2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Robust optimization on receding horizon of processes with storages and periodic production and consumption contracts

Lubomír Baramov, and Vladimír Havlena, *Member, IEEE*

Abstract—**This paper presents a real-time optimizer for processes with material storages and periodic targets on integrals of optimized variables. A problem of robust target attainment with respect to uncertainties in future disturbance trajectory estimates was formulated. This robustness is achieved by means of soft constraints forcing future trajectories to a time-varying set from which reaching the target is feasible. An algorithm for computing this set is outlined, which is tractable for problems of medium size. The application area for this optimizer is assumed to be in (but not limited to) heat and power plant optimization with multiple fuels with periodic contracts on electric energy delivered and fuel gas consumed during the contract period.**

I. Introduction

THIS paper deals with real-time optimization of processes involving media (material or energy) flows that are subject to transformation in processing units. Flows entering the process are referred to as sources or raw materials, while those at the opposite end of the process are denoted as products. The flows are assumed to be one-directional and without recycles. Some of the product flows must satisfy external demands while some source flows are fixed by external supply. These fixed flows can be looked at as process disturbances. Moreover, some of the sources are received from, and some products are supplied to distribution networks. The total amounts of these sources consumed and products supplied over specific time intervals are subject to contracts with network operators. These contracts are periodically re-occurring with possibly re-negotiated targets values. Failures to fulfill the contracts are heavily penalized. The process may involve storages for some flows at additional costs of material deposition, storing and retrieval. Models of processing units as relations among media flows are assumed to be static and linear. The linearity assumption is restrictive in many cases; the effects of non-linearities can be handled as an uncertain disturbance. On the other hand, the process dynamics can usually be neglected on the optimization time scale. The goal of the optimization is to maximize the profit over a time horizon given by the difference of sale prices of products and the total costs, involving costs of materials, processing and storage costs as well as penalties for not fulfilling contracts. The re-occurrence of contracts requires solving the problem on a horizon of a multiple of the longest contracting interval.

The main application area for this optimizer is the production in combined heat and power plants combusting multiple fuels. One of the fuels may be a locally produced low grade fuel – e.g. biofuel, black liquor (pulp and paper industry), refinery byproducts. Other fuels usually include a high grade one, as e.g. natural gas, obtained via a distribution network and subject to 24-hour contracts. To guarantee combustion stability and emission limits, low-grade to high-grade fuel flow ratio is restricted. Products are steam to be timely delivered to other processes of the plant or to external customers and electric power subject to the contract with the grid operator, typically with a 1-hour period.

The above formulation of the profit maximization leads us to solving a *linear programming* (LP) problem on the receding horizon. LP-based optimizers tend to produce optimized variables changing significantly from one sampling interval to another, which is an unwelcome property in process optimization. Therefore, it is a common practice to re-formulate it as a *quadratic programming* (QP) problem by adding some conditioning penalty terms. This problem is routinely solved in advanced process control and dynamic real-time optimization, see e.g. [1], [2]. This formulation requires estimates of future disturbance trajectories. These estimates are made using production models which are not perfectly reliable; the simplest one assumes that the present value is constant on the optimization horizon, (the random walk disturbance model); more sophisticated models consider periodic load profiles as in [3]. While crude prediction models work well in control applications, where a poor estimate of future disturbance causes temporarily a sloppier response, in our case, it may cause failure to fulfill the contract, resulting in heavy penalties. Therefore, uncertainties in future disturbance trajectories are considered. Finding the best solution for the worst-case uncertainty in receding horizon is highly computationally demanding, as known from robust MPC control, see e.g., [4] – [8]. We propose an approach of solving robust target attainment where robustness is introduced by adding soft constraints. A robust reachability set for contracted variable trajectories is found so that if these trajectories belong to this set, feasible trajectories of optimization variables exist and the contract can be fulfilled for any future disturbance from the pre-defined set. Computing this robust reachability set is still demanding, but tractable. The robustness soft constraints thus force the

L. Baramov is with Honeywell Prague Laboratory, V Parku 18, Praha 4, 14800, Czech Republic (phone: +420-234-625-959; e-mail: lubomir.baramov@honeywell.com).

V. Havlena is with Honeywell Prague Laboratory and with Faculty of Electrical Engineering, Czech Technical University in Prague e-mail: vladimir.havlena@honeywell.com).

process trajectories, for each instant on the optimization horizon, to the region from which the target can be attained in the remaining time regardless of the uncertainty. As for some uncertainties a feasible robust trajectory may not exist, the problem formulation based on soft robustness bounds allows obtaining a solution of the lowest risk of missing the target. In this paper, we shall restrict ourselves to processes with up to two media under contract, covering most practical problems in the power and energy sectors. A simpler optimizer considering only the electricity generation contract without media storages was developed earlier, see, [9].

II. Problem Formulation

Let the free optimization variables be aggregated in vector $\mathbf{x}=\left[x_1 \quad \cdots \quad x_m\right]^T$ where the components are (non-negative) flows. Process is described by balance equations in nodes, as e.g. in Figure 1. Balance equation for node n is of the form

$$\sum_{i\in I_n} K_{ni} x_i(k) - \sum_{j\in O_n} K_{nj} x_j(k) = \sum_{l=1}^{n_p} K_{pl} p_l(k) - \sum_{m=1}^{n_s} K_{sm} s_m(k) \,. \quad (1)$$

There, I_n and O_n are, respectively, sets of flows entering and leaving node n. Variables p_l, are product flows fixed by external demand; correspondingly, s_m denotes external sources with fixed supply. Variables K_{ni}, K_{nj}, K_{pl} and K_{sm} denote non-negative transformation gains. Equations for all nodes are written in matrix form as

$$\tilde{\mathbf{A}}_{\mathbf{E}}\mathbf{x}(k) = \mathbf{b}_{\mathbf{E}}(k) \,. \quad (2)$$

Matrix $\tilde{\mathbf{A}}_{\mathbf{E}}$, is of full row rank, its non-zero elements correspond to the interconnection graph in the process topology. Recall that there are no flow recycles. Assume further that the process involves n_S storages, described by

$$V_{Sp}(k+1) = V_{Sp}(k) + T_s \cdot \left(\sum_{i\in I_p} x_i(k) - \sum_{j\in O_p} x_j(k) \right), \; p = 1,\dots,n_S \,. \quad (3)$$

There, V_{Sp} is the storage state, the amount of media stored; I_p and O_p are sets of flows into and out-of the storage, respectively. T_S denotes the sampling interval. Similarly, states are defined for each contract on media as

$$V_{Cr}(k+1) = \begin{cases} T_s x_{i_{cr}}(k) & \text{for } k \in K_r \\ V_{Cr}(k) + T_s x_{i_{cr}}(k) & \text{otherwise} \end{cases}, \; r = 1,\dots,m_c \,. \quad (4)$$

There, K_r is the set of end-of-contract times. Further, there are box constraints on flow variables and storages

$$0 \le \mathbf{x_L} \le \mathbf{x}(k) \le \mathbf{x_H}, \quad 0 \le V_{SpL} \le V_{Sp}(k) \le V_{SpL}, \quad p = 1,\dots,n_S \,. \quad (5)$$

Additional inequality constraints can be considered, as e.g., restricting the ratio of two flows,

$$x_i(k) - r_{ij} x_j(k) \le 0, \quad i \in I_r \quad (6)$$

The goal of the optimization problem is to minimize the following cost on a horizon of length N:

$$J\left(\mathbf{X}(k)\right) = J_B\left(\mathbf{X}(k)\right) + J_C\left(\mathbf{X}(k)\right) + J_S\left(\mathbf{X}(k)\right). \quad (7)$$

There, $\mathbf{X}(k) = \left[\mathbf{x}(k)^T \quad \dots \quad \mathbf{x}(k+N)^T\right]^T$ is the vector of current and future flow variables; the term $J_B\left(\mathbf{X}(k)\right)$ is the balance between the cost of sources and the price of products

$$J_B\left(\mathbf{X}(k)\right) = \sum_{i=0}^{N}\sum_{j=1}^{m} c_j x_j(k+i) \,. \quad (8)$$

Typically, costs of sources are non-negative numbers while prices of products are non-positive ones with possible exceptions; e.g., a product is a waste whose disposal costs money. Intermediate products may have positive costs associated with processing involved, that is not included in product prices, as e.g. the liquefaction cost of the gas to be temporarily stored. The cost of not fulfilling the contract is

$$J_C\left(\mathbf{X}(k)\right) = \sum_{j=1}^{m_c}\sum_{l\in K_j} c_{Cjl}\left|V_{Cj}(l) - t_{jl}\right|, \quad (9)$$

where $K_j = \left\{K_{j1},\dots,K_{jn_c}\right\}$ is the set of end-of-contract-instants, $0 \le K_{j1} < \dots < K_{jn_c} \le N$ and t_{jl} is the target for the amount of j^{th} contracted medium during the period ending at K_l. It is straightforward to show that penalty terms involving absolute values can be transformed, using appropriately constrained auxiliary variables into linear terms. Finally, term $J_S\left(\mathbf{X}(k)\right) = \sum_{p=1}^{n_S}\sum_{i=0}^{N} c_{Sp} V_{Sp}(k+i)$ penalizes storage costs.

To achieve a smoother real-time operation, it is worth including a low-weighted quadratic penalty term

$$J_D(\mathbf{X}(k)) = \sum_{i=0}^{N}\sum_{j=1}^{n} r_j \cdot \left(x_j(k+i) - x_j(k+i-1)\right)^2 \,. \quad (10)$$

Constraints are obtained by aggregating those of (2), (5), (6) for times $k, k+1, \dots, k+N$ into the matrix form

$$\mathbf{A_E X}(k) = \mathbf{B_E}(k), \quad \mathbf{AX}(k) \le \mathbf{B}(k) \,. \quad (11)$$

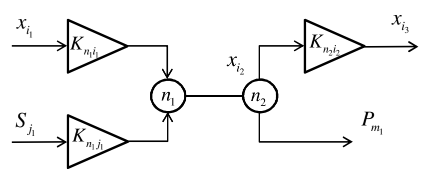

Figure 1 Flow balances in two nodes

III. Reformulation for Robust Target Attainment

A. Feasibility regions

Feasibility region for the above problem is given by

$$F\left(\mathbf{B_{EN}}(k)\right) = \left\{\mathbf{X} \mid \mathbf{A_E X} = \mathbf{B_E}(k), \mathbf{AX} \le \mathbf{B}(k)\right\}; \quad (12)$$

it is a polytope in the space of optimization variables. Vector $\mathbf{B}(k)$ depends only on the state of the storage at time k and is known and constant on the optimization horizon. Vector $\mathbf{B_E}(k)$ depends on future disturbances and can be decomposed, using (1), as follows

$$\mathbf{B_E}(k) = \mathbf{B_{E0}} + \mathbf{D_{Ep}P}(k) - \mathbf{D_{Es}S}(k) \,, \quad (13)$$

where $\mathbf{P}(k)$ and $\mathbf{S}(k)$ are, respectively, fixed future source supplies / product demands on the optimization horizon, as $\mathbf{P}(k)=\left[p_1(k) \quad \ldots \quad p_1(k+N) \quad \cdots \quad p_{n_p}(k) \quad \ldots \quad p_{n_p}(k+N)\right]^T$. The future disturbances are not known; they must be replaced by estimates and hence, are highly uncertain. Matrices $\mathbf{D_{Ep}}$ and $\mathbf{D_{Es}}$ have one positive element per column as follows from (1). $\mathbf{B_E}$ can be represented as

$$\mathbf{B_E}(k)=\mathbf{B_{EN}}(k)+\begin{bmatrix}\mathbf{D_{Ep}} & -\mathbf{D_{Es}}\end{bmatrix}\begin{bmatrix}\mathbf{\Delta_P}(k)\\ \mathbf{\Delta_S}(k)\end{bmatrix}, \qquad (14)$$

where $\mathbf{B_{EN}}(k)$ is the nominal vector obtained from (13) using estimates made at time k. Uncertainty $\mathbf{\Delta}=\begin{bmatrix}\mathbf{\Delta_P^T} & \mathbf{\Delta_S^T}\end{bmatrix}^T$ is assumed to be box-constrained, $\mathbf{\Delta_{min}}(k)\le\mathbf{\Delta}(k)\le\mathbf{\Delta_{max}}(k)$ for all k. *Robust feasibility region* is then given by

$$R_F\left(\mathbf{B_{EN}},\mathbf{\Delta_{min}},\mathbf{\Delta_{max}}\right)=\bigcap_{\mathbf{\Delta_{min}}\le\mathbf{\Delta}\le\mathbf{\Delta_{max}}} F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta}\right) \qquad (15)$$

where. $\mathbf{D_E}=\begin{bmatrix}\mathbf{D_{Ep}} & -\mathbf{D_{Es}}\end{bmatrix}$. As follows from convexity of the sets under discussion, it is sufficient to make the intersection in (15) only for corners of the box-shaped uncertainty region. But exploiting the problem inherent structure would reveal that only *two corners* need to be considered,

$$R_F\left(\mathbf{B_{EN}},\mathbf{\Delta_{min}},\mathbf{\Delta_{max}}\right)=F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_A}\right)\cap F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_B}\right), \qquad (16)$$

$$\mathbf{\Delta_A}=\begin{bmatrix}\mathbf{\Delta_{Pmax}}\\ \mathbf{\Delta_{Smin}}\end{bmatrix},\quad \mathbf{\Delta_B}=\begin{bmatrix}\mathbf{\Delta_{Pmin}}\\ \mathbf{\Delta_{Smax}}\end{bmatrix}. \qquad (17)$$

Deriving this result is omitted here for space consideration.

B. Target reachability sets

Targets are volumes of specific media produced or consumed during given periods specified by contracts with distribution network operators. These contracts are concerned a restricted set of flows x_i, $I_C=\{i_1,\cdots,i_{mc}\}$; the volumes of the media are time integrals of the corresponding flows starting from the end of the previous period, as in (4). The reachability sets are regions in the space of contracted volumes $\mathbf{V_c}(k)=\left[V_{ci_1}(k) \quad \cdots \quad V_{ci_{mc}}(k)\right]^T$. They are defined as follows: Assume at time K the volumes $\mathbf{V_c}(K)$ are required to belong to *a priori* defined target set $\overline{T}(K)$ (can be a single point). *Target reachability set* for time $k<K$ denoted as $T(k)$ is a region in $\mathbb{R}^{m_c}$ so that for any $\mathbf{V_c}(k)\in T(k)$ there exist a sequence of flows $\mathbf{x}(k),\ldots$, $\mathbf{x}(K-1)$ that are feasible satisfying (2), (5) and (6), such that $\mathbf{V_c}(K)\in\overline{T}(K)$. Set $T(k)$ is *robust target reachability set* if the target is reached for any feasible flow sequence under any perturbation in(14).

Reachability set is computed recursively. Set $T(K)=\overline{T}(K)$ and, for $j=K-k-1,\ldots,1,0$,

$$T(k+j)=\left\{Z-T_sY \mid Z\in T(k+j+1), Y\in C_{k+j}(F)\right\}. \qquad (18)$$

There, $C_{k+j}(F)$ are projections of set F defined in the space of optimized flows on the full horizon to the subspace of contracted flows at time $k+j$. To stress that the reachability sets depend on F, they shall be denoted as $T(k+i,F)$. Robust reachability sets are obtained as

$$\begin{aligned}T\left(k+j,R_F\left(\mathbf{B_{EN}},\mathbf{\Delta_{min}},\mathbf{\Delta_{max}}\right)\right)&=T\left(k+j,F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_A}\right)\right)\\ &\cap T\left(k+j,F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_B}\right)\right)\end{aligned} \qquad (19)$$

The optimization horizon may span over multiple contracting periods. Then recursion (18) is then reset at the end of every contract. Also, between the end of the last contract and the end of optimization horizon, reachability is set to $\mathbb{R}^{m_c}$.

C. Formulation of the robust optimization problem

The original optimization problem is robustified by adding a penalty term, imposing penalties on the squared distance of contract variable trajectories from the robust reachability regions. In particular, the robustified cost function is given by

$$\begin{aligned}&J_R\left(\mathbf{X}(k),\mathbf{Z_A}(k),\mathbf{Z_B}(k)\right)=\\ &J\left(\mathbf{X}(k)\right)+W\sum_{i=1}^{K}\left\|\begin{bmatrix}\mathbf{V_c}(k+i)\\ \mathbf{V_c}(k+i)\end{bmatrix}-\begin{bmatrix}\mathbf{z_A}(k+i)\\ \mathbf{z_B}(k+i)\end{bmatrix}\right\|^2,\end{aligned} \qquad (20)$$

where W is *a priori* chosen weight and $\mathbf{z_A}$ and $\mathbf{z_B}$ are auxiliary vectors; constraints on these variables are imposed for all $i=1,\ldots,N$ as follows:

$$\begin{aligned}\mathbf{z_A}(k+i)&\in T\left(k+i,F(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_A})\right)\\ \mathbf{z_B}(k+i)&\in T\left(k+i,F(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_B})\right).\end{aligned} \qquad (21)$$

Constraints (21) are convex polytopic regions in the $\mathbb{R}^{m_c}$ space; hence, they can be represented by a standard set of linear inequalities of the form

$$\left.\begin{aligned}\mathbf{A_{CAi}}\mathbf{z_{Ai}}(k+i)&\le\mathbf{B_{CAi}}\\ \mathbf{A_{CBi}}\mathbf{z_{Bi}}(k+i)&\le\mathbf{B_{CBi}}\end{aligned}\right\} i=1,\ldots,K_{nc} \qquad (22)$$

This formulation may resemble the concept of range control known from process control, see [2] , [10]– [12]. The difference is, that while the range control concept uses box-shaped regions in each time-frame, (22) is more general. Further, the region is not operator-defined, but computed from uncertainty bounds. Note also that this formulation can be feasible even if the intersection in (19) is empty. Then, the optimal trajectory passes between these disjoint sets, as the best trade-off for robustness. The target can still be attained when the worst-case scenario does not happen. Finding the regions in (21), or their suitable approximations will be dealt with in the following sections.

IV. Computing Target Reachability Sets

A. Approximation of the robust feasibility set projections

As was shown in the previous section, to compute target reachability sets needed for robustification of the optimization problem, set-sequence $\left\{C_{k+j}(F)\right\}_{j=0}^{N}$ is needed, where $C_{k+j}(F)$ is the projection of feasibility region F to the space of contracted flows at time $k+j$. Set F is a convex polytope described by equations and inequalities (12). To obtain its projections, a vertex representation of this polytope is needed. The projection is then obtained as a projection of vertices, which is straightforward. Finding the set of vertices (called vertex enumeration problem, see [13] and [14]) in spaces of high dimension and a large number of inequalities is computationally highly demanding. Here we shall look for a suitable set $\tilde{C}_{k+j}(F) \subset C_{k+j}(F)$ whose computation is tractable and as close to $C_{k+j}(F)$ as possible. The media under contract are restricted to one source and one product, denoted as S and P, respectively.

It is first assumed that there exist points $\mathbf{X}^{*}_{\mathbf{Cmax}}(k+i,F)$ and $\mathbf{X}^{*}_{\mathbf{Cmin}}(k+1,F)$ in $\tilde{C}_{k+i}(F)$; the former is defined as

$$\mathbf{X}^{*}_{\mathbf{Cmax}}(k+i,F)=\begin{bmatrix}\overline{S}^{*}_{k+i}\\ \overline{P}^{*}_{k+i}\end{bmatrix}, \tag{23}$$

$$\overline{S}^{*}_{k+i}=\max\left\{S \mid \begin{bmatrix}S\\P\end{bmatrix}\in \tilde{C}_{k+i}(F)\right\}, \overline{P}^{*}_{k+i}=\max\left\{P \mid \begin{bmatrix}S\\P\end{bmatrix}\in \tilde{C}_{k+i}(F)\right\}. \tag{24}$$

The point $\mathbf{X}^{*}_{\mathbf{Cmin}}$ is defined analogously. This assumption is justifiable in the given context – we can expect that the largest product demand can be satisfied under the largest input supply and vice-versa. Then, set $\tilde{C}_{k+i}(F)$ is bounded by a union of two piecewise-affine boundary segments:

$$B_{k+i}(F)=B^{1}_{k+i}\cup B^{2}_{k+i} \tag{25}$$

where

$$B^{1}_{k+i}=\left\{\left[\underline{S}_{k+i}(P)\ \ P\right]^{T}\middle|\ P\in\mathbb{R}\right\}\cup\left\{\left[S\ \ \overline{P}_{k+i}(S)\right]^{T}\middle|\ S\in\mathbb{R}\right\}, \tag{26}$$

$$B^{2}_{k+i}=\left\{\left[\overline{S}_{k+i}(P)\ \ P\right]^{T}\middle|\ P\in\mathbb{R}\right\}\cup\left\{\left[S\ \ \underline{P}_{k+i}(S)\right]^{T}\middle|\ S\in\mathbb{R}\right\}, \tag{27}$$

$$\overline{S}_{k+i}(P)=\max\left\{S \mid \left[S\quad P\right]^{T}\in\tilde{C}_{k+i}(F)\right\}; \tag{28}$$

$\underline{S}_{k+i}$, $\overline{P}_{k+i}$ and $\underline{P}_{k+i}$ obtained analogously. Both segments B^{1}_{k+i} and B^{2}_{k+i} share the extreme points $\mathbf{X}^{*}_{\mathbf{Cmax}}(k+i,F)$ and $\mathbf{X}^{*}_{\mathbf{Cmin}}(k+i,F)$. These points and boundary segments are shown on a sample projection in Figure 2. For the robust reachability set we need the intersection $\tilde{C}_{k+i}\left(F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_A}\right)\right) \cap \tilde{C}_{k+i}\left(F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_B}\right)\right)$. From the nature of the problem it follows that the intersection is bounded by the boundary segment $B^{2}_{k+i}\left(F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta}_{B}\right)\right)$ from one side and by $B^{1}_{k+i}\left(F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_A}\right)\right)$ from the other as is shown in Figure 2. Hence, only these two boundary segments need to be computed.

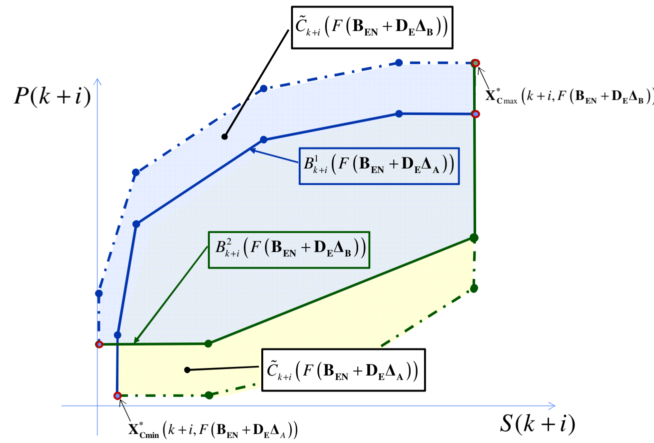

Figure 2 Sets $\tilde{C}_{k+i}\left(F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta}_{A}\right)\right)$ and $\tilde{C}_{k+i}\left(F\left(\mathbf{B_{EN}}+\mathbf{D_E}\mathbf{\Delta_B}\right)\right)$, their boundaries and extreme points.

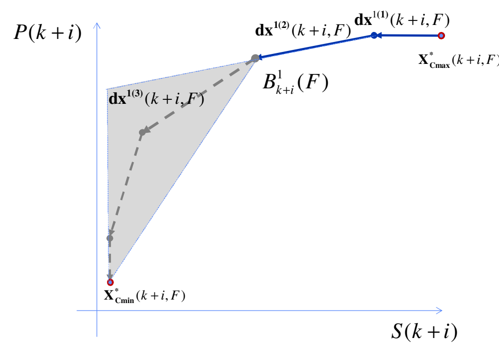

Figure 3 Boundary set computation

These boundaries are obtained by solving a series of optimization problems. It is necessary to realize that the overall feasibility must be satisfied; thus these optimizations are solved on the full flow space and over the optimization horizon. The coupling of the individual time frames is due to the constraints on internal storages. If there were not internal storages, the problem can be addressed separately for each time-frame, saving a tremendous amount of computation.

The algorithm traces the boundaries from one extreme point to another, searching for vertices, as in Figure 3. The goal is to get as close to the boundaries of $\left\{C_{k+i}(F)\right\}_{i=1}^{N}$ as possible. Additional constraints are imposed to guarantee convexity and extreme point properties. Details are omitted here.

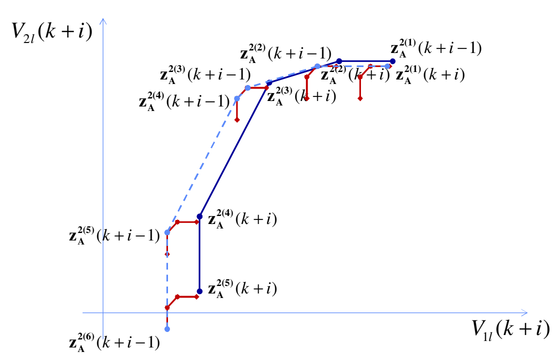

Figure 4 Iterating boundary of the reachability set in time backwards.

B. Transforming feasibility sets to target reachability sets

Target reachability sets were defined above in Section III.B. Here we shall describe algorithm for obtaining this set from projections of the feasibility set on the contracted flows subspace outlined in the previous paragraph. These projections are approximated by polytopes bounded by segments $B^1_{k+i}(F(\mathbf{B}_{\mathbf{EN}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{A}}))$ and $B^2_{t+k}(F(\mathbf{B}_{\mathbf{EN}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{B}}))$. They are represented as sets of vertices,

$$\begin{aligned} B^1_{k+i}(F(\mathbf{B}_{\mathbf{EN}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{B}})) &= \left\{\mathbf{x}^{\mathbf{1(0)}}_{\mathbf{CA}}(k+i),....,\mathbf{x}^{\mathbf{1(M_{1A})}}_{\mathbf{CA}}(k+i)\right\}, \\ B^2_{k+i}(F(\mathbf{B}_{\mathbf{EN}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{B}})) &= \left\{\mathbf{x}^{\mathbf{2(0)}}_{\mathbf{CB}}(k+i),....,\mathbf{x}^{\mathbf{2(M_{2B})}}_{\mathbf{CB}}(k+i)\right\}. \end{aligned} \tag{29}$$

These boundary segments in the disturbance flow spaces are transformed to boundary segments of robust target reachability set in the space of corresponding volumes. These boundary segments are denoted as

$$\begin{aligned} D^2_{k+i}(F(\mathbf{B}_{\mathbf{E0}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{A}})) &= \left\{\mathbf{z}^{\mathbf{2(0)}}_{\mathbf{A}}(k+i),....,\mathbf{z}^{\mathbf{2(n_{2A})}}_{\mathbf{A}}(k+i)\right\}, \\ D^1_{k+i}(F(\mathbf{B}_{\mathbf{E0}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{B}})) &= \left\{\mathbf{z}^{\mathbf{1(0)}}_{\mathbf{B}}(k+i),....,\mathbf{z}^{\mathbf{1(n_{1B})}}_{\mathbf{B}}(k+i)\right\}. \end{aligned} \tag{30}$$

These boundaries are known *a priori* for or the end-of-contract times $i = K_l$. The recursion (18) is done, for segment $D^2_{k+i}(F(\mathbf{B}_{\mathbf{E0}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{A}}))$ in the vertex representation for $i = K_l - 1,..., K_{l-1}+1$ as follows:

1. Get intermediate sets
$$\begin{aligned} &\bar{D}^2_{k+i}(F(\mathbf{B}_{\mathbf{E0}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{A}})) = \\ &\left\{\mathbf{z}^{\mathbf{2(s)}}_{\mathbf{A}}(k+i+1) - T_s \cdot \mathbf{x}^{\mathbf{1(r)}}_{\mathbf{cA}}(k+i)\right\}_{\substack{s=1,...,n_{2A} \\ r=0,...,M_{1A}}}; \end{aligned} \tag{31}$$
2. Obtain $D^2_{k+i}(F(\mathbf{B}_{\mathbf{E0}}+\mathbf{D}_{\mathbf{E}}\boldsymbol{\Delta}_{\mathbf{A}}))$ as the smallest vertex sub-sets of (31), creating its convex hull, see Figure 4.
3. Eliminate redundant vertices to prevent uncontrolled growth of their number as i decreases: those whose adjacent edges are nearly co-linear as well as those lying on an edge that is in an infeasible region.
4. Transform the vertex representation of the boundary to one given by inequalities as in (22).

The algorithm is analogous for the other boundary segment.

V. Example – CHP Optimization

An example of a process in combined heat and power plant (CHP) is shown in Figure 5. It includes one boiler with three fuels and a turbo-generator. In fact, they may be virtual units aggregating multiple parallel boilers and turbines, respectively. There are three sources – boiler fuels: First, natural gas, denoted as F_1, as the high grade fuel of the highest price. It is subject to periodic contracts with distributor. It can be stored in a liquefaction tank, which adds some freedom for optimization, but it is expensive: gas liquefaction needs energy, as well as cooling the tank. The second fuel, denoted as F_2 is oil; as a liquid it can be stored cheaply so that it does not pose tight constraints, beyond the range for its flow. Finally, the third fuel is CO gas, which is a refinery waste that cannot be stored or processed in other way than burning (for safety reasons, due to its toxicity): either in the boiler, or uselessly in the flare. At the product side, there is heat in the form of process steam P_1; it has to follow external demands. The other product P_2, is electric power supplied to the grid; it is subject to contract with the grid operator. Product P_3 is added formally, it is CO gas burned in flare. Some intermediates arise in the process: I_1 is the volume of natural gas supplied to burners, while I_2 and I_3 are flows of this gas to be stored and retrieved from the storage, respectively. These have a unit price attached as storage/retrieval requires energy.

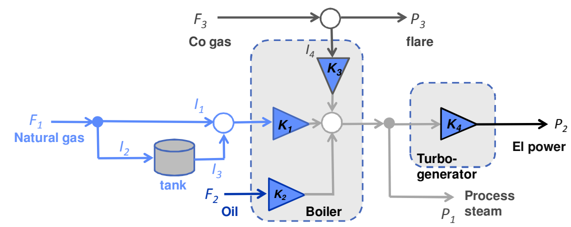

Figure 5 Process topology

Balance equations corresponding to the process topology in Figure 5 are as follows

$$\begin{aligned} &F_1 - I_1 - I_2 = 0, \quad I_4 + P_3 = F_3, \\ &K_4\left(K_1(I_1+I_3) + K_2F_2 + K_3I_4\right) - P_2 = K_4P_1 \end{aligned} \tag{32}$$

Coefficients K_1 to K_4 are transformation gains. Contracts on natural gas and electricity are periodic; in simulations, the contracting period was set to 50 time steps and is synchronized for both media; target values are updated for the second contracting period within the horizon. Current values of CO gas and process steam flows are prescribed and their future trajectories are estimated. The uncertainties for the future values were chosen to a bound of 20% of their ranges around the nominal estimates from step 25 of the horizon forward; the uncertainty box increases linearly from zero for the current time to the maximum size at step 25 and beyond. Horizon length is equal to 100 steps. However, the so called blocking strategy is applied: from prediction step 25 on, the flows are allowed to change every 4th step, to reduce complexity. Further, the inequality constraints include ranges for all flows involved, for the storage state as well as a bound on the fuel mixture ratio in order to stabilize combustion and to reduce emissions

$$-(I_1+I_3) + R_{\min}I_4 \le 0\,. \tag{33}$$

Figure 6 shows predicted trajectories of accumulated volumes of contracted flows as well as the storage state. The evolution of robust reachability sets for accumulated volumes of contracted media is in Figure 7. It shows robust reachability sets for three future time points: $i = 25$, 35 and 45; the first two preceding the first contract time end, the last one follows shortly after it. It can be observed that the robust optimizer is able to steer the process close to the target.

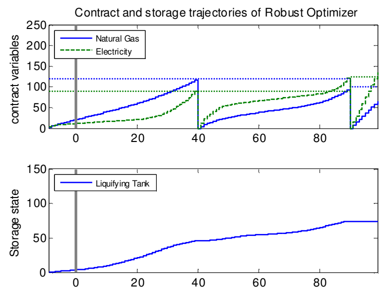

Figure 6 Predicted trajectories of the contract variables and the storage state.

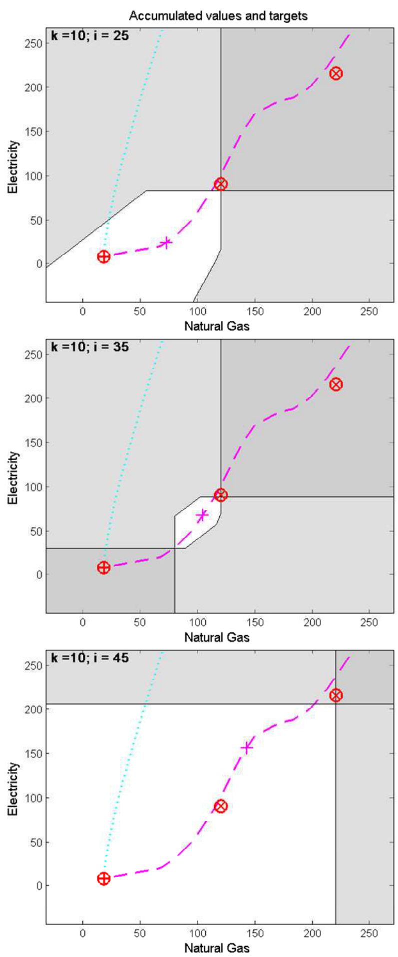

Figure 7 Robust target reachability sets at three points of the horizon (white area). Combined xo-marks: starting point and targets. + mark: trajectory point for the robust optimizer. Magenta, dashed line: Projection of the trajectory of contract variables.

ACKNOWLEDGMENT

This research was supported from Grant TA01020871 of the Technology Agency of the Czech Republic.

REFERENCES

[1] Qin, S.J., and Badgwell, T.A.: A survey of industrial model predictive control technology. *Control Engineering Practice*, Vol 11, pp. 733—764, 2003.

[2] Havlena, V. and Lu. J. "A Distributed Automation Framework For Plant-Wide Control, Optimization, Scheduling and Planning," *Proc 16th IFAC World Congress,* Prague, Czech Republic, 2005.

[3] Dotzauer, E.: Simple model for prediction of loads in district-heating systems, *Applied Energy* 73, 2002, pp 277—284.

[4] Bemporad, A. Morari, M., "Robust Model Predictive Control: A Survey", *Lecture Notes in Control and information Sciences 245*, pp. 207—226, 1999.

[5] Lee, J.H. and Yu, Z.: "Worst-case Formulation of Model Predictive Control for Systems with Bounded Parameters", *Automatica*, Vol 33, No 5, pp 763—781, 1997.

[6] Scokaert, P.O.M and Mayne, D.Q.: Min-Max Feedback Model Predictive Control for Constrained Linear Systems. IEEE Trans Auto Contr, Vol 43, No 8, pp1136—1142, 1998.

[7] Munoz de la Pena, D., Alamo, T. Ramirez, D. R. and Camacho, E.F., "Min-Max Model predictive control as a quadratic program" in *Proc 16th IFAC World Congress,* Prague, Czech Republic, 2005.

[8] Ramirez, D. R., Alamo, T. and Camacho, E.F., "Computationally Efficient Min-Max MPC" in *Proc 16th IFAC World Congress,* Prague, Czech Republic, 2005.

[9] Havlena, V. and Findejs. J., "*Predictive Contract System and Method"*, US Patent 2007/0067068

[10] Havlena, V. and Findejs, J." "Application of model predictive control to advanced combustion control," *Control Engineering Practice*, 13 (6), pp 671—680, 2005

[11] Lu, J.Z. and Escarcega, J.: RMPCT: Robust MPC Technology Simplifies APC. *AIChE 1997 Spring Meeting, Houston*.

[12] Lu, J.Z.: "Challenging control problems and emerging technologies in enterprise optimization," 6th IFAC Symposium on Dynamics and Control Process Systems, Cheju, Korea, 2001.

[13] Avis, D. Fukuda, K. "A pivoting algorithm for convex hulls and vertex enumeration of arrangements and polyhedra". *Discrete and Computational Geometry* **8** ,295–313, 1992.

[14] Kvasnica, M., Grieder, P. and Baotic, "*Multi-Parametric Toolbox (MPT)"*, http:\\ http://control.ee.ethz.ch/~mpt/, 2004.