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Terminal Sliding Mode Impedance Control for Bilateral Teleoperation under Unknown Constant Time Delay and Uncertainties

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Abstract— **Finite time control of a teleoperation system under unknown constant time delay and parametric uncertainties is considered using terminal sliding mode and impedance control. Terminal sliding mode controller is a finite time control scheme that provides finite time convergence to the equilibrium point as well as fast response and high tracking precision in comparison with conventional sliding mode controllers, which are widely used for control of teleoperators. The proposed control scheme for bilateral teleoperation under unknown constant time delay is a sliding mode-impedance control for master robot and a terminal sliding mode control for slave robot. These two control laws can provide perfect tracking and transparency that are main goals in bilateral teleoperations. Simulation results verify the effectiveness of the proposed scheme for bilateral teleoperation under time delay.**

I. INTRODUCTION

Teleoperation indicates operating at a distance and defines the idea of a user interacting with and operating on a remote environment [1], [2]. The application of teleoperation systems ranges from space robots, unmanned underwater vehicles, operating in hazardous environments, mobile robots to telesurgery. It is desired that a teleoperation system has two main characteristics, *stability* and *transparency*. The system must remain stable despite various behaviors of the operator or the environment. *Transparency* or *telepresence* defines the sense of telepresence of the operator in the environment [1].

Another challenge in the control of teleoperators is the time-delay existing in the communication channel that enables remote operation. Many controller architectures have been proposed for dealing with these problems [3], [4]. Passivity based methods have been studied extensively and can be classified in three groups of scattering based, damping injection and adaptive controllers (see [4] for a recent historical survey). The methods based on input-to-state stability (ISS) can guarantee boundedness of the tracking errors in the case of non-vanishing perturbations and non-passive human operation [5], [6], [7]. Beside these methods, sliding mode controllers have been used as an effective tool for control of teleoperation systems with time-delay [3]. Sliding mode controllers ensure asymptotic stability of the closed-loop system even in the presence of non-idealities of the model of the system and parametric uncertainties and hard nonlinearities. Park and Cho [8] proposed an impedance controller for the master and a sliding mode controller for the slave of a teleoperation system with time-varying delay, in which the parameters of the sliding mode controller are independent of the time delay. In [9], they proposed a sliding-mode-based impedance controller to reduce the contact forces between slave and environment. Also, in [10] the signum function in the sliding mode controller is replaced with a saturation function to avoid chattering in the control signal. As a result, the tracking performance degrades and the system loses asymptotic stability. Garcia-Valdovinos et al. [11] proposed a second order sliding mode controller with linear observers that can guarantee asymptotic stability against constant time delays and provide a free of chattering control signal. The proposed scheme does not achieve perfect transparency. Gonzalez et al. [12] also proposed a second order sliding mode control with a nonlinear observer estimating velocity and acceleration designed via the super twisting algorithm.

In this paper, a finite time convergent sliding mode controller called terminal sliding mode is proposed for a teleoperation system with time delay and parametric uncertainties. Terminal sliding mode controllers are a class of finite time controllers that yield fast and finite time convergent of the response to the desired path and high tracking precision as well as robustness properties [13]. This approach has been effectively applied to control robot manipulators [14], [15]. The terminal sliding mode controller used in this paper [15] is continuous and therefore chattering-free. The proposed control scheme is a wise combination of terminal sliding mode control and impedance control that can provide perfect tracking and transparency in the presence of time delay. This is the first time that finite time controllers with terminal sliding mode are used effectively for both of the master and slave systems to control a teleoperation system having both parametric uncertainties and time delays and bring about both transparency and perfect tracking for the teleoperation system.

This paper is organized as follows. In Section II the teleoperation system and its dynamic equation are defined. The terminal sliding mode and impedance control are proposed in Section III. The stability analysis is done in Section IV. The effectiveness of the proposed control scheme is examined in Section V, and finally, the concluding remarks and contributions of this work are enlightened in the last section.

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II. TELEOPERATION SYSTEM

In a teleoperation system, the human operator is in contact with the master manipulator and applies force on it. As

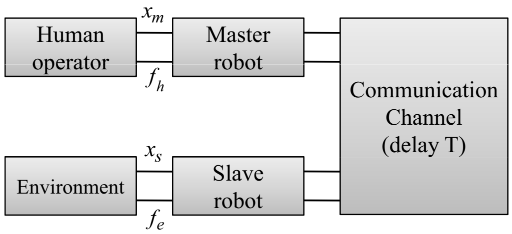

Fig. 1. Teleoperation system scheme

a result, the master is displaced and at the slave side, the slave manipulator mimics this displacement. The necessary information from the master and the operator is transmitted via the communication channel. The slave manipulator motion is affected by this information and the environment. The information of force, position and velocity of both of the manipulators can be transmitted *bilaterally* through the communication channel. The teleoperation system scheme is shown in Fig. 1. The dynamics of the 1-dof master/slave systems are as the following [1]

$$M_m \ddot{x}_m + B_m \dot{x}_m = u_m + f_h \tag{1}$$

$$M_s \ddot{x}_s + B_s \dot{x}_s = u_s - f_e \tag{2}$$

where x_i, $\dot{x}_i$ and $\ddot{x}_i$ denote position, velocity and acceleration respectively; M_i and B_i are mass and viscous friction coefficient, respectively; u_i is the control signal, with i=m, s which denote the master and slave. f_h is the force imposed by the human on the master, and f_e is the force exerted on the slave by the environment. The parameters of the master and slave systems are uncertain and change in the following range:

$$\underline{M}_m < M_m < \overline{M}_m, \;\; \underline{B}_m < B_m < \overline{B}_m$$

$$\underline{M}_s < M_s < \overline{M}_s, \;\; \underline{B}_s < B_s < \overline{B}_s$$

The desired position and contact force for master and slave are

$$x_{sd}(t) = x_m(t - T), \tag{3}$$

$$f_{ed}(t) = f_h(t - T) \tag{4}$$

where T denotes the delay induced by the communication channel and the subscript d denotes *desired*.

Remark 1: It must be noted that the desired characteristics in Eqs. (3) and (4) are equivalent to desired characteristics of a teleoperation system, i.e. perfect tracking and transparency. The first equation says that the slave position must be equal to the master position after passing the time delay T and It is is obvious that this is the definition of perfect tracking in a teleoperation system with time delay. To show that the second equation is the result of perfect transparency in the system, we first introduce some parameters. $Z_e = \frac{F_e}{V_s}$ is the environment impedance and $Z_t = \frac{F_h}{V_m}$ is the impedance transmitted to or felt by the operator, where F_e, F_h, V_s andV_m are Laplace transforms of f_e, f_h, $\dot{x}_s$ and $\dot{x}_m$ respectively. Now the transparency condition is defined as [16]

$$Z_t = Z_e \tag{5}$$

From the definition of the impedances, we can write the above condition in the form

$$\frac{F_h}{V_m} = \frac{F_e}{V_s}. \tag{6}$$

From Eq. (3), $V_s = V_m e^{-sT}$ and therefore,

$$\frac{F_h}{V_m} = \frac{F_e}{V_m e^{-sT}}, \tag{7}$$

that is

$$F_e = F_h e^{-sT}. \tag{8}$$

The equivalent representation of Eq. (8) in time domain is Eq. (4).

III. Controller Design

A. Sliding Mode-Impedance Controller for the Master

Master controller produces a desired dynamic behavior between the human operator and the master robot, robust to parametric uncertainties and unknown time delay. If we choose the desired impedance the same as environment, the operator will have the sense of direct operation on the environment. The desired master impedance is $M_e s + B_e + K_e 1/s$ and the desired dynamics of the master is

$$\frac{F_h}{V_m} = M_e s + B_e + K_e \frac{1}{s} \tag{9}$$

and in the time domain

$$M_e \ddot{x}_m + B_e \dot{x}_m + K_e x_m = f_h \tag{10}$$

where M_e, B_e, K_e are the inertia, damping coefficient and stiffness of the environment which is the desired impedance. Therefore, the system impedance error is

$$I_e(t) = M_e \ddot{x}_m + B_e \dot{x}_m + K_e x_m - f_h \tag{11}$$

Now, the sliding surface is defined as follows

$$s = \frac{1}{M_e} \int_0^t I_e(\tau) d\tau = 0 \tag{12}$$

Substituting (11) in (12), and integrating, we obtain

$$s = \dot{x}_m + \frac{B_e}{M_e} x_m + \frac{K_e}{M_e} \int_0^t x_m(\tau) d\tau - \frac{1}{M_e} \int_0^t f_h(\tau) d\tau \tag{13}$$

When the system is in the sliding mode, it has reached the surface, the state trajectory continues to be on the sliding surface (i.e. $s = 0$) so that $\dot{s} = 0$. In this case, from (13) we obtain $I_e = M_e \dot{s} = 0$.

Consider the following master control law

$$\begin{aligned} u_m &= u_{0m} + u_{1m} \qquad (14) \\ u_{0m} &= (\frac{\hat{M}_m}{M_e} - 1) f_h + (\hat{B}_m - \frac{\hat{M}_m B_e}{M_e}) \dot{x}_m - \frac{\hat{M}_m K_e}{M_e} x_m \\ u_{1m} &= -\hat{M}_m K_{cm} sign(s) \end{aligned}$$

where $sign(.)$ denotes the signum operator. Here $\hat{M}_m$, $\hat{B}_m$ are nominal values of M_m and B_m, respectively and are assumed to be

$$\hat{M}_m = \sqrt{\overline{M}_m \underline{M}_m},\ \hat{B}_m = \frac{\overline{B}_m + \underline{B}_m}{2}$$

and the parameter K_{cm} is

$$\begin{aligned} K_{cm} &= \sqrt{\frac{\overline{M}_m}{\underline{M}_m}} \Bigg(\eta_1 + \frac{1}{M_e} (\sqrt{\frac{\underline{M}_m}{\overline{M}_m}} + 1) \, |f_h| \\ &+ \left(\frac{1}{\overline{M}_m} (\frac{\overline{B}_m - \underline{B}_m}{2}) + \left| \frac{B_e}{M_e} \right| (\sqrt{\frac{\underline{M}_m}{\overline{M}_m}} + 1) \right) |\dot{x}_m| \\ &+ \left| \frac{K_e}{M_e} \right| (\sqrt{\frac{\underline{M}_m}{\overline{M}_m}} + 1) \, |x_m| \Bigg) \end{aligned} \quad (15)$$

in which $\eta_1 > 0$ and $|.|$ denotes the absolute value operator.

B. Terminal Sliding Mode Controller for the Slave

The slave control design is based on a terminal sliding mode (TSM) controller introduced in [15]. This slave controller is designed to render the tracking error between the delayed master position and the slave position to zero in finite time. In conventional sliding mode (SM) controller, the time of reaching the tracking error to zero is infinite, however the time of reaching to the sliding surface is finite. In other words, the tracking error converges to zero asymptotically in conventional sliding mode but terminal sliding can provide convergence in a finite time. The tracking error is as follows

$$e = x_{sd}(t) - x_s(t) \quad (16)$$

$$e = x_m(t - T) - x_s(t) \quad (17)$$

The nonsingular terminal sliding surface is

$$s = e + \beta |\dot{e}|^{\gamma} sign(\dot{e}) = 0 \quad (18)$$

where $\beta > 0$ and $1 < \gamma < 2$. This is equivalent with

$$\dot{e} + \beta' |e|^{\gamma'} sign(e) = 0 \quad (19)$$

where $\beta' = \beta^{-1/\gamma} > 0$ and $1/2 < \gamma' = 1/\gamma < 1$. The sliding surface (18) is continuous and differentiable although the absolute and signum operators are involved [15]. The first derivative of the sliding surface is

$$\dot{s}(t) = \dot{e} + \beta\gamma |\dot{e}|^{\gamma - 1} \ddot{e} \quad (20)$$

The slave controller is designed as

$$\begin{aligned} &u_s = u_{0s} + u_{1s} \\ &u_{0s} = \hat{M}_s(\beta^{-1}\gamma^{-1}|\dot{e}|^{1-\gamma}\dot{e} + \ddot{x}_m(t-T)) + \hat{B}_s\dot{x}_s + f_e \\ &u_{1s} = \hat{M}_s K_{cs} sign(s) \end{aligned} \quad (21)$$

where $k_1, k_2 > 0$, $0 < \rho < 1$. Here $\hat{M}_s$, $\hat{B}_s$ are nominal values of M_s and B_s, respectively and are assumed to be

$$\hat{M}_s = \sqrt{\overline{M}_s \underline{M}_s},\ \hat{B}_s = \frac{\overline{B}_s + \underline{B}_s}{2}.$$

In the absence of acceleration sensor, we can rewrite the controller as

$$\begin{aligned} u_s &= u_{0s} + u_{1s} \quad (22) \\ u_{0s} &= \hat{M}_s \beta^{-1}\gamma^{-1}|\dot{e}|^{1-\gamma}\dot{e} + \hat{B}_s\dot{x}_s + f_e \\ &+ \frac{\hat{M}_s}{M_e}(f_h(t-T) - B_e\dot{x}_m(t-T) - K_e x_m(t-T)) \\ u_{1s} &= \hat{M}_s K_{cs} sign(s) \end{aligned}$$

where

$$\begin{aligned} K_{cs} &= \sqrt{\frac{\overline{M}_s}{\underline{M}_s}} \Bigg(\eta_2 + \Big((\sqrt{\frac{\underline{M}_s}{\overline{M}_s}} + 1) |\ddot{x}_m(t-T)| \\ &+ \frac{1}{\underline{M}_s} (\frac{\overline{B}_s - \underline{B}_s}{2})) |\dot{x}_s| \Big) \\ &+ \beta^{-1}\gamma^{-1} (\sqrt{\frac{\underline{M}_s}{\overline{M}_s}} + 1) |\dot{e}|^{2-\gamma} \Bigg) \end{aligned} \quad (23)$$

in which $\eta_2 > 0$. The proposed controller guarantees the convergence of the tracking error e to zero in finite time.

C. Discussion

Remark 2: It must be noted that the slave controller guarantees precise tracking of the master displacement by the slave controller, and the master controller guarantees the transparency of the teleoperation system by putting $\frac{F_h}{V_m} = Z_e$, which is equivalent with the transparency condition in Eq. (5). Therefore, the proposed controllers for the master and slave can provide main requisites of a teleoperation system, i.e. *stability*, *tracking* and *transparency*. The sliding mode controller for the slave can be replaced with a sliding mode-based impedance control to reduce the impact forces between the slave and the environment, but in this case, we'll lose the perfect tracking of the master by the slave.

Remark 3: It is known that the signum function $sign(.)$, used in Eqs. (14) and (22), can cause chattering in the control signal. To alleviate this chattering, signum function $sign(s)$ can be replaced by the saturation function $sat(s/\varepsilon)$:

$$sat(s/\varepsilon) = \begin{cases} \frac{s}{|s|}, & |s| \geq \varepsilon \\ \frac{s}{\varepsilon}, & |s| < \varepsilon \end{cases} \quad (24)$$

where $\varepsilon > 0$.

Remark 4: The terminal sliding controllers can be designed in a way that the phase of reaching the trajectories to the sliding surface happens faster. This is called fast terminal sliding reaching law. In this case, the master controller in Eq. (14) is redesigned as:

$$u_{1m} = -\hat{M}_m K_{cm}(sign(s) + k_{1m}|s|^{\rho_1} sign(s) + k_{2m}s) \quad (25)$$

where $k_{1m}, k_{2m} > 0$ and $1/2 < \rho_1 < 1$. As well, the slave controller in Eq. (22) can be redesigned as:

$$u_{1s} = \hat{M}_s K_{cs}(sign(s) + k_{1s}|s|^{\rho_2} sign(s) + k_{2s}s) \quad (26)$$

where $k_{1s}, k_{2s} > 0$ and $1/2 < \rho_2 < 1$.

Remark 5: It is obvious from transparency condition in Eq. (5) that the parameters of the environment must be

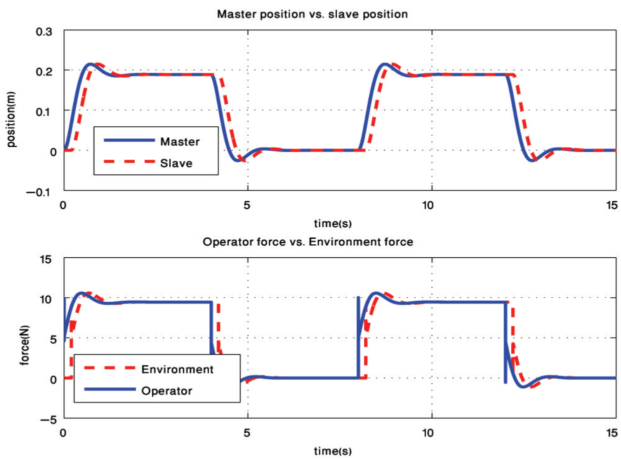

Fig. 2. Position and force tracking results for terminal sliding mode controllers with one-way time delay $T = 200ms$

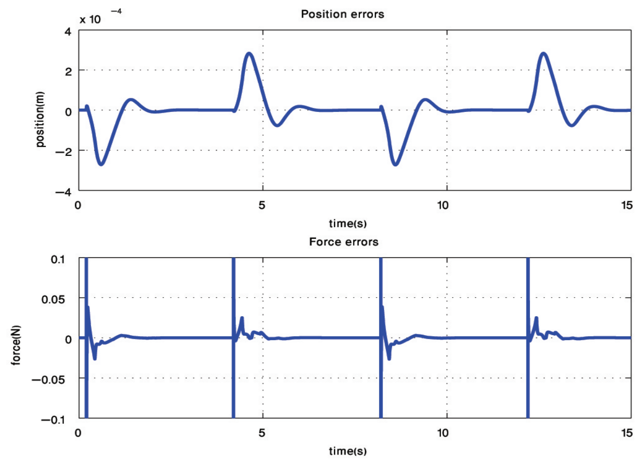

Fig. 3. Position and force errors for terminal sliding mode controllers with one-way time delay $T = 200ms$

known for achieving a perfectly transparent teleoperation system. In the case of not having the exact value of the environment parameters, one can replace them by parameters of a desired impedance but perfect transparency is lost.

IV. Stability Analysis

In this section, the stability analysis of the closed-loop dynamics of both master and slave systems is studied using the Lyapunov theory. First we define finite time stability and its Lyapunov description.

Definition 1: A system is said to be finite time stable if the system trajectories converge to the equilibrium point in finite time τ, and remain on it for the future time $t \geq \tau$ [17]. One of the Lyapunov results for finite time stability is that for a positive definite function $V(t)$, we have the following inequality:

$$\dot{V} + cV^{\alpha} \leq 0 \tag{27}$$

where $c > 0$ and $0 < \alpha < 1$.

A. Stability of the Master

Consider the Lyapunov candidate $V = s^2/2$. By differentiating V with respect to time and using Eqs. (12) and (11), we have

$$\dot{V} = \frac{1}{M_e} s(M_e \ddot{x}_m + B_e \dot{x}_m + K_e x_m - f_h) \tag{28}$$

and substituting $\ddot{x}_m$ from Eq. (1), and the control law from Eq. (14), we have

$$\dot{V} = s\left(\frac{1}{M_m}(-B_m \dot{x}_m + u_m + f_h) + \frac{B_e}{M_e}\dot{x}_m + \frac{K_e}{M_e}x_m - \frac{1}{M_e}f_h\right) \tag{29}$$

$$\begin{aligned} \dot{V} \;=\; & s\Bigg(\frac{1}{M_e}(\frac{\hat{M}_m}{M_m} - 1)f_h + \frac{1}{M_m}(-B_m + \hat{B}_m)\dot{x}_m \\ & + \frac{B_e}{M_e}(1 - \frac{\hat{M}_m}{M_m})\dot{x}_m + \frac{K_e}{M_e}(1 - \frac{\hat{M}_m}{M_m})x_m \\ & - K_{cm}\frac{\hat{M}_m}{M_m} sign(s)\Bigg) \end{aligned} \tag{30}$$

$$\begin{aligned} \dot{V} \;\leq\; & \Bigg(+ \frac{1}{M_e}(\sqrt{\frac{\underline{M}_m}{\overline{M}_m}} + 1)\,|f_h| \\ & + \Big(\frac{1}{\overline{M}_m}(\frac{\overline{B}_m - \underline{B}_m}{2}) + \left|\frac{B_e}{M_e}\right|(\sqrt{\frac{\underline{M}_m}{\overline{M}_m}} + 1)\Big)\,|\dot{x}_m| \\ & + \left|\frac{K_e}{M_e}\right|(\sqrt{\frac{\underline{M}_m}{\overline{M}_m}} + 1)\,|x_m|\Bigg)|s| - K_{cm}\sqrt{\frac{\underline{M}_m}{\overline{M}_m}}|s| \end{aligned} \tag{31}$$

Then, from (15), we have:

$$\dot{V} \leq -\eta_1 |s| = -\sqrt{2}\eta_1\sqrt{V} \tag{32}$$

where $\eta_1 > 0$. This inequality in accordance with (27), $\alpha = 0.5$ and $c = \sqrt{2}\eta_1$, proves the finite time convergence of the dynamics of the master system to the desired impedance.

B. Stability of the Slave

Consider the Lyapunov candidate $V = s^2/2$. By differentiating V with respect to time and using Eq. (20), we have

$$\dot{V} = s(\dot{e} + \beta\gamma|\dot{e}|^{\gamma-1}\ddot{e}) \tag{33}$$

Using Eq. (17), we have

$$\dot{V} = s\left(\dot{e} + \beta\gamma|\dot{e}|^{\gamma-1}(\ddot{x}_m(t-T) - \ddot{x}_s(t))\right) \tag{34}$$

TABLE I

System Parameters

Parameters	Mass (kg)	Damping $(N.m.s)$	Stiffness $(N.m)$
Master	0.1	0.6	-
Salve	5	1.5	-
Operator Hand	1	1	3
Environment	1	10	50

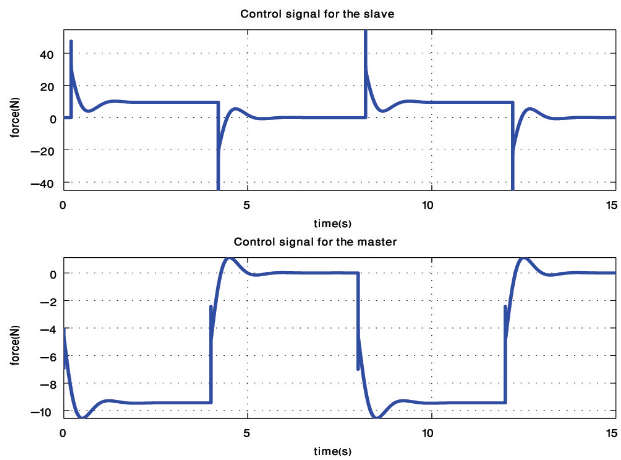

Fig. 4. Control signals for slave and master robots with one-way time delay $T = 200ms$

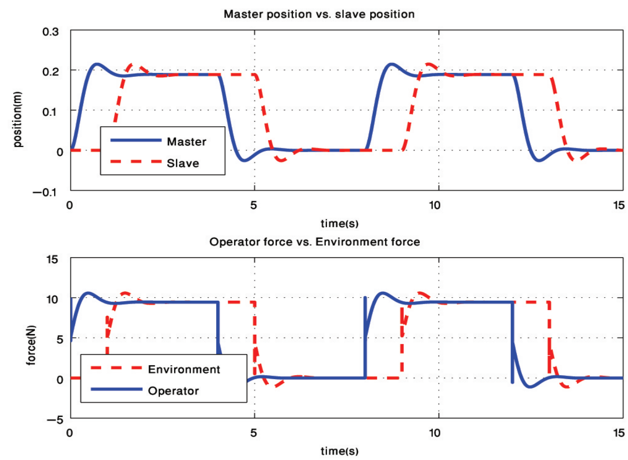

Fig. 5. Position and force tracking results for terminal sliding mode controllers with one-way time delay $T = 1s$

Then, we substitute $\ddot{x}_s(t)$ from Eq. (2) and use the control law in Eq. (22)

$$\begin{aligned}\dot{V} &= s\Big((1-\frac{\hat{M}_s}{M_s})\dot{e}+\beta\gamma|\dot{e}|^{\gamma-1}(1-\frac{\hat{M}_s}{M_s})\ddot{x}_m(t-T)\\ &\quad -\frac{1}{M_s}\beta\gamma|\dot{e}|^{\gamma-1}(\hat{B}_s-B_s)\dot{x}_s\\ &\quad -\beta\gamma|\dot{e}|^{\gamma-1}\frac{\hat{M}_s}{M_s}K_{cs}sign(s)\Big) \end{aligned} \tag{35}$$

$$\begin{aligned}\dot{V} &\le \Big((\sqrt{\frac{\overline{M}_s}{\underline{M}_s}}+1)|\dot{e}| \\ &\quad +\beta\gamma|\dot{e}|^{\gamma-1}\Big((\sqrt{\frac{\overline{M}_s}{\underline{M}_s}}+1)|\ddot{x}_m(t-T)| \\ &\quad +\frac{1}{\underline{M}_s}(\frac{\overline{B}_s-\underline{B}_s}{2})|\dot{x}_s|-\sqrt{\frac{\underline{M}_s}{\overline{M}_s}}K_{cs}\Big)\Big)|s| \end{aligned} \tag{36}$$

Then, from (23), we have

$$\dot{V} \le -\eta_2|s| = -\sqrt{2}\eta_2\sqrt{V} \tag{37}$$

where $\eta_2 > 0$. This with finite time stability of the sliding surface guarantees finite time stability of the closed-loop system.

V. SIMULATIONS

The parameters of the system are assumed to be as Table I. The master and slave parameters vary in the following range:

$$0.1 < M_m < 0.9, \;\; 0.1 < B_m < 1.1$$

$$M_s = 5, \;\; 1 < B_s < 3$$

The controller parameters are set to $\beta = 10$, $\gamma = 1.5$, $\eta_1 = \eta_2 = 2$, $k_{1m} = k_{1s} = 4$, $k_{2m} = k_{2s} = 10$ and $\rho_1 = \rho_2 = 5/6$. The time delay is $T = 200\ ms$. The operator's force on the master, f_h is defined as

$$f_h = f_h^* - (M_h\ddot{x}_m + B_h\dot{x}_m + K_h x_m) \tag{38}$$

where f_h^* is the exogenous hand force force operator and is chosen to be a pulse with period of 8 seconds and amplitude of 10 N. The notations M_h, B_h and K_h are the parameters of operator's hand impedance and are shown in Table I. Fig. 2 shows the master and slave positions and forces for TSM controller. Perfect position and force tracking is achieved. It must be noted that the shift of the slave position relative to the master position is because of the inherent delay in the communication channel and is not because of the inaccuracy of the controllers, so this shift is unavoidable and acceptable in a teleoperation system. Therefore, the proposed scheme acquires the desired performance properties of precise tracking and transparency in accordance with Eqs. (3) and (4). Control forces of the applied controllers are shown in Fig. 4, where the control laws are chattering free due to the use of the saturation function instead of signum function. The position and force tracking errors is shown in Fig. 3. The position and force tracking errors of TSM controllers is calculated by Eq. (17) and Eqs. (3) and (4).

$$e_p = x_m(t-T) - x_s(t) \tag{39}$$

$$e_f = f_h(t-T) - f_e(t) \tag{40}$$

where e_p and e_f denote the position and force errors, respectively. From this figure, it can be seen that the position and force errors converge to zero in a finite time.

Another simulation is performed with one way time delay of $T = 1\ s$. The master and the slave contact-mode positions and torque tracking profiles are shown in Fig. 5. The contact-mode behavior of the teleoperation system is stable and the environment can have the sense of direct interaction with in the environment by receiving the operator force after a constant time delay.

VI. CONCLUSIONS

A robust, fast and finite time convergent control is proposed for a teleoperation system under time delay that can achieve perfect tracking and transparency. The control scheme is a wise combination of terminal sliding mode control and impedance control that can provide finite time stability and not only asymptotic stability and perfect transparency in the presence of time delay and parametric uncertainties. Finite time convergence of the errors to zero point is proved through stability analysis introducing Lyapunov candidates. Perfect position tracking of the master by the slave robot with impedance control in the master side causes direct application of the operator force on the environment and hence sense of telepresence of the operator in the environment. Simulation results show the robustness of the terminal sliding-mode impedance controller to unknown constant time delays and parametric uncertainties.

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