then crossed by the K_M -trajectories (see Figure 1). For a sake of clarity, $\bar{\eta}$ is used instead of $\bar{\eta}(\gamma)$ in the sequel. Consider the worst case, i.e. the case for which the delay of z_1 -sign switching detection equals one sampling period. In [3], it has been shown that there is convergence to the domain defined by (5) if

$$|z_1(T_s^i - \gamma T_e)| - |z_1(T_s^i + T_e)| \geq 0. \quad (8)$$

It yields

$$2\eta^i[\gamma - 3] - \gamma^2 + \gamma + 2 \leq 0. \quad (9)$$

From [3], the value $\bar{\eta}$ is the value for which

$$z_1(T_s + T_e) = z_1(T_s - \gamma T_e),$$

under the assumption $T_d = T_e$, i.e. the worst case. The aforementioned case is also the case depicted on Figure 1. Considering the worst case and supposing that the system is starting from the fourth quadrant (bottom-right-hand side of the phase plane), i.e. $z_o = z(T_s^i + T_e)$, once the system trajectories have converged to the domain (5), the K_m -trajectories will take $2\bar{\eta} - \gamma - 1$ sampling times to reach z_2 -axis. Then, denoting

$$K_{T_e} = 2\bar{\eta} - \gamma - 1, \quad (10)$$

the z_1 -sign switching detection occurs after *at most* $\text{ceil}(K_{T_e})$ sampling periods after T_s^i . Next, the limit of reduction of K_{T_e} , as a function of the parameter γ is obtained through the examination of the relationship between $\bar{\eta}$ and γ . From (6), one gets

$$K_{T_e} = \frac{\gamma^2 - \gamma - 2}{\gamma - 3} - \gamma - 1 = \frac{\gamma + 1}{\gamma - 3}. \quad (11)$$

Recalling that $\gamma > 3$, it yields

$$K_{T_e} > 1, \forall \gamma > 3. \quad (12)$$

The function K_{T_e} is defined for $\gamma > 3$, and is monotonous and decreasing. It is clear, from (6), that it converges to 1 for $\gamma \rightarrow \infty$. Furthermore, it is also clear that K_{T_e} is smaller than 2 for $\gamma \geq 7$. Thereby, it is obvious that

$$T_s^i - T_s^{i-1} \leq (\text{ceil}(K_{T_e}) + 1) \cdot T_e \leq 3T_e,$$

for $\gamma > 7$. ■

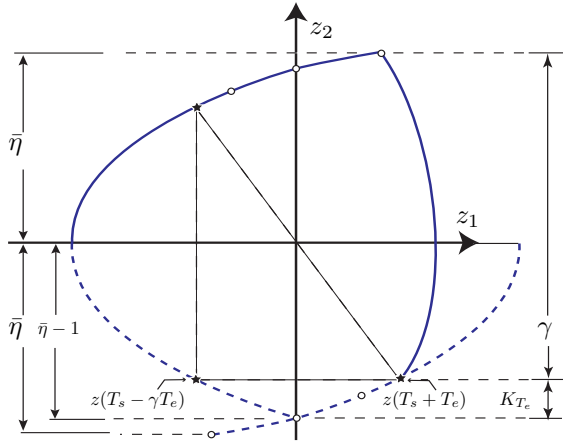


Fig. 1. $\gamma > \bar{\eta}(\gamma)$.

B. Preliminary result for a first attempt of adaptive control solution

Now, for the perturbed system (1) with $\omega \neq 0$, the next result is claimed

Lemma 1. Consider system (1)-(2) with $|\omega| < \delta$. There always exists a finite gain K_m and a finite parameter γ which ensure the establishment of a practical second order sliding mode such that

$$T_s^i - T_s^{i-1} \leq 3T_e.$$

Remark 1. Lemma 1 is the basis for the adaptation law developed in the sequel. The condition $T_s^i - T_s^{i-1} \leq 3T_e$ is fulfilled in the practical sliding mode context. ■

Proof of Lemma 1. The proof is divided into two parts

- First, it is shown that, knowing that $K_M = \gamma K_m$ with γ constant, it is always possible to ensure convergence by increasing K_m .
- Then, once convergence is ensured, the second part of the proof establishes that, once the real sliding mode is established, $T_s^{i-1} - T_s^i \leq 3T_e$.

As a matter of fact, any point in the phase plane can be seen, forward or backwards in time, as a point belonging to a trajectory that crosses the z_1 -axis, given that $K_m > \delta > |\omega|$.

Thus, without loss of generality, consider a trajectory with the following initial condition at $t = t_o$ ($z_{1o} > 0$)

$$z_o = z(t_o) = [z_{1o} \ 0]^T.$$

Then, by supposing that the control law (2) has been tuned to ensure that the trajectories of (1) reach in a finite time the domain defined by (5), it is sure that, in the phase plane, the trajectory will cross the z_2 -axis at a finite time t_1 , i.e. at $t = t_1$, $z_1(t_1) = 0$. Moreover, the maximum/minimum values of $z_2(t_1)$ can be computed. Introduce the next notations for the perturbed system (1)

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= u^*, \end{aligned} \quad (13)$$

with $u^* = u + \omega$, u being defined according to (2). Now, define K_m^* and K_M^* respectively as

$$\begin{aligned} K_m^* &= K_m + \omega, \\ K_M^* &= \gamma K_m + \omega, \end{aligned}$$

with $|\omega| \leq \delta$. Furthermore, define $\theta > 1$ such that

$$K_m = \theta \delta.$$

Then, the gains K_m^* and K_M^* fulfill the next inequalities

$$\begin{aligned} \delta(\theta - 1) &\leq K_m^* \leq \delta(\theta + 1), \\ \gamma\delta(\theta - 1) &\leq K_M^* \leq \gamma\delta(\theta + 1). \end{aligned} \quad (14)$$

As previously exposed, consider that the trajectory is starting from the right half phase plane ($z_{1o} > 0$). For the unperturbed case, the trajectories read as

$$\begin{aligned} z_2(t) &= -K_m(t - t_o), \\ z_1(t) &= -\frac{K_m}{2}(t - t_o)^2 + z_{1o}. \end{aligned} \quad (15)$$

Given the previous definition of t_1 and $z_1(t)$, on has

$$z_1(t_1) = 0 = -\frac{K_m}{2}t_1^2 + z_{1o}, \quad (16)$$

which gives

$$t_1 = \sqrt{\frac{2z_{1o}}{K_m}}, \quad (17)$$

hence, $z_2(t_1)$ reads as

$$z_2(t_1) = -\sqrt{2K_m z_{1o}}. \quad (18)$$

Then, for the perturbed case ($\omega \neq 0$), by using the upper and lower bounds of K_m^* in (14), one has

$$\sqrt{2z_{1o}\delta(\theta - 1)} \leq |z_2(t_1)| \leq \sqrt{2z_{1o}\delta(\theta + 1)}. \quad (19)$$

Denote z_{2dist} , the distance between the upper and lower bounds of inequality (19)

$$z_{2dist} = \sqrt{2z_{1o}\delta} \left(\sqrt{\theta + 1} - \sqrt{\theta - 1} \right). \quad (20)$$

The above equation reveals that z_{2dist} decreases as θ increases and, as expected, increases with z_{1o} and δ . In order to be sure that there is convergence for the perturbed case, the gain K_M

has to have a sufficient magnitude to reach a family of K_m -trajectories which are closer from the origin than the K_m -trajectories on which the system is evolving before detection of a z_1 -sign change (see Figure 2). Then, from (14), one gets

$$\gamma\delta(\theta - 1) \geq \sqrt{2z_{1o}\delta} \left(\sqrt{(\theta + 1)} - \sqrt{(\theta - 1)} \right). \quad (21)$$

Furthermore, while z_{2dist} decreases when θ increases, the magnitude of K_M^* is increasing proportionally to θ . Then, it becomes obvious that convergence is always obtained for some finite value of θ , then for a finite K_m . This concludes the first part of the proof.

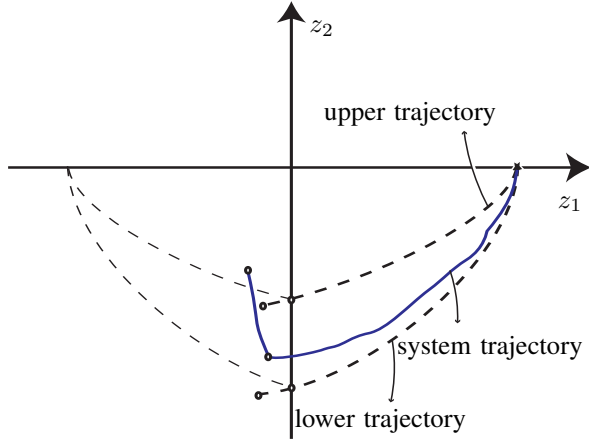


Fig. 2. Ensuring convergence.

Now, one demonstrates that, once a sliding mode is established, the inequality $T_s^i - T_s^{i-1} \leq 3T_e$ can always be established for some finite γ . Define γ^* as

$$K_M^* = \gamma^* K_m^*. \quad (22)$$

Note that, from Proposition 1, for the unperturbed case ($\omega = 0$), the inequality $T_s^i - T_s^{i-1} \leq 3T_e$ is established for any $\gamma > 7$, if $K_m > 0$. This result can be transposed to the uncertain system (1): then, if $K_m^* > 0$ and $\gamma^* > 7$, a similar result can be obtained for the perturbed case: from (14), one gets

$$\gamma^* \geq \frac{\gamma(\theta - 1)}{\theta + 1}. \quad (23)$$

Then, a sufficient condition to ensure the establishment of a real second order sliding mode and $T_s^i - T_s^{i-1} \leq 3T_e$ is

$$\gamma^* \geq \frac{\gamma(\theta - 1)}{\theta + 1} > 7. \quad (24)$$

It yields that for finite $\gamma > 7$ there always exists a finite parameter θ which verifies the previous inequality. The parameter to be selected for design is γ which defines the minimum θ to fulfill (23). That is the minimum finite K_m which fulfills (23) depends on the selection of the finite $\gamma > 7$. ■

Let us point out the importance of Lemma 1. Suppose that the parameter γ has been fixed to a constant value $\gamma = \gamma_c > 7$.

Then, according to Lemma 1, a practical second order sliding mode can always be established by tuning/increasing K_m up to some finite value. Moreover, if after convergence the gain K_m is such that

$$K_m = \theta\delta \text{ with } \theta > \frac{\gamma_c + 7}{\gamma_c - 7}, \quad (25)$$

then $T_s^i - T_s^{i-1} \leq 3T_e$ is established. This point is detailed through the following simulations.

Simulations. Consider the uncertain nonlinear system (1) with the perturbation term defined as

$$\omega = 4 \sin(t) + 4 \text{sign}(z_2),$$

It yields $\delta = 8$. The system (1) is initialized at

$$z(0) = [z_1(0) \ z_2(0)]^T = [3.6 \ 6]^T.$$

The objective of these simulations is to evaluate the result claimed in Lemma 1. Setting $\gamma = 8$: in this case, from (25), one can reach $T_s^i - T_s^{i-1} \leq 3T_e$ when $K_m = \theta\delta$ with $\theta > 15$ and $\delta = 8$, i.e. $K_m > 120$. K_m is forced to keep growing if $T_s^i - T_s^{i-1} > 3T_e$ and is defined as the sampled output of the continuous system

$$\dot{K}_{mc} = \begin{cases} \Gamma & \text{if } T_s^i - T_s^{i-1} > 3T_e \\ 0 & \text{if } T_s^i - T_s^{i-1} \leq 3T_e \end{cases} \quad (26)$$

with $\Gamma = 20$ and the sampling time $T_e = 0.01s$. Figure 3-top shows how the gain $K_m(t)$ is growing until, as expected, $T_s^i - T_s^{i-1} \leq 3T_e$, and it is confirmed that K_m stops to grow when it attains 120, as expected previously. Given the gain dynamics, K_m can not be reduced, but is maintained to a constant value once the second order sliding mode is established¹. Figure 3-bottom displays the number of sampling period between two consecutive z_1 -sign commutation detection, i.e. the ratio $\frac{T_s^i - T_s^{i-1}}{T_e}$ with $i \in \mathbb{N}$ being as a clock counter. As pursued, once the real second order sliding mode is established, this ratio is lower or equal to 3. The evolution of the state variable z_1 and z_2 of system (1), and the perturbation ω are plotted in Figure 4. It is clear that there is the establishment of a real second order sliding mode; however, the accuracy versus z_2 is not good, due to the fact that the gain is too large. The next subsection removes this drawback thanks to a new gain adaptation law.

C. An adaptive controller for perturbed system

The following result allows to adjust the gain $K_m(t)$ with respect to the establishment (or not) of a second order sliding mode. The detection of this latter is based on Proposition 1, i.e. there is second order sliding mode if $T_s^i - T_s^{i-1} \leq 3T_e$. Furthermore, Lemma 1 has shown that there always exists gain $K_m(t)$ allowing the establishment of a second order sliding mode; it yields that a manner to get a second order

¹This drawback, which engenders an over-tuning of the gain versus the uncertainties and perturbations, will be removed with the gain adaptation law given in the sequel of the paper.

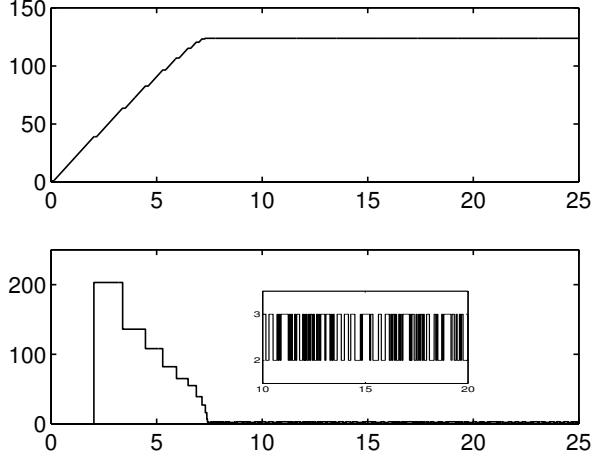


Fig. 3. **Top.** Gain K_m versus time (sec). **Bottom** Ratio $(T_s^i - T_s^{i-1})/T_e$ versus time (sec).

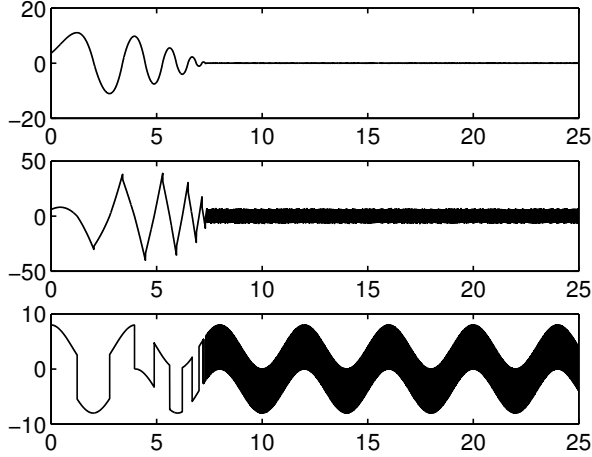


Fig. 4. **Top.** State variable z_1 versus time (sec). **Middle.** State variable z_2 versus time (sec). **Bottom.** Perturbation ω versus time (sec).

sliding mode is to increase $K_m(t)$ until this sliding mode appears. Then, consider the following adaptation law for the gain K_m

$$\dot{K}_m = \Gamma \text{sign}(t - T_s^i - 3T_e), \quad (27)$$

with $\Gamma > 0$ and $K_m(0) > 0$. The main result is claimed by the next theorem.

Theorem 2. Consider system (1) controlled by (2). The gain $K(t)$ is defined as

$$K(t) = \begin{cases} K_m & \text{if } t \notin \mathcal{T} \\ K_M & \text{if } t \in \mathcal{T} \end{cases} \quad (28)$$

with $\mathcal{T} = \{t \mid (k-1)T_e \leq t < kT_e \text{ and } \text{sign}(z_1((k-2)T_e)) \neq \text{sign}(z_1((k-1)T_e)), k \in \mathbb{N}, k \geq 2\}$,

$$\dot{K}_m = \Gamma \text{sign}(t - T_s^i - 3T_e), \quad (29)$$

with $\Gamma > 0$, $K_m \geq 0$. There exists a sufficiently large parameter $\gamma > 7$ such that if $K_M = \gamma K_m(t)$ then, the control law (2)-(28) ensures the establishment of a real second order sliding mode with respect to z_1 .

Remark 2. If the bound δ of the perturbation is *a priori* known, the previous result can be modified as follows. Suppose that the desired maximum value for K_m is fixed (for technological reasons) at K_m^M , which gives a maximum value for θ , θ^M , defined as

$$\theta^M = \frac{K_m^M}{\delta}.$$

If the parameter γ fulfills

$$\gamma > \frac{7(\theta^M + 1)}{\theta^M - 1}, \quad (30)$$

and given that $K_M = \gamma K_m(t)$, then the control law (2)-(28) ensures the establishment of a real second order sliding mode with respect to z_1 . ■

Proof of Theorem 2.

The theorem is a direct consequence of Lemma 1 and its proof. Recalling $K_m = \theta\delta$ ($\theta > 1$) and given that the condition on γ to ensure $T_s^i - T_s^{i-1} \leq 3T_e$ reads as

$$\gamma > \frac{7(\theta + 1)}{\theta - 1}, \quad (31)$$

one gets

$$\gamma > \frac{7K_m/\delta}{K_m/\delta - 1} = \frac{7K_m}{K_m - \delta}.$$

Then, a sufficiently large parameter γ can fulfill this condition; note that it is obviously larger than 7. Then, given Lemma 1 and Theorem 1, convergence can always be ensured with finite K_m for any finite initial conditions and bounded perturbation ω . Secondly, once the convergence is achieved and while $K_m > |\omega|$ and γ fulfilling (31), the inequality $T_s^i - T_s^{i-1} \leq 3T_e$ will be established and kept.

Once the inequality $T_s^i - T_s^{i-1} \leq 3$ is obtained, from (27), K_m will decrease at least until the inequality $K_m > \theta\delta$ is broken. In other words, after convergence $K_m > \theta\delta$ can not be maintained as it implies $\dot{K}_m < 0$. Then, the inequality $T_s^i - T_s^{i-1} \leq 3$ can be broken, which implies that K_m is increasing again, and so on.

Thus, trajectories are attracted to a vicinity of the origin and the gain K_m is reduced to be below an upper bound directly related to the bound δ of the perturbation ω . However, a very positive point is that the gain $K_m(t)$ is *on-line* adjusted which allows that it is closer from ω . Note that Theorem 2 ensures the establishment of a practical sliding mode but the accuracy is not given: this latter point will be a topic of future works.

V. EXAMPLE

Consider the next system

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= u + \omega \end{aligned} \quad (32)$$

with $\omega = 4 \sin(t) + 4 \text{sign}(z_2)$. The control law u is defined by (2)-(28)-(29) and $K_M = \gamma K(t)$. The initial conditions were selected as $z(0) = [3.6 \ 6]^T$, whereas the controller parameters have been tuned as $\gamma = 8$ and $\Gamma = 20$. The sampling time equals $T_e = 10 \text{ ms}$. Results from simulations are shown in Figures 5 and 6. One verifies that the state variables z_1 and z_2 are converging towards a vicinity of the origin, with a better accuracy than in previous section (in which the gain K_m can not be reduced): this feature clearly highlights the interest of the adaptation law for K_m and K_M . Figure 6 displays the evolution of the gain K_m : this latter is increasing during the convergence phase, then is reduced once the system trajectories have reached a vicinity of the origin. The gain is *on-line* adjusted in order to maintained a real sliding mode.

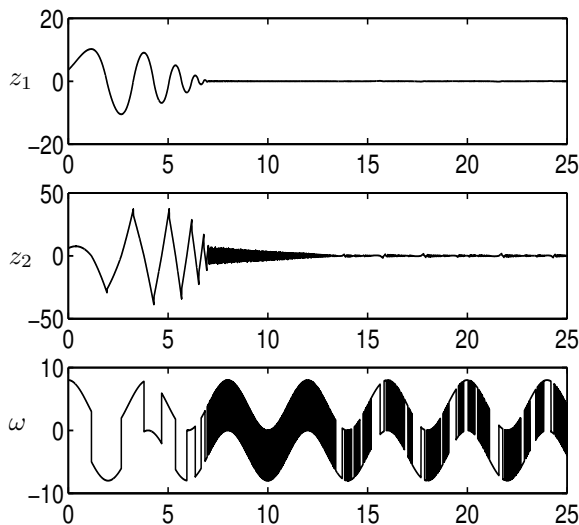


Fig. 5. **Top.** State variable z_1 versus time (sec). **Middle.** State variable z_2 versus time (sec). **Bottom.** Perturbation ω versus time (sec).

VI. CONCLUSION

A new adaptive-gain output feedback controller is presented in this paper. Finite-time real second order sliding mode is obtained in spite of bounded perturbations with unknown bound. The controller uses only the information of the sliding variable, its derivative is not required to adapt the gain and ensure the establishment of a real second order sliding mode.

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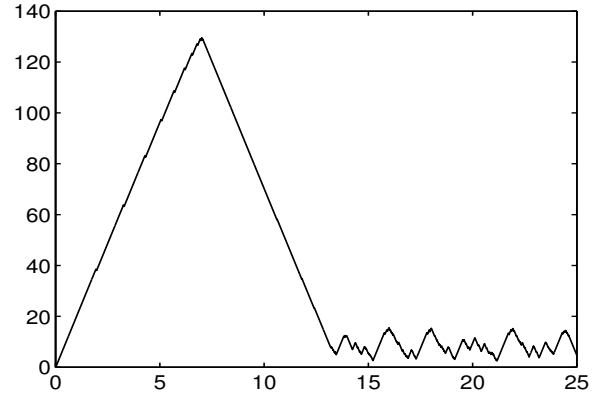


Fig. 6. Gain K_m for the adaptive scheme.

REFERENCES

- [1] C. Edwards, and S. Spurgeon, Sliding Mode Control: Theory and Applications, *Francis and Taylor*, 1998.
- [2] A. Levant *Sliding order and sliding accuracy*, International Journal of Control, vol. 58, pp.1247-1263, 1993.
- [3] A. Estrada, and F. Plestan *Second order sliding mode output feedback control: impulsive gain and extension with adaptation*, IEEE Conference on Decision and Control, Maui, Hawaii, USA, 2012.
- [4] L. Fridman *Chattering analysis in sliding mode systems with inertial sensors*, International Journal of Control, vol.76, no.9, pp.906-912, 2003. bibitemGua05 Z.H Guan, D. J. Hill and X. Shen *On hybrid impulsive and switching systems and application to nonlinear control*, IEEE Trans. Autom. Control, vol.50, no.7, pp.1058-1062, 2005.
- [5] A. Glumineau, Y. Shtessel, and F. Plestan *Control of a double integrator via discontinuous-impulsive feedback with adaptation* IFAC World Congress, Milano, Italy, 2011.
- [6] A. Levant *Higher order sliding mode, differentiation and output feedback control*, International Journal of Control, vol.76, pp.924-941, 2003.
- [7] Z. G. Li, C. Y. Wen, and Y. C. Soh *Analysis and Design of Impulsive Control Systems*, IEEE Trans. Autom. Control, vol.46, no.6, pp.894-897, 2001.
- [8] Y. Orlov. Discontinuous Systems: Lyapunov Analysis and Robust Synthesis Under uncertainty Conditions. *Springer*, London, 2008.
- [9] F. Plestan, E. Moulay, A. Glumineau and T. Cheviron *Robust output feedback sampling control based on second order sliding mode*, Automatica, vol.46, no.6, pp.1096-1100, 2010.
- [10] F. Plestan, Y. Shtessel, V. Brégeault, and A. Poznyak *New methodologies for adaptive sliding mode control*, International Journal of Control, vol.83, no.9, pp.1907-1919, 2010.
- [11] Y. Shtessel, M. Taleb, and F. Plestan *A novel adaptive-gain supertwisting sliding mode controller: methodology and application*, Automatica, to appear, 2012.
- [12] Vadim Utkin, Alex Poznyak and Patricio Ordaz *Adaptive Super-Twist Control with Minimal Chattering Effect*, IEEE Conference on Decision and Control - European Control Coinference, Orlando, Florida, USA, 2011.
- [13] Jose Kochalummootil, Yuri Shtessel, Jaime A. Moreno, and Leonid Fridman *Adaptive Twist Sliding Mode Control: a Lyapunov Design* IEEE Conference on Decision and Control - European Control Coinference, Orlando, Florida, USA, 2011.
- [14] R. Santiesteban, Leonid Fridman, and Jaime A. Moreno *Finite time convergence analysis for twisting controller via a strict Lyapunov function* Proceedings of 11th Workshop on Variable Structure Systems, pp 1-6, 2010.
- [15] T. Yang *Impulsive control*, IEEE Trans. Autom. Control, vol.44, no. 5, pp.1081-1083, 1999.